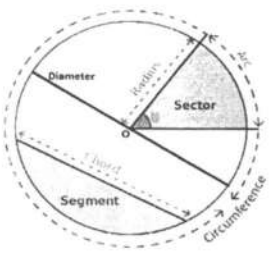
CHAPTER-9

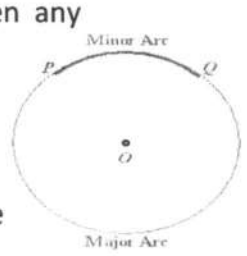
CIRCLES

MIND-MAP

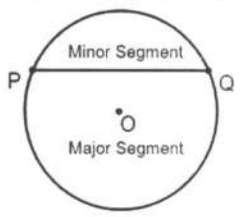
Terms related to Circle



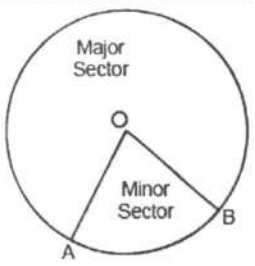
Sections formed between any two points on circum ference



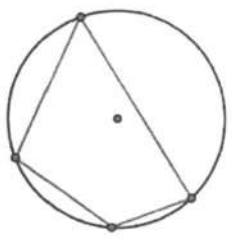
Sections formed between chord & arc



Sections formed between radii and arc



Cyclic Quadrilateral : A quadrilateral having its all four vertices on the circumference of a circle



Some theorems on circle

- Equal chords of a circle subtends equal angles at the centre of a circle.
- The perpendicular from the centre of a circle bisects the chord.
- Equal chords of a circle are equidistant from the centre of a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180^0.
- The angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.

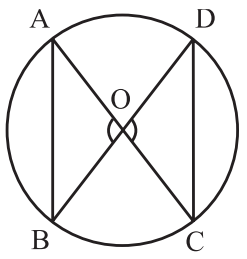
Key points

The collection of those points in a plane which are at a fixed distance from a given fixed point is called a circle. The fixed point is called centre of the circle and the fixed distance is called radius.

Theorem : Equal chords of a circle subtends equal angles at centre.

If $AB = CD$ then

$\angle AOB = \angle COD$

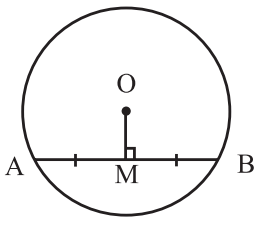


Converse : If angles subtended by chord at centre are equal, then chords are equal.

Theorem : The perpendicular from centre to a chord of a circle, bisects the chord.

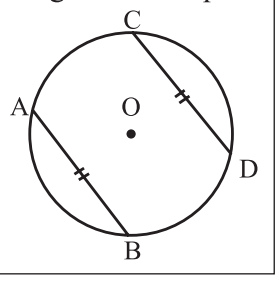
If $OM \perp AB$ then

$AM = BM$



Converse : The line joining the mid-point of the chord to the centre of a circle is prependicular to the chord.

Property : If two chords of a circle are equal then corresponding arcs are equal.

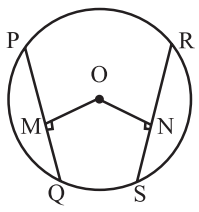


Converse : If arcs of a circle are equal then corresponding chords are also equal.

Theorem : Equal chords of a circle are equidistant from centre.

If PQ = RS

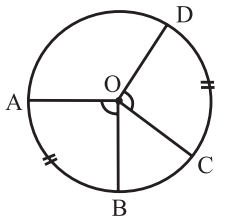
then OM = ON



Converse : Chords equidistant from centre are equal in length.

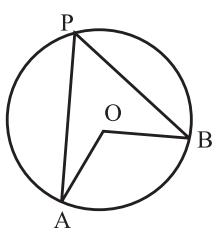
Property : Congruent arcs or equal arcs of a circle subtend equal angle at the centre.

$\Rightarrow \angle AOB = \angle COD$



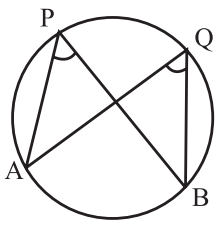
Theorem : The angle subtended by an arc at the centre of circle is twice the angle which is subtended at remaining part of the circle.

$\Rightarrow \angle AOB = 2\angle APB$



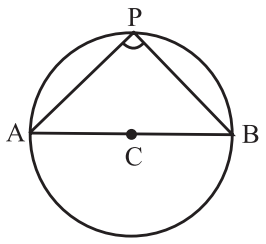
- Any two angles in the same segment of the circle are equal.

$\angle APB = \angle AQB$



- Angle in a semicircle is a right angle.

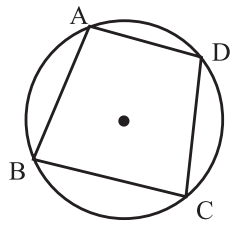
$\Rightarrow \angle APB = 90^{o}$



Theorem : In a cyclic quadrilateral, the sum of opposite angles is 180°.

$\angle A + \angle C = 180°$

$\angle B + \angle D = 180°$



Converse : If sum of the opposite angles of a quadrilateral is 180° then the quadrilateral is cyclic.

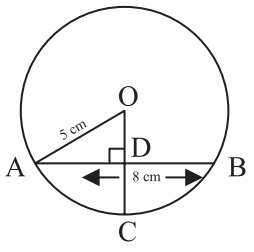
Very Short Answer Questions (1 Mark)

1. The angles in the same segment of a circle are:

(a) Equal (b) Complementary

(c) Supplementary (d) Vertically Opposite Angles

2. In fig, $OA = 5$ cm, $AB = 8$ cm and OD is perpendicular to AB. CD is equal to:



(a) 2 cm (b) 3 cm

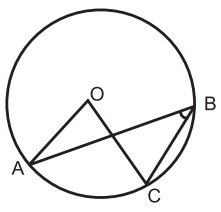
(c) 4 cm (d) 5 cm

3. The radius of a circle is 13 cm and the length of one of its chords is 10cm. The distance of the chord from the centre is:

(a) 11.5 cm (b) 12 cm

(c) $\sqrt{69}$ cm (d) 23 cm

4. In fig. if $\angle ABC = 20°$, then $\angle AOC$ is equal to:



(a) 20° (b) 40°

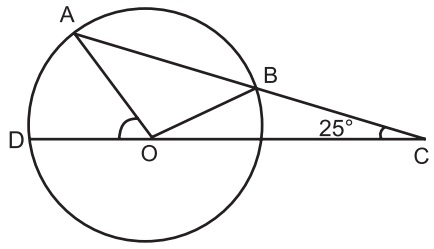
(c) 60° (d) 10°

5. AB and BC are chords of a circle such that $AB = 12$ cm, $BC = 16$ cm and AB is perpendicular to BC. The radius of the circle passing through the point A,B and C is:

(a) 6 cm (b) 8 cm

(c) 10 cm (d) 12 cm

6. In the given figure, AB is chord of a circle with centre O and AB is produced to C such that $BC = OB$. Also, CO is joined and produced to meet the circle in D. If $\angle ACD = 25°$, then $\angle AOD = ?$



(a) 50° (b) 75°

(c) 90° (d) 100°

7. For the points A, B, C and D to be concyclic, $\angle BDC$ and $\angle BAC$ should be equal to:

(a) 180° (b) 90°

(c) 45° (d) 100°

8. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm the distance of AB from the centre of the circle is:

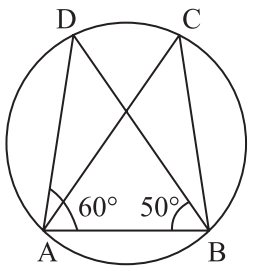
(a) 17 cm (b) 15 cm

(c) 4 cm (d) 8 cm

9. In the given figure, $\angle DAB = 60°$ and $\angle ABD = 50°$ then $\angle ACB$ = ?

(a) 50° (b) 60°

(c) 70° (d) 80°



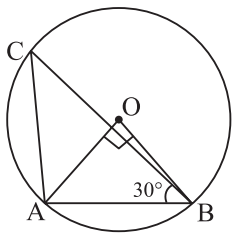
10. In figure $\angle AOB = 90°$ and $\angle CBA = 30°$, then $\angle CAO$ is equal to:

(a) 30°

(b) 45°

(c) 90°

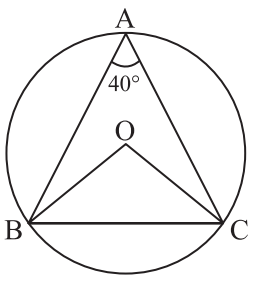
(d) 60°



11. In the given figure O is the centre of a circle and $\angle BAC = 40°$, then $\angle OBC$ = ?

(a) 40° (b) 50°

(c) 80° (d) 20°



12. An equilateral triangle of side 9 cm is inscribed in a circle. The radius of the circle is:

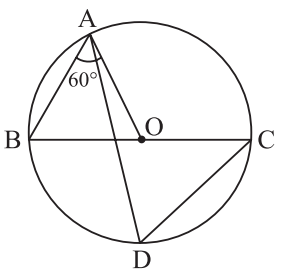
(a) 3 cm (b) $3\sqrt{2}$ cm

(c) $3\sqrt{3}$ cm (d) 6 cm

13. In fig. BC is a diameter of the circle and $\angle BAO = 60°$, then $\angle ADC$ is equal to:

(a) 30° (b) 60°

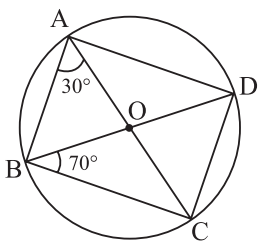
(c) 120° (d) 45°



14. In the given figure, the measure of $\angle BCD$ is :

(a) 80° (b) 30°

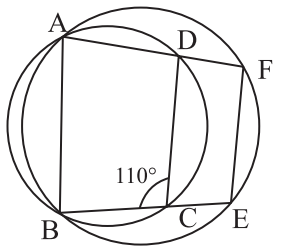
(c) 70° (d) 100°



15. In the given figure $ABCD$ and $ABEF$ are cyclic quadrilaterals. If $\angle BCD = 110°$ then $\angle BEF = ?$

(a) 110° (b) 55°

(c) 90° (d) 70°



16. $ABCD$ is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and $\angle ADC = 140°$, then $\angle BAC$ is equal to:

(a) 80° (b) 30°

(c) 50° (d) 40°

17. The length of the chord which is at a distance of 12 cm from the centre of a circle of radius 13 cm is:

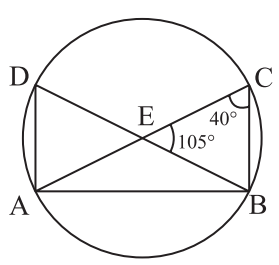
(a) 5 cm (b) 10 cm

(c) 12 cm (d) 13 cm

18. In the given figures, $\angle ECB = 40°$ and $\angle CEB = 105°$, then $\angle EAD = ?$

(a) 35° (b) 20°

(c) 50° (d) 40°



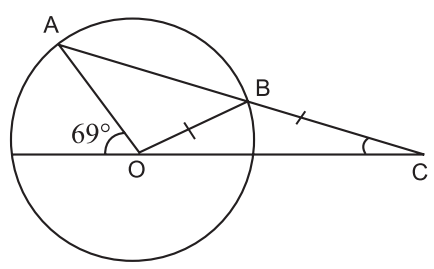
19. In the following figure, BC = radius OB. Find the value of $\angle OCB$.

(a) 69°

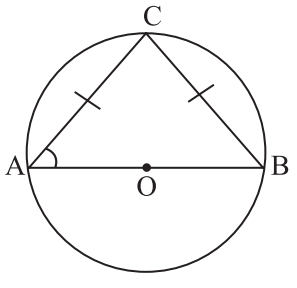
(b) 46°

(c) 92°

(d) 23°



20. In the given figure, AOB is the diameter of the circle and $AC = BC$. Find $\angle CAB$.



(a) 60° (b) 46°

(c) 45° (d) 70°

21. A segment of a circle is the region between an arc and a _____ of the circle.

22. An arc of a circle is called a _____ if the ends of the arc lie on the ends of a diameter.

23. The degree measure of a semi circle is _____.

24. A circle divides the plane into _____ parts.

25. The diameter is the _____ chord of the circle.

26. Circles having the same centre and different radii are called _____ circle.

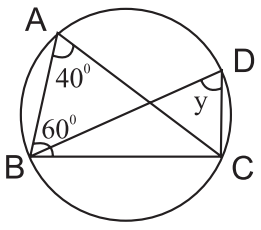
27. Angle in a semicircle is a _____ angle.

28. If two chords of a circle are equal then the corresponding arcs are _____.

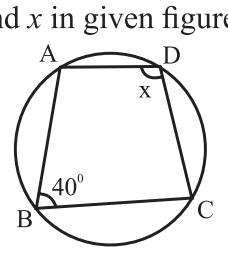
29. If the sum of a pair of opposite angles of a quadrilateral is 180°, then quadrilateral is _____.

30. A round pizza is cut into 4 equal pieces. Each piece represent a _____.

31. Find y in given figure.



32. Find x in given figure.

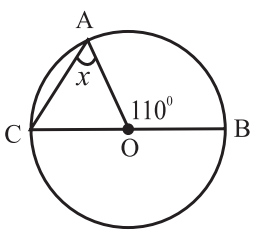


33. AD is a diameter of a circle andAB is a chord. IfAD = 34 cm, AB = 30 cm then find BD.

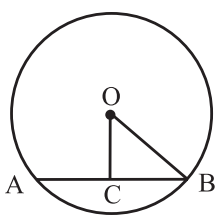
34. Given two concentric circles with centre O. A line cut the circle at A, B, C and D respectively. If AB = 10 cm, then find the length of CD.

Short Answer Type-I Questions (2 Marks)

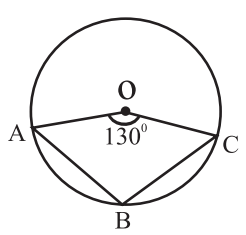
35. Find x in given figure.



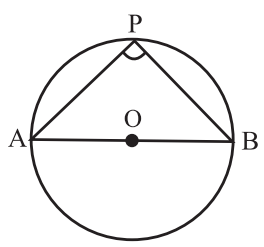
36. In given figure, OC is perpendicular segment drawn from centre$O$ on chord AB. If OB = 5 cm and OC = 3 cm then find length of AB.



37. In given figure, O is centre of circle. If $\angle AOC = 130°$ then find $\angle$ABC.



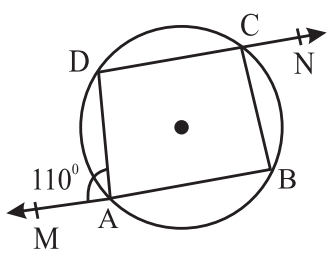
38. In given figure, AOB is diameter of circle & P is any point on the circle. Find $\angle APB$.



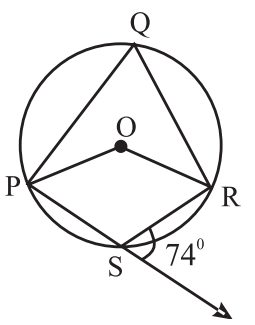
39. Prove that the cyclic parallelogram is a rectangle.

40. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

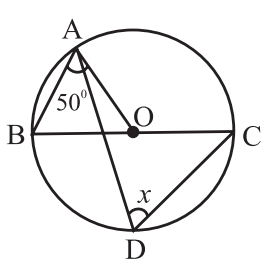
41. In the following figure, find the value of $\angle BCN$.



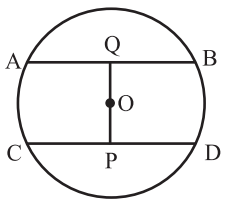
42. In the given figure, find the value of reflex angle POR.



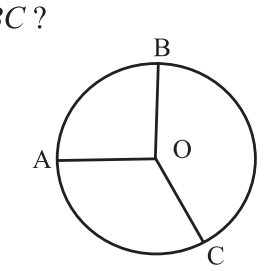
43. Find the value of x in figure if O is centre of circle and $\angle OAB = 50°$



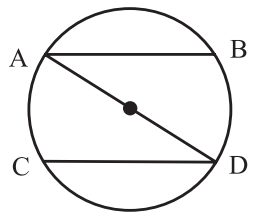
44. In the given figure, O is centre of the circle with radius 5 cm, $OP \perp CD$, $OQ \perp AB$, $AB \parallel CD$, $AB = 6$ cm and $CD = 8$ cm. Determine PQ.



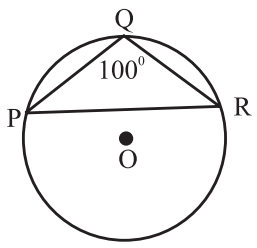
45. In the given figure, O is the centre of a circle and $\angle AOB = 90°$, $\angle BOC = 120°$. What is measure of $\angle ABC$?



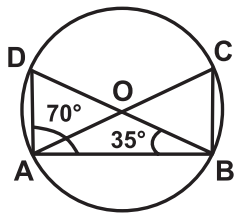
46. In the given figure, AB and CD are parallel chords and length of arc AC = 14 cm. What is length of arc BD?



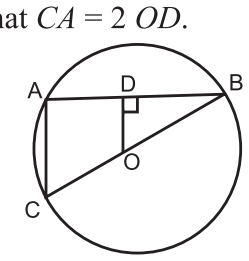
47. In given figure, $\angle PQR = 100°$ where P, Q & R are points on the circle with centre O. Find $\angle OPR$.



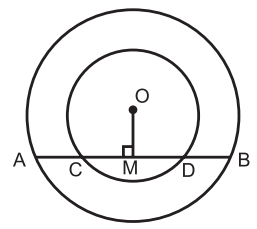
48. In the given figure, O is centre of circle. If $\angle ABD = 35°$ and $\angle BAD = 70°$, find $\angle ACB$.



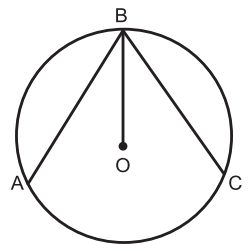
49. In fig. OD is perpendicular to the chord AB of a circle whose centre is O and BC is a diameter. Show that $CA = 2\ OD$.



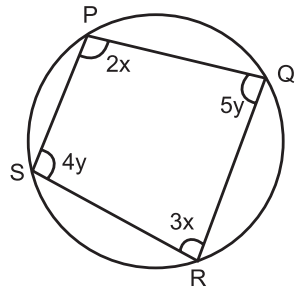
50. Two concentric circles with centre O where AB is chord of outer circle which intersects the inner circle at C and D are shown in figure. If $AB = 12$ cm and $CD = 8$ cm, find AD.



51. In figure, $AB = BC$ and O is the centre of the circle. Prove that BO bisects $\angle ABC$.

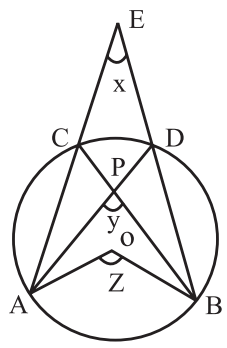


52. In figures, $PQRS$ is a cyclic quadrilateral. Find the value of x and y.



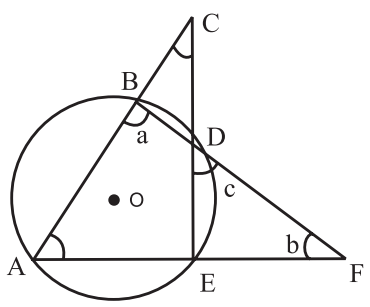
Short Answer type-II Questions (3 marks)

53. In the given figure, O is the centre of a circle. Prove that $\angle x + \angle y = \angle z$.

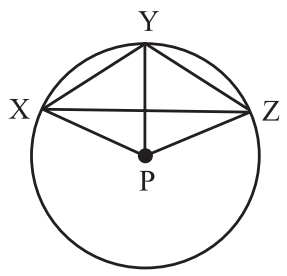


54. If two non parallel sides of a trapezium are equal then prove that it is cyclic quadrilateral.

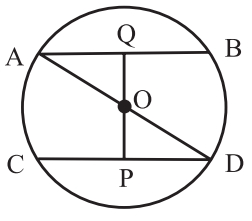
55. In the given figure, determine *a, b* and c if $\angle BCD = 43°$ and $\angle BAF = 62°$.



56. In the figure, P is the centre of a circle. Prove that $\angle XPZ = 2\,(\angle XZY + \angle YXZ)$.

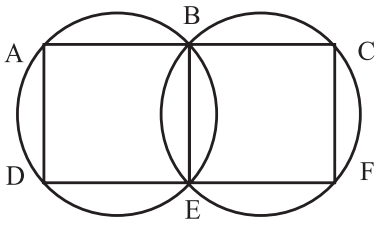


57. In the given figure, AD is diameter of the circle whose centre is O and $AB \parallel CD$. Prove that $AB = CD$.

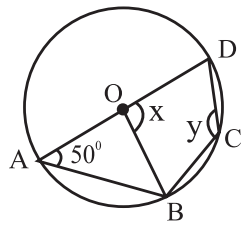


58. In an equilateral triangle, prove that the centroid and the circumcentre coincide.

59. In the given figures, *A, B, C* and *D, E, F* are two sets of collinear points. Prove that *AD* || *CF*.



60. In given figure, *O* is centre of circle and *DAB* = 50°. Calculate the value of *x* and *y*.



61. If two equal chords of a circle intersect within the circle, then prove that the segment of one chord is equal to corresponding segment of other chord.

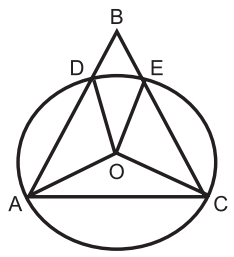
62. Prove that if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

63. The bisector of angle *A, B* and *C* of a Δ*ABC* intersect its circum circle at *D, E* and *F* respectively. Prove that the angles of a triangle*DEF* are $90° - \frac{1}{2} \angle A$, $90° - \frac{1}{2} \angle B$, $90° - \frac{1}{2} \angle C$ respectively.

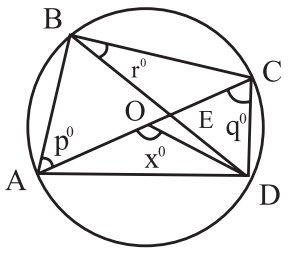
64. Find the sum of the angles in the four segments exterior to a cyclic quadrilateral.

65. Let the vertex$B$ of a triangleABC be located outside a circle and let the sides AC and CB of the triangle intercepts equal chordsAD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angle subtended by the chords AC and DE at the centre.

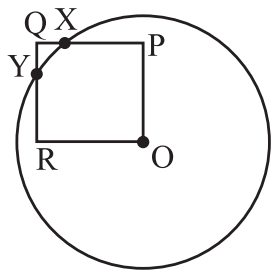
$$\angle ABC = \frac{1}{2}(\angle DOE - \angle AOC)$$



66. In the adjoining figure AC is diameter of a circle with centre O and chord $BD \perp AC$, intersecting each other at E. Find out the values of p, q, r in terms of x if $\angle AOD = x°$, $\angle BAC = p°$, $\angle ACD = q°$, $\angle CBD = r°$.

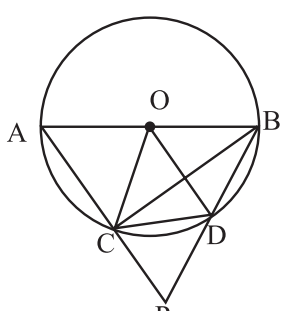


67. In the given figure $OPQR$ is a square.A circle drawn with centre O cuts the square in X and Y. Prove that $QX = QY$.

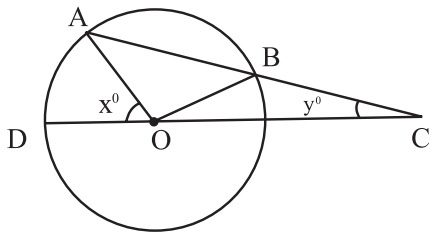


68. Prove that the opposite angles of a cyclic quadrilateral are supplementary.

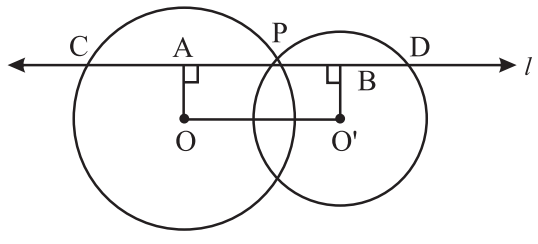
69. In the given figure, AB is a diameter of a circle (O, r) and chord CD = radius OC. AC and BD when produced meet at P. Prove that $\angle APB$ is 60°.



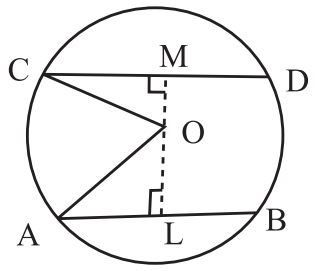
70. In the given figure, AB is chord of a circle with centre O and AB is produced to C such that $BC = OB$. Also, CO is joined and produced to meet the circle in D. If $\angle ACD = y°$ and $\angle AOD = x°$ then prove that $x = 3y$.



71. Two circles whose centres are O and O' intersect at P. Through P, a line l parallel to OO', intersecting the circle at C and D is drawn. Prove that $CD = 2OO'$.



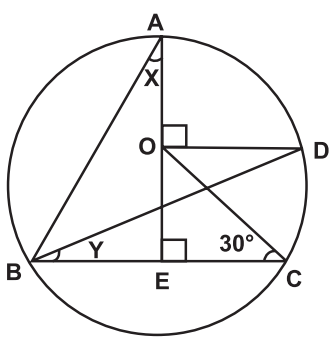
72. AB and CD are two parallel chords of a circle which are on opposite sides of the centre O such that AB = 10 cm, CD = 24 cm and the distance between AB and CD is 17 cm. Find the radius of the circle.



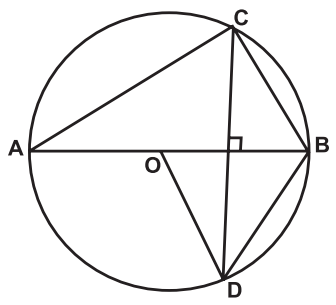
Long Answer type Questions (5 marks)

73. AB and AC are two chords of a circle of radius r such that $AB = 2AC$. If p and q are the distance of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$

74. In figure, O is the centre of the circle, $\angle BCO = 30°$, $AE \perp BC$ and $DO \perp AE$. Find x and y.



75. In figure, O is the centre of the circle, $BD = OD$ and $CD \perp AB$. Find $\angle CAB$.



76. Prove that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

77. Show that if two chords of a circle bisect one another they must be diameters.

78. Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

Chatper - 9

CIRCLE

Answers

1. (a) equal

2. (a) 2 cm

3. (b) 12 cm

4. (b) 40°

5. (c) 10 cm

6. (b) 75°

7. (b) 90°

8. (d) 8 cm

9. (c) 70°

10. (d) 60°

11. (b) 50°

12. (c) $3\sqrt{3}$ cm

13. (b) 60°

14. (a) 80°

15. (a) 110°

16. (c) 50°

17. (b) 10 cm

18. (a) 35°

19. (d) 23°

20. (c) 45°

21. chord

22. semi-circle

23. 90°

24. three

25. longest

26. concentric

27. right

28. equal

29. cyclic

30. sector

31. $y = 40°$

32. 140°

33. 16 cm

34. CD = 10 cm

35. 35°

36. $AB = 8$ cm

37. $\angle ABC = 115°$

38. $\angle APB = 90°$

40. Angle by minor arc = 30°

Angle by major arc = 150°

41. $\angle BCN = 70°$

42. $\angle PSR + \angle RST = 180° \Rightarrow \angle PSR = 106°$

reflex $\angle POR = 2\angle PSR = 212°$

43. $\angle AOB = 80°$

$\angle AOB + \angle AOC = 180°$

$\angle AOC = 100°$

$\angle ADC = \frac{1}{2} \angle AOC$

$\Rightarrow x = 50°$

44. $AQ = \frac{1}{2} AB \Rightarrow AQ = 3$ cm

In ΔAOQ

$(OA)^2 = (AQ)^2 + (OQ)^2$

$\Rightarrow OQ = 4$ cm

Similarly $OP = 3$ cm

$PQ = 7$ cm

45. $\angle AOB + \angle BOC + \angle AOC = 360°$

$\Rightarrow \angle AOC = 150°$

$\angle ABC = \frac{1}{2} \angle AOC = 75°$

46. $BD = 14$ cm

47. $\angle OPR = 10°$

48. $\angle ABD + 70° + 35° = 180°$

$\Rightarrow \angle ADB = 75°$

$\angle ACB = \angle ADB = 75°$

49. $OD \parallel AC$

$\Rightarrow OD = \frac{1}{2}$CA

$\Rightarrow CA = 2\ OD$

50. $AM = 6$ cm

$MD = 4$ cm

$AD = (6 + 4)$ cm $= 10$ cm

51. Δ's $AOB \cong \Delta COB$ (by SSS)

$\Rightarrow \angle OBA = \angle OBC$

$\Rightarrow \angle OBC$ bisects $\angle ABC$

52. $2x + 3x = 180° \Rightarrow x = 36°$

$y + 45y = 180°$

$y = 20°$

53. $\angle ACB = \angle ADB$ (Angles in same segment)

$\angle z = 2\angle ACB$

$\Rightarrow \angle z = \angle ACB + \angle ADB$

$\angle y = \angle ACB + \angle CAD$

$\Rightarrow \angle z = \angle y - \angle DAC + \angle ADB$

But, $\angle ADB - \angle DAC = \angle x$

$\Rightarrow \angle x + \angle y = \angle z$

55. $a = 105°$, $b = 13°$, $c = 62°$

56. $\angle XPY = 2\angle XZY$

$\angle YPZ = 2\angle YXZ$

$\angle XPZ = 2(\angle XZY + \angle YXZ)$

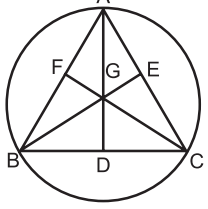
57. $\Delta AOQ \cong \Delta POD$

$\Rightarrow OQ = OP$ (by *CPCT*)

$\Rightarrow AB = CD$

58.

A



$\Delta BEC \cong \Delta BFC$

$\Rightarrow BE = CF$

sim. $\Delta CAF \cong \Delta CAD$

$\Rightarrow\ CF = AD$

So $AD = BE = CF$

$\frac{2}{3}AD = \frac{2}{3}BE = \frac{2}{3}CF$

$GA = GB = GC$

Hence centroid and circumcentre are coincident

59. $\angle DAB + \angle BED = 180°$

But $\angle BED = \angle BCF$

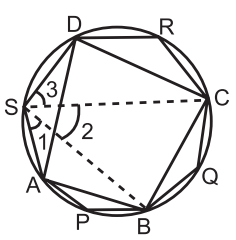
$\Rightarrow \angle DAB + \angle BCF = 180°$

Hence $AD \parallel CF$

60. $\angle AOB = 80°$

$\Rightarrow x = 100°, y = 130°$

64.



$\angle 1 + \angle P = 180°$

$\angle 2 + \angle Q = 180°$

$\angle 3 + \angle R = 180°$

$\angle 1 + \angle P + \angle 2 + \angle Q + \angle 3 + \angle R = 3 \times 180°$

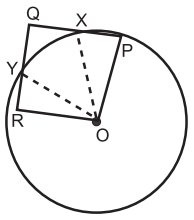
$\Rightarrow \angle P + \angle Q + \angle R + \angle S = 6 \times 90°$

66. $p = 90° - \frac{1}{2}x$, $q = \frac{1}{2}x$

$r = \frac{1}{2}(180° - x)$

$= 90° - \frac{1}{2}x$

67.



$QR = QP$ ($\because$ OPQR is square)

$\Delta ORY \cong \Delta OPX$

$\therefore\ RY = PX$

$\Rightarrow QR - RY = QP - PX$

$\Rightarrow QY = QX$

70. $BC = OB$

$\Rightarrow \angle BOC = y$

$\angle ABO = 2y, \angle OAB = 2y$

$(2y) + (2y) + (180 - x - y) = 180°$

$\Rightarrow x = 3y$

71. $CA = AP$

$\Rightarrow\ CP = 2AP$

Similarly $BP = BD$

$\Rightarrow PD = 2\ PB$

$CD = 2\ AP + 2\ PB$

$CD = 2OO'$

72. In ΔALO

$OA^2 = OL^2 + AL^2$

$r^2 = x^2 + 5^2$ ---(1)

In ΔOMC

$OC^2 = OM^2 + CM^2$

$r^2 = (17 - x)^2 + (12)^2$ ---(2)

from (1) & (2)

$34x = 408$

$x = 12$ cm

$\therefore\ r = 13$ cm

In ΔABE,

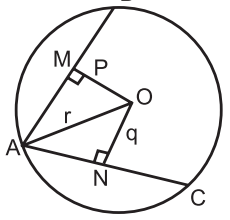
$60° + x + 90° = 180°$

$x = 30°$

$x = 30°$, $y = 15°$

B

73.



In ΔAMO

$OA^2 = OM^2 + AM^2$

$\Rightarrow \left(\frac{AB}{2}\right)^2 = r^2 - p^2 \ (OM \perp AB)$

$(AB)^2 = 4r^2 - 4P^2$

Similarly $AC^2 = 4r^2 - 4q^2$

$AB^2 = 4AC^2$ ($\because$ AB = 2AC)

$\Rightarrow\ 4r^2 - 4p^2 = 4(4r^2 - 4q^2)$

$\Rightarrow\ 4q^2 = p^2 + 3r^2$

74. $\angle EOC = 180° - 30° - 90° = 60°$

$\angle COD = 180° - 60° - 90° = 30°$

$\angle COB = 2\angle CBD = 1$

$2y = 30° \Rightarrow y = 15°$

Similarly

$\angle ABC = \frac{1}{2}\angle AOC$

$\Rightarrow\ \angle ABC = \frac{1}{2}(90° + 30°)$

$\Rightarrow\ \angle ABC = 60°$

75. Since $OB = OD = BD$

$\Rightarrow\ \Delta OBD$ is an equilateral triangle

$\therefore\ \angle BOD = 60° \Rightarrow \angle AOD = 120°$

Now $\angle ACD = 1/2\angle AOD = 60°$

$\Rightarrow \angle CBA = 60°$

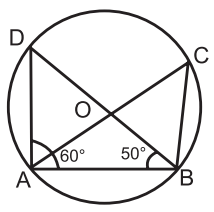
Hence $CAB = 30°$

(Angle sum property)

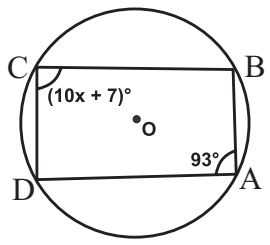
CHAPTER-9
Practice Test
Circles

Time : 1 hr. **M.M. 20**

1. In fig, $\angle DAB = 60°$ and $\angle ABD = 50°$. Find $\angle ACB$. (1)

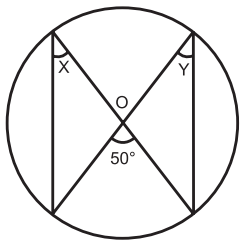


2. A circle passes through *A, B, C* and *D* as shown in figure. If $\angle BAD = 93°$ find *x*. (1)



3. The chord of a circle is equal to its radius. Find the angle subtended by this chord at the minor arc of the circle. (2)

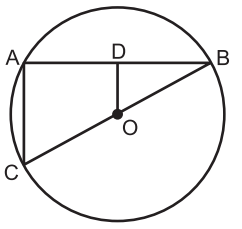
4. In the given figure, find x where O is the centre of the circle. (2)



5. Prove that equal chords of a circle subtend equal angles at the centre. (3)

6. Prove that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°. (3)

7. In the given figure, OD is perpendicular to chord AB of a circle with centre O. If BC is a diameter then show that AC = 2OD (3)



8. In figure, *AB* is a chord of a circle with centre *O* and *AB* is produced to *C*. Also, *CO* is joined and produced to meet the circle in *D*. If $\angle ACD = y°, \angle AOD = x°$ and $x = 3y$, then prove that $BC = OB$. (5)