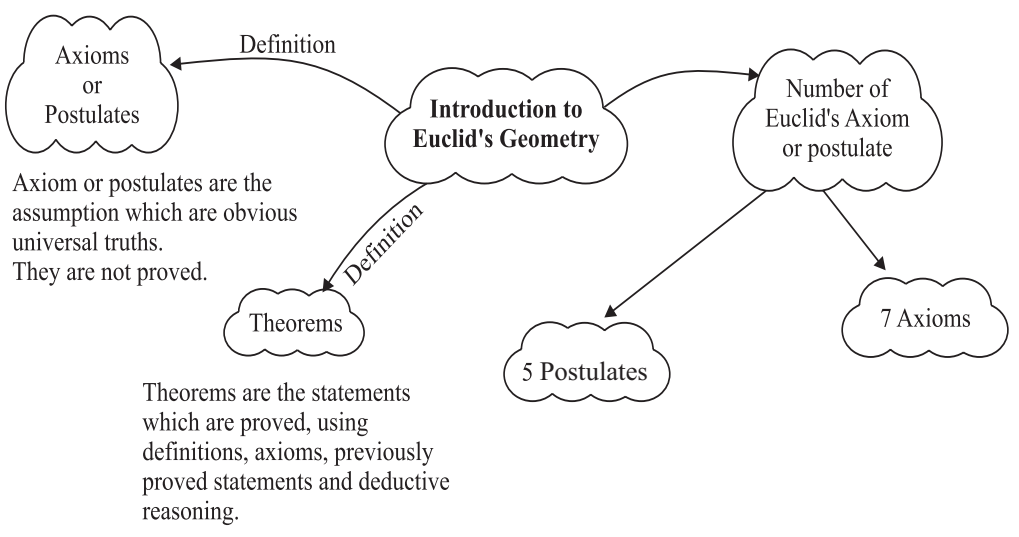
Chapter-5
INTRODUCTION TO EUCLID'S GEOMETRY



Key points

- **Introduction:** Euclidean geometry, which is taught today is named after Euclid – he is known as "the father of geometry". Euclid also studied and contributed in other areas of mathematics, including number theory and astronomy.
- **Axiom or Postulates:** Axiom or Postulates are the assumptions which are obvious universal truths. They are not proved.
- **Theorems:** Theorems are statements which are proved, using definitions, axioms, previously proved statements and deductive reasoning.

Some of Euclids Axioms

1. Things which are equal to the same thing are equal to one another.
2. If equals are added to equals, the wholes are equal.
3. If equals are subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.
6. Things which are double of the same things are equal to one another
7. Things which are halves of the same things are equal to one another.

Euclid's Postulates and Definitions

- **Postulates 1:** A straight line may be drawn from any one point to any other points.
- **Postulate 2:** A terminated line can be produced indefinitely.
- **Postulate 3:** A circle can be drawn with any centre and any radius.
- **Postulate 4:** All right angles are equal to one-another.
- **Postulate 5:** If a straight line falling on two straight line makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produces indefinitely, meet on that side on which the sum of angles is less than two right angles.

Definitions

1. A Point is that which has no part.
2. A line is breadthless length.
3. The ends of a line are points
4. A straight line is a line which lies evenly with the points on it self.
5. A surface is that which has length and breadth only.
6. The edges of a surface are lines.
7. A plane surface is a surface which lies evenly with the straight lines on it self.

Very Short Answer type Questions (1 Marks)

1. **Through two points:**
   (a) A unique line can be drawn (b) No line can be drawn
   (c) Two lines can be drawn (d) More than two lines can be drawn
2. **Euclid arranged all known work in the field of mathematics in his treatise called:**
   (a) Elements (b) Axioms
   (c) Theorems (d) Postulates
3. **Things which are double of the same things are:**
   (a) Halves of the same thing (b) Double of the same thing
   (c) Equals (d) Four times of the same thing

4. A mathematical statement whose truth has been logically established is called:

(a) An Axiom (b) A postulate

(c) A Theorem (d) None of the above

5. Two lines having a common point are called:

(a) parallel lines (b) intersecting lines

(c) coincident (d) None of the above

6. A proof is required for _____ (Postulate, Axioms, Theorem)

7. The number of line segments determined by three collinear points is _____ (Two, three, only one)

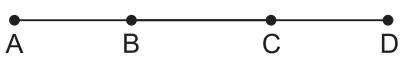
8. Euclid stated that if Equals are subtracted from equal then the remainders are equal is the form of _____ (an axiom, a definition, a postulate)

9. A point has _____ dimensions.

10. There are _____ number of Euclid's postulates.

11. Write the number of dimensions, that a surface contain.

12. In given figure $AB = CD$ then AC and BD are equal or not?



13. How many lines can pass through a single point?

14. Write Euclid's fifth postulate.

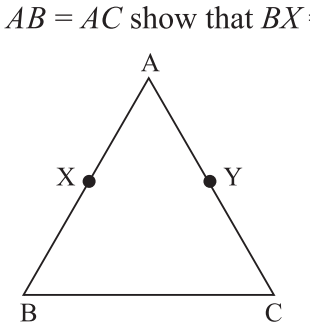
15. If $a + b = 15$ and $a + b + c = 15 + c$

which axiom of Euclid does the statement illustrate?

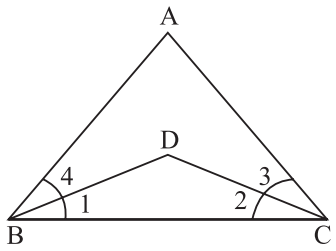
Short Answer type-I Questions (2 Marks)

16. If $x + y = 10$ and $x = z$ then show that $z + y = 10$

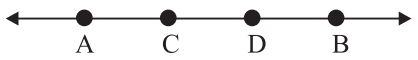
17. In given figure $AX = AY$, $AB = AC$ show that $BX = CY$



18. In the given figure $\angle ABC = \angle ACB$, $\angle 3 = \angle 4$ show that $\angle 1 = \angle 2$

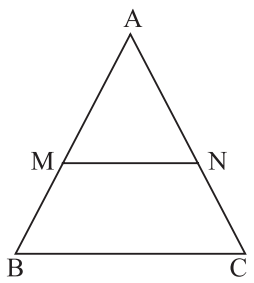


19. In the given figure if $AD = CB$ then prove that $AC = BD$

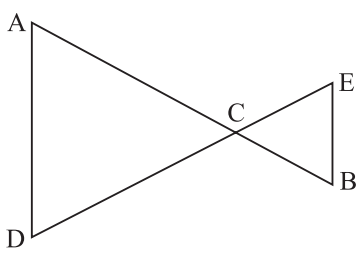


20. Solve the equation $x - 10 = 15$, state which axiom do you use here.

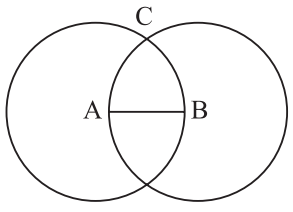
21. In the given figure if $AM = \frac{1}{2} AB$, $AN = \frac{1}{2} AC$ and $AM = AN$ then show that $AB = AC$



22. In the given figure $AC = DC$, $CB = CE$ then show that $AB = DE$



23. In figure, A and B are centres of the two intersecting circles, which intersect at C. Prove that $AB = AC = BC$

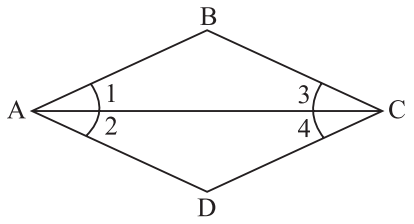


24. Prove that every line segment has one and only one mid point.

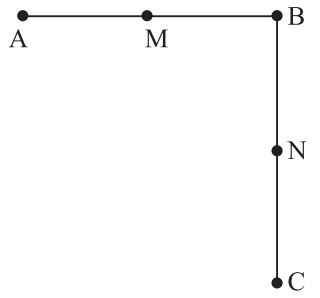
25. Kartik and Himank have the same weight. If they each gain weight by 3 kg how will their new weight be compared? State Euclid's axiom used.

Short Answer type-II Questions (3 Marks)

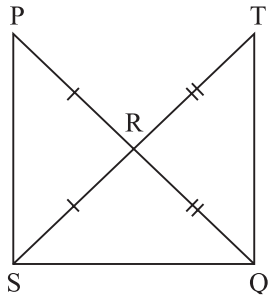
26. In the given figure $\angle 1 = \angle 2$ and $\angle 2 = \angle 3$ then show that $\angle 1 = \angle 3$



27. In the given figure $AB = BC$, M is the mid point of AB and N is the mid-point of BC. Show that $AM = NC$



28. In the given figure $PR = RS$ and $RQ = RT$. Show that $PQ = ST$ and write the Euclid's axiom to supports this.



29. An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all the angles are 60° each.

Can you justify that all the sides and all the angles are equal in equilateral triangle?

30. Ram and Shyam are two students of class IX. They have given equal donation to a blind school in the month of March. In April each student double their donation.

(a) compare their donation in April.

(b) which mathematical concept have been covered in this question?

CHAPTER-5

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Answers

1. (a) A unique line can be drawn

2. (a) Elements

3. (c) Equals

4. (a) An axiom

5. (b) Intersecting lines

6. Theorem

7. only one

8. An axiom

9. Zero

10. Five

11. Two

12. Equal

13. Infinite

14. If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

15. Second axiom

16. Given $x + y = 10$ --- (1)

and $x = z$ --- (2)

on subtracting y from both sides, of eq^{n} (1)

$x + y - y = 10 - y$ [by axiom 3]

$z = 10 - y$ [from eq 2]

on adding y on both sides, we get

$z + y = 10 - y + y$ [by axiom 2]

$z + y = 10$

17. $AB = AC$ --- (1)

$AX = AY, AY$ --- (2)

According to Euclid's axiom (3), if equals are subtracted from equals then remainders are also equal

Subtracting equation (2) from equation (1)

$$AB - AX = AC - AY$$

$$BX = CY \qquad \text{(Hence proved)}$$

18. $\angle ABC = \angle ACB$ --- (1)

$\angle 4 = \angle 3$ --- (2)

eq^n (1) – eq^n (2)

$$\angle ABC - \angle 4 = \angle ACB - \angle 3 \qquad \text{[using axiom 3]}$$

$$\angle 1 = \angle 2$$

19. $AD = CB$

$$AC + CD = CD + DB$$

on subtracting CD from both sides

$$AC + CD - CD = CD + DB - CD \qquad \text{(using axiom 3)}$$

$$AC = DB$$

20. $x - 10 = 15$

Adding 10 both sides

$$x - 10 + 10 = 15 + 10 \qquad \text{[by axiom 2]}$$

$$x = 25$$

21. Given; $AM = \frac{1}{2} AB$ --- (i)

$$AN = \frac{1}{2} AC \qquad \text{--- (ii)}$$

$$AM = AN \qquad \text{--- (iii)}$$

from eq^{n}s (i), (ii) & (iii), we get

$$\frac{1}{2} AB = \frac{1}{2} AC$$

$$AB = AC \qquad \text{[by axiom 7]}$$

22. $AC = DC$ --- (1)

$CB = CE$ --- (2)

By Euclid's axiom 2

If two equals are added to equals, then the wholes are equal.

Adding eq^{n} (1) and eq^{n} (2)

$$AC + CB = DC + CE$$

$$AB = DE$$

23. $AB = AC$ --- (1) [Radius of the same circle]

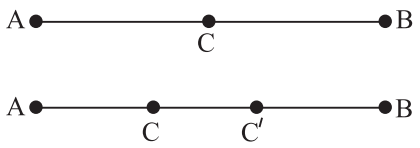
$BC = AB$ --- (2) [Radius of the same circle]

from eq^{n} (1) and eqn (2)

$$AB = AC = BC \text{[by axiom 1]}$$

24. We have C as the mid point of the line segment AB, so $AC = BC$

Let there are two mid-point C & C' of AB



Then, $AC = \frac{1}{2} AB$ $AC' = \frac{1}{2} AB$

$\Rightarrow AC = AC'$ [by axiom 1]

which is possible only when C coincides C', so point C lies on C'.

25. Kartik's weight = Himank's weight

Kartik's weight + 3 kg = Himank's weight + 3kg [by axiom 2]

Their new weight will be equals By Euclid's second axiom. If equals are added to equals then wholes are equal.

26. $\angle 1 = \angle 2$ --- (1)

$\angle 2 = \angle 3$ --- (2)

from equation (1) and (2)

$\angle 1 = \angle 3$ [By axiom 1]

27. $AB = BC$

$$AM + BM = BN + CN$$

$AM = 2CN$ 2

[M & N are mid-point of AB & BC respectively]

$AM = CN$ [By Euclid's axiom 6]

28. $PR = RS$ --- (1)

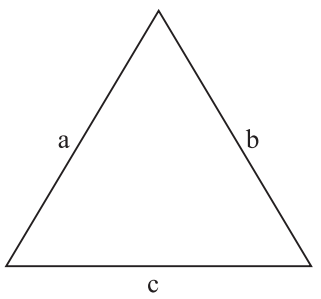
$RQ = RT$ --- (2)

Adding equation (1) and (2)

$$PR + RQ = RS + RT$$

$PQ = ST$ [By axiom 2]

29.



$a = b$ and $b = c$

$\Rightarrow$ $a = b = c$ [By axiom 1]

All sides of triangle are equal since all the angles are of 60° in an equilateral triangle so they must be equal to one another.

30. Ram's donation in March = Shyam's donation in March --- (1)

Ram's donation in April = 2 × Ram's donation in March --- (2)

Shyam's donation in April = 2 × Shyam's donation in March --- (3)

Using equation (1), (2) & (3)

$\Rightarrow$ Ram's donation in April = Shyam's donation in April [using axiom 6]