CHAPTER - 14

PROBABILITY

KEY POINTS

- Random Experiment: If an experiment has more than one possible outcome and it is not possible to predict the outcome in advance then experiment is called random experiment.
- Sample Space: The collection or set of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space (set) is called a sample point.
- **Event:** A subset of the sample space associated with a random experiment is called an event.
- **Elementary or Simple Event:** An event which has only one sample point is called a simple event.
- Compound Event: An event which has more than one sample point is called a Compound event.
- Sure Event: If an event is same as the sample space of the experiment, then event is called sure event. In other words an event which is certain to happen is sure event.
- **Impossible Event:** Let S be the sample space of the experiment, $\phi \subset S$, ϕ is called impossible event. In other words an event which is impossible to happen is the impossible event.
- Exhaustive and Mutually Exclusive Events: If Events E₁, E₂, E₃......E_n are n events of a sample space S such that
 - (i) $E_1U E_2UE_3U....UE_n = S$ then Events E_1 , E_2 , $E_3.....E_n$ are called exhaustive events.

- (ii) E_i ∩E_j =φ for every i ≠ j then Events E₁, E₂, E₃......E_n are called mutually exclusive.
- **Probability of an Event:** For a finite sample space S with equally likely outcomes, probability of an event A is defined as:

$$P(A) = \frac{n(A)}{n(S)}$$

where n(A) is number of elements in A and n(S) is number of elements in set S and $0 \le P(A) \le 1$

(a) If A and B are any two events then

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

= $P(A) + P(B) - P(A \text{ and } B)$

- (b) A and B are mutually exclusive events, thenP(A U B) = P(A) + P(B) (since P(A ∩ B) = 0 for mutually exclusive events)
- (c) $P(A) + P(\overline{A}) = 1 \text{ or } P(A) + P(\text{not } A) = 1$
- (d) P(Sure event) = P(S) = 1
- (e) P (impossible event) = $P(\phi) = 0$
- (f) $P(A B) = P(A) P(A \cap B) = P(A \cap \overline{B})$
- (g) $P(B-A) = P(B) P(A \cap B) = P(\overline{A} \cap B)$
- (h) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 P(A \cup B)$
- (i) $P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B}) = 1 P(A \cap B)$

Addition theorem for three events

Let A, B and C be any three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

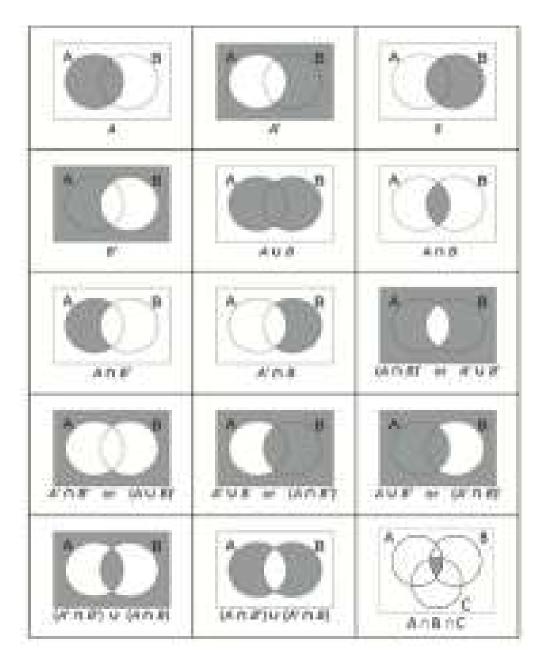
Axiomatic Approach to Probability:

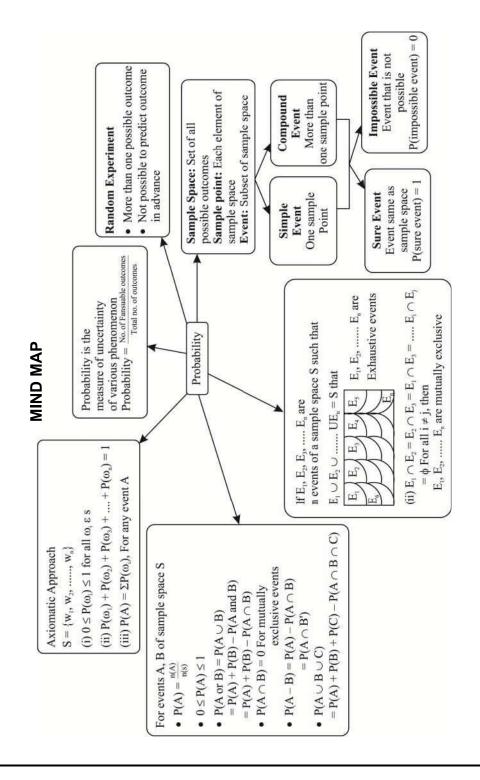
Let S be a sample space containing elementary outcomes w_1 , w_2 ,, w_n

i.e.
$$S = \{w_1, w_2, \dots, w_n\}$$

- (i) $0 \le P(w_i) \le 1$, for all $w_i \in S$
- (ii) $P(w_1) + P(w_2) + P(w_3) + \dots + P(w_n) = 1$
- (iii) P (A) = $\sum P(w_i)$, for any event A containing elementary events w_i .

VENN DIAGRAM OF DIFFERENT SETS





VERY SHORT ANSWER TYPE QUESTIONS

- Describe the Sample Space for the following experiments:
 A coin is tossed twice and number of heads is recorded.
- 2. A card is drawn from a deck of playing cards and its colour is noted.
- 3. A coin is tossed repeatedly until a tail comes up.
- 4. A coin is tossed. If it shows head, we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.
- 5. Two balls are drawn at random in succession without replacement from a box containing 1 red and 3 identical white balls.
- 6. A coin is tossed n times. Find the number of element in its sample space.
- 7. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?
- 8. What is the probability that a given two-digit number is divisible by 15?
- 9. If $P(A \cup B) = P(A) + P(B)$, then what can be said about the events A and B?
- 10. If $P(A \cup B) = P(A \cap B)$, then find relation between P(A) and P(B).

SHORT ANSWER TYPE QUESTIONS

- 11. Let A and B be two events such that P(A) = 0.3 and $P(A \cup B) = 0.8$, find P(B) if $P(A \cap B) = P(A) P(B)$.
- 12. Three identical dice are rolled. Find the probability that the same number appears on each of them.
- 13. In an experiment of rolling of a fair die. Let A, B and C be three events defined as under:

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A : a number which is a perfect square

B: a prime number

C: a number which is greater than 5.

Is A, B, and C exhaustive events?

14. Punching time of an employee is given below:

DAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
TIME (AM)	10:35	10:20	10:22	10:27	10:25	10:40

If the reporting time is 10:30 a.m, then find the probability of his coming late.

- 15. A game has 18 triangular blocks out of which 8 are blue and rest are red and 19 square blocks out of which 7 are blue and rest are yellow. One piece is lost. Find the probability that it was a square of blue colour.
- 16. A card is drawn from a pack of 52 cards. Find the probability of getting:
 - (i) a jack or a queen
 - (ii) a king or a diamond
 - (iii) a heart or a club
 - (iv) either a red or a face card.
 - (v) neither a heart nor a king
 - (vi) neither an ace nor a jack
 - (vii) a face card
- 17. In a leap year find the probability of
 - (i) 53 Mondays and 53 Tuesdays
 - (ii) 53 Mondays and 53 Wednesday
 - (iii) 53 Mondays or 53 Tuesdays
 - (iv) 53 Mondays or 53 Wednesday
- 18. In a non-leap year, find the probability of
 - (i) 53 Mondays and 53 Tuesdays.

- (ii) 53 Mondays or 53 Tuesdays.
- 19. Three candidates A, B, and C are going to play in a chess competition to win FIDE (World Chess Federation) cup this year. A is thrice as likely to win as B and B is twice as likely as to win as C. Find the respective probability of A, B and C to win the cup.

LONG ANSWER QUESTIONS

- 20. Find the probability that in a random arrangement of the letters of the word UNIVERSITY two I's come together.
- 21. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x.
- 22. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.
- Find the probability of at most two tails or at least two heads in a toss of three coins.
- 24. A, B and C are events associated with a random experiment such that

$$P(A) = 0.3,$$

$$P(B) = 0.4$$
, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$ and $P(A \cap B \cap C) = 0.09$. If

P(A U B U C) \geq 0.75 Then prove that P(B \cap C) lies in the interval [0.23, 0.48].

- 25. $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probability of three mutually exclusive events. Then find the set of all values of p.
- 26. An urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from

- urn B and placed in urn A. Now if one ball is drawn at random from urn A then find the probability that it is found to be red.
- 27. If three distinct numbers are chosen randomly from the first 100 natural numbers, then find the probability that all three of them are divisible by both 2 and 3.
- 28. $S = \{1, 2, 3, \dots, 30\}, A = \{x : x \text{ is multiple of } 7\}, B = \{x : x \text{ is multiple of } 5\}, C = \{x : x \text{ is a multiple of } 3\}.$

If x is a member of S chosen at random find the probability that

- (i) $x \in A \cup B$
- (ii) $x \in B \cap C$
- (iii) $x \in A \cap \overline{C}$
- 29. One number is chosen at random from the number 1 to 100. Find the probability that it is divisible by 4 or 10.
- 30. If A and B are any two events having $P(A \cup B) = \frac{1}{2}$ and $P(\overline{A}) = \frac{2}{3}$, then find the $P(\overline{A} \cap B)$.
- 31. Three of the six vertices of a regular hexagon are chosen at random. What is probability that the triangle with these vertices is equilateral?
- 32. A typical PIN (Personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?
- 33. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random. Find the probability that the balls are of same colour.
- 34. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that
 - (i) it will get at least one of the two awards
 - (ii) it will get only one of the awards.

- 35. A girl calculates that the probability of her winning the first prize in a lottery is 0.02. If 6000 tickets were sold, how many tickets has she bought?
- 36. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9?
- 37. All the face cards are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 and similar value for other cards. Find the probability of getting a card with value less than 7.
- 38. If A,B and C are three mutually exclusive and exhaustive events of an experiment such that 3P(A) = 2P(B) = P(C), then find the value of P(A).

CASE STUDY TYPE QUESTIONS

39. To make a healthy routine and to do some physical exercise during lockdown a family decided to roll a dice and based on the outcomes, they will decide activities to be done.



• If the outcome is 2, 4 or 6, they will do 30 minutes walk on the roof.

- If it shows 1 or 3 on the dice, 15 minutes meditation to be done.
- If outcome is 5, then they will toss a coin. If it shows "Head", the family will do 5 minutes of rope skipping. If there is "Tail", family will do 20 minutes of Yoga.
- i. How many elements are there in the sample space?
- ii. What is the probability of doing walking?
- iii. What is the probability of doing rope skipping?
- iv. What is the probability of doing yoga or meditation?
- v. Two activities having the same probability are
- 40. In a class of 60 students, hobbies were discussed. 30 liked reading, 32 liked singing and 24 liked about reading and singing.



- i. Find the probability that the student liked reading or singing.
 - $(a)\frac{17}{30}$
- (b) $\frac{19}{30}$
- $(c)\frac{23}{30}$
- $(d)\frac{29}{30}$
- ii. How many students neither like reading nor singing?
 - (a) 30
- (b) 28
- (c) 22
- (d) 38

iii.	Find the probability reading?	that the studen	t neither like	singing	nor
	$(a)\frac{11}{30}$	(b) $\frac{13}{30}$			

(c) $\frac{7}{30}$ (d) $\frac{1}{30}$

iv. Find the probability that a student like singing but not reading?

(a) $\frac{4}{15}$ (b) $\frac{7}{15}$ (c) $\frac{1}{15}$ (d) $\frac{2}{15}$

v. Find the probability that a student like reading only.

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{7}{10}$ (d) 0

Multiple Choice Questions

41. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is -

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{5}{9}$ (d) $\frac{1}{30}$.

42. Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits?

(a) $\frac{1}{16}$ (b) $\frac{16}{25}$

(c) $\frac{1}{65}$ (d) $\frac{1}{25}$.

43.	The probability that a non-leap year selected at random will have				
	52 Sundays is -				
	(a) 0	(b) 1			
	(c) $\frac{1}{7}$	(d) $\frac{2}{7}$.			
44.	The probability that a non-leap year selected at random will have				
	53 Sundays is -				
	(a) 0	(b) 1			
	(c) $\frac{1}{7}$	(d) $\frac{2}{7}$.			
	7	(d) 7.			
45.	The probability that a leap year selected at random will have 54				
	Sundays is				
	(a) 0	(b) 1			
	(c) $\frac{1}{7}$	(d) $\frac{2}{7}$.			
46.	Three unbiased coins are tosse	ed. If the probability of getting at			
	least 2 tails is p, Then the value of 8p -				
	(a) 0	(b) 1			
	(c) 3	(d) 4.			
47.	Four unbiased coins are tossed. If the probability of getting odd				
	number of tails is p, then the value of 16p -				
	(a) 1	(b) 2			
	(c) 4	(d) 8			
48.	From 4 red balls, 2 white balls and 4 black balls, four balls are				
	selected. The probability of getting 2 red balls is p, then the				

value of 7p -

(a) 1

(b) 2

(c) 3

(d) 4

49. If A and B are mutually exclusive events, then

(a) $P(A) \leq P(\overline{B})$

(b) $P(A) \ge P(\overline{B})$

(c) $P(A) < P(\overline{B})$

(d) None of these

50. The probability that atleast one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{\mathbb{A}}) + P(\overline{\mathbb{B}})$ is

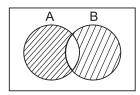
(a) 0.4

(b) 0.8

(c) 1.2

(d) 1.6

51. In the following Venn diagram circles A and B represent two events:



The probability of the union of shade region will be

(a)
$$P(A) + P(B) - 2P(A \cap B)$$

(b)
$$P(A) + P(B) - P(A \cap B)$$

(c)
$$P(A) + P(B)$$

(d)
$$2P(A) + 2P(B) - P(A \cap B)$$

52. A bag contains 10 balls, out of which 6 balls are white and the others are non-white. The probability of getting a non-white is

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $\frac{1}{2}$

(d) $\frac{2}{3}$

53. Two dice are thrown together. The probability of getting the sum of digits as a multiple of 4 is:

(a) $\frac{1}{9}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{5}{9}$

54. The probability of getting sum more then 8 when a pair of dice are thrown is:

(a) $\frac{7}{36}$

(b) $\frac{5}{18}$

(c) $\frac{7}{18}$

(d) $\frac{5}{36}$

55. If A and B are two events, such that

$$P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}, P(\overline{A}) = \frac{2}{3},$$

Then P(B) is given by:

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{1}{9}$

(d) $\frac{2}{9}$

56. A die is rolled. Let E be the event 'die shows 4 and F be the event 'die shows an even number".

- (a) mutually exclusive
- (b) exhaustive
- (c) mutually exclusive and exhaustive
- (d) None of these

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four

Alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

- (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
- (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.
- (c) Assertion is correct, reason is incorrect.
- (d) Assertion is incorrect, reason is correct.
- 57. **Assertion:** Probability of getting a head in a toss of an unbiased coin is $\frac{1}{2}$.

Reason: In a simultaneous toss of two coins, the probability of getting 'no tails' is $\frac{1}{4}$.

58. **Assertion:** In tossing a coin, the exhaustive number of cases is $2 \times 2 = 4$.

Reason: If a pair of dice is thrown, then the exhaustive number of cases is $6 \times 6 = 36$.

59. **Assertion:** If $A \cap B = \phi$, then $P(A \cap B) = 0$

Reason: For mutually exclusive events A and B, $P(A \cap B) = 0$

60. Consider a single throw of die and two events.

A = the number is even = $\{2, 4, 6\}$

B = the number is a multiple of $3 = \{3, 6\}$

Assertion:
$$P(A \cup B) = \frac{4}{6} = \frac{2}{3} \text{ and } P(A \cap B) = \frac{1}{6}$$

Reason:
$$P(\overline{A} \cap \overline{B}) = 1 - \frac{1}{6} = \frac{5}{6}$$

ANSWERS

6.
$$2^n$$
 [eg: $2^1 = \{H, T\}, 2^2 = \{HH, HT, TH, TT\}, 2^3 = \{HHH, TTT\}$]

7.
$$\frac{8}{21}$$

8.
$$\frac{1}{15}$$

10.
$$P(A) = P(B)$$
 [Possible only if $A = B$]

11.
$$\frac{5}{7}$$
 [Hint: P(A \cup B) = P(A) + P(B) – P(A)P(B)]

12.
$$\frac{1}{36} \left[= \frac{6}{216} \right]$$

14.
$$\frac{1}{3}$$

15.
$$\frac{7}{37}$$

16. (i)
$$\frac{2}{13}$$
 [Hint: P(J \cup Q) = P(J) + P(Q) - P(J \cap Q)]

(ii)
$$\frac{4}{13}$$

(iii)
$$\frac{1}{2}$$

(iv)
$$\frac{8}{13}$$

(v)
$$\frac{9}{13}$$
 [P($\overline{H \cup K}$)=1-P($H \cup K$)]

(vi)
$$\frac{11}{13}$$
 [P($\overline{A \cup J}$) = 1 - {P(A) + P(J) - P(A \cap J)}

(vii) $\frac{3}{13}$ [J, Q, K of diamond, heart, club and spade are 12 face cards]

17. (i)
$$\frac{1}{7}$$

(iii)
$$\frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$$

(iv)
$$\frac{2}{7} + \frac{2}{7} - 0 = \frac{4}{7}$$

- 18. Non leap year = 52 weeks and 1 day
 - (i) 0 (both together not possible)

(ii)
$$\frac{1}{7} + \frac{1}{7} - 0 = \frac{2}{7}$$

19. [Hint: P(C) = x, P(B) = 2P(C) = 2x P(A) = 3P(B) = = 6x]
$$\frac{2}{3}, \frac{2}{9}, \frac{1}{9}$$

20.
$$\frac{1}{5} \left[\text{Hint} : \frac{9!}{10!/2!} = \frac{9!}{5 \times 9!} \right]$$
 21.3 $\left[\text{Hint} : \frac{{}^{5}\text{C}_{2}}{{}^{5+x}\text{C}_{2}} = \frac{5}{14} \right]$

21.3 Hint:
$$\frac{{}^{5}C_{2}}{{}^{5+x}C_{2}} = \frac{5}{14}$$

22.
$$\frac{23}{28}$$
 Hint: $\frac{{}^{8}C_{3} - {}^{5}C_{3}}{{}^{8}C_{3}}$

23.
$$\frac{7}{8}$$
 [Hint: P(A \cup B)]

 $0.23 \le P(B \cap C) \le 0.48$ [Hint: $0.75 \le P(A \cup B \cup C) \le 1, 0.75 \le 1.23$ 24. $-x \leq 1$

25.
$$\frac{-1}{3} \le p \le \frac{-1}{3}$$

$$\left[\text{Hint: } 0 \le P(A) \le 1, \ 0 \le P(B) \le 1 \ \text{Multually Exclusive} \right]$$

$$0 \le P(C) \le 1 \ 0 \le P(A) + P(B) + P(C) \le 1$$

Hint:

Case I: A
$$\xrightarrow{\text{Red}}$$
 B, B $\xrightarrow{\text{Red}}$ A, $\frac{6}{10} \times \frac{5}{11} \times \frac{6}{11}$

Case II: A $\xrightarrow{\text{Red}}$ B, B $\xrightarrow{\text{Black}}$ A, $\frac{6}{10} \times \frac{6}{11} \times \frac{5}{10}$

Case III: A $\xrightarrow{\text{Black}}$ B, B $\xrightarrow{\text{Red}}$ A, $\frac{4}{10} \times \frac{4}{11} \times \frac{7}{10}$

Case IV: A $\xrightarrow{\text{Black}}$ B, B $\xrightarrow{\text{Black}}$ A, $\frac{4}{10} \times \frac{7}{11} \times \frac{6}{10}$

27.
$$\frac{4}{1155} \left[\text{Hint} : \frac{{}^{16}\text{C}_3}{{}^{100}\text{C}_3} \right]$$

28. (i)
$$\frac{1}{3} \left[\frac{n(A \cup B)}{n(s)} = \frac{n(A) + n(B) - n(A \cap B)}{n(s)} \right]$$

(ii)
$$\frac{1}{15}$$
 [Hint: B \cap C = {15, 30}]

(iii)
$$\frac{1}{10} \left[\text{Hint: A} \cap \overline{C} = \{7, 14, 28\} \right]$$

29.
$$\frac{3}{10}$$
 $\left[\text{Hint: P(A $\cup B) = P(A) + P(B) - P(A \cap B) = } \frac{25}{100} + \frac{10}{100} - \frac{5}{100} \right]$$

30.
$$\frac{1}{6}$$
 [Hint: $P(\overline{A} \cap B) = P(A \cap B) - P(A)$]

31.
$$\frac{1}{10}$$
 Hint: $\frac{2}{{}^{6}C_{3}}$

32.
$$\frac{1231}{7776} \left[\text{Hint: } 1 - \frac{36 \times 35 \times 34 \times 33}{(36)^4} \right]$$

33.
$$\frac{63}{190} \left[\text{Hint} : \frac{{}^{9}\text{C}_{2}}{{}^{20}\text{C}_{2}} + \frac{{}^{7}\text{C}_{2}}{{}^{20}\text{C}_{2}} + \frac{{}^{4}\text{C}_{2}}{{}^{20}\text{C}_{2}} \right]$$

34. (i) 0.64 [Hint: P(D U M)]

(ii) 0.44
$$= P(D \cap \overline{M}) \cup P(\overline{D} \cap M)$$
$$= P(D \cup M) - P(D \cap M)$$

36.
$$\frac{5}{12}$$

37.
$$\frac{3}{5}$$
 Hint: = $\frac{4 \times {}^{6}C_{1}}{{}^{40}C_{1}}$

38.
$$\frac{2}{11}$$
 Hint:=Let P(C) = x $\frac{x}{3} + \frac{x}{2} + x = 1$

39.
$$S = \left\{ \frac{2, 4, 6}{\text{walking}}, \frac{1, 3}{\text{meditation}}, \frac{5H}{\text{Rope skipping}}, \frac{5T}{\text{Yoga}} \right\}$$

ii.
$$\frac{3}{7}$$

iii.
$$\frac{1}{7}$$

i. 7 ii.
$$\frac{3}{7}$$
 iii. $\frac{1}{7}$ iv. $\frac{1}{7} + \frac{2}{7} = \frac{3}{7}$

v. Yoga and rope skipping

40. i. (b)
$$\frac{19}{30}$$
 ii. (c) 22 iii. (a) $\frac{11}{30}$ iv. (d) $\frac{2}{15}$ v. (a) $\frac{1}{10}$

iii. (a)
$$\frac{11}{30}$$

iv. (d)
$$\frac{2}{15}$$
 \

v. (a)
$$\frac{1}{10}$$

41. (c)
$$\frac{5}{9} \left[\frac{4}{18} + \frac{6}{18} = \frac{10}{18} \right]$$

- 42. (d) $\frac{1}{25} \left[= \frac{4}{4 \times 5 \times 5} \right]$
- 43. (b) 1

44. (c) $\frac{1}{7}$

45. (a) 0

46. (d) 4

47. (d) 8

48. (c) 3

49. (a) $P(A) \le P(\overline{B})$

50. (c) 1.2

51. (a) $P(A) + P(B) - 2P(A \cap B)$

52. (a) $\frac{2}{5}$

53. (c) $\frac{1}{4}$

54. (b) $\frac{5}{18}$

- 55. (b) $\frac{2}{3}$
- 56. (d) None of these (neither mutually exclusive nor exhaustive)
- 57. (b)

58. (d)

59. (a)

60. (c)