CHAPTER - 13

STATISTICS

KEY POINTS

● Range of Ungrouped Data and Discrete Frequency Distribution.

● RANGE = Largest observation – smallest observation.

● Range of Continuous Frequency Distribution.

● Upper Limit of Highest Class – Lower Limit of Lowest Class.

● **Mean deviation for ungrouped data or raw data:**

$$M.D. \text{ (about mean)} = \frac{\sum |x_i - \bar{x}|}{n}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D. \text{ (about median)} = \frac{\sum |x_i - M|}{n}, \text{ where } M \text{ is the Median.}$$

● **Mean deviation for grouped data (Discrete frequency distribution and Continuous frequency distribution):**

$$M.D. \text{ (about mean)} = \frac{\sum f_i |x_i - \bar{x}|}{N}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D. \text{ (about mean)} = \frac{\sum f_i |x_i - M|}{N}, \text{ where } M \text{ is the Median.}$$

Note: $N = \sum f_i$

● Variance is defined as the mean of the squares of the deviations from mean.

- Standard deviation 'σ' is positive square root of variance.

$$\sigma = \sqrt{\text{Variance}}$$

- Variance 'σ^2'and standard deviation (SD) σ for ungrouped data

$$\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2 \implies \boxed{S.D. = \sigma = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}}$$

- Standard deviation of a discrete frequency distribution

$$S.D. = \sigma = \sqrt{\frac{1}{N}\sum f_i(x_i - \bar{x})^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - (\sum f_i x_i)^2}$$

- Short cut method to find variance and standard deviation

$$\text{Variance} = \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2 \right]$$

$$S.D. = \sigma = \frac{h}{N}\sqrt{N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2}$$

where $y_i = \dfrac{x_i - A}{h}$, A = Assumed mean

- If each observation is multiplied by a positive constant k then variance of the resulting observations becomes k^2 times of the original value and standard deviation becomes k times of the original value.

- If each observation is increased by k, where k is positive or negative, then variance and standard deviation remains same.

- Standard deviation is independent of choice of origin but depends on the scale of measurement.

- The mean of first 'n; natural number is $\dfrac{n+1}{2}$.

- The mean of first 'n' even natural numbers = (n + 1)
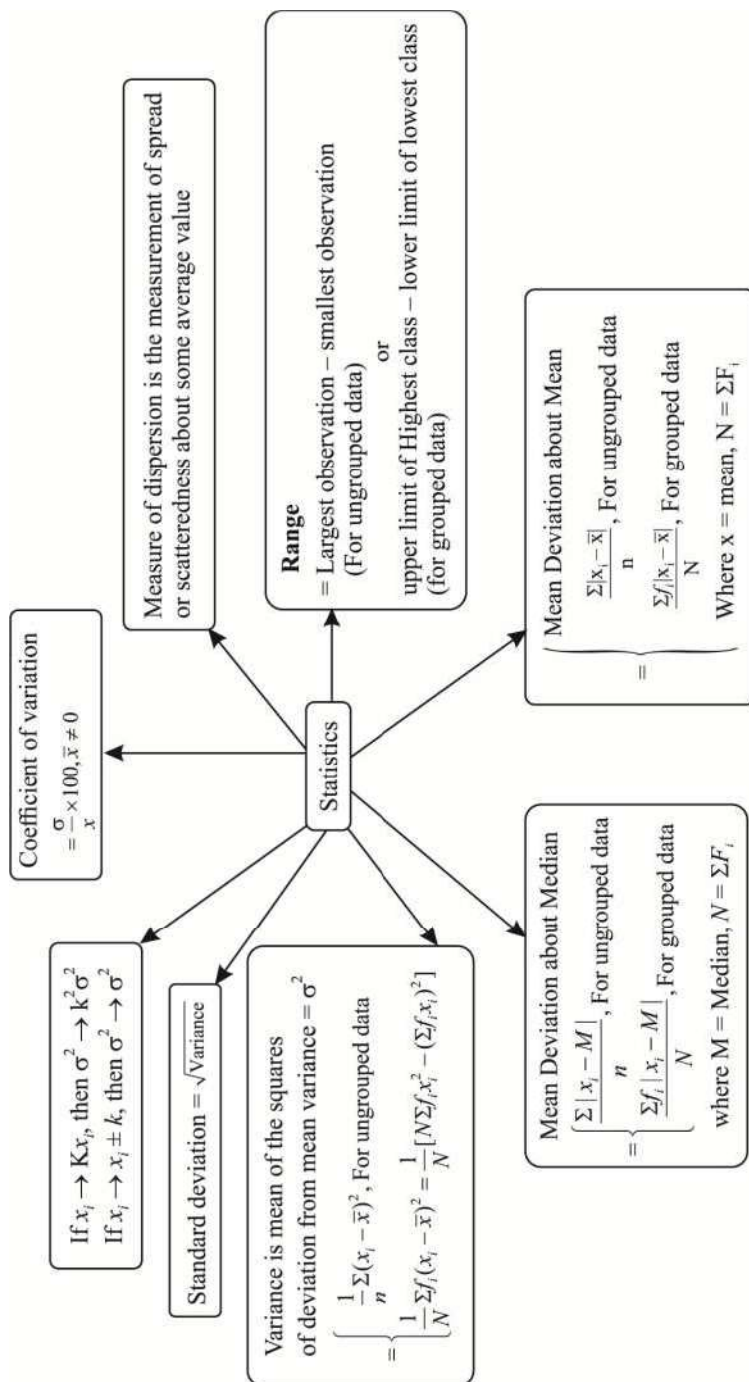
VERY SHORT ANSWER TYPE QUESTIONS

1. The sum of the squares of deviation for 10 observations taken from their mean 50 is 250. Find Standard Deviation.

2. The sum of the squares of deviation for 10 observations taken from their mean 25 is 500. Find Variance.

3. If the variance of 14, 18, 22, 26, 30 is 'k', then find the variance of 28, 36, 44, 52, 60.

SHORT ANSWER TYPE QUESTIONS

4. Find the Variance of First 10 Natural Numbers.

5. Find the Variance of First 5 Multiples of 6.

6. Find the Standard Deviations of First 10 Even Natural numbers.

7. Find the Standard deviation for the following data:

   10, 20, 30, 40, 50, 50, 60, 70, 80, 90

8. Find the variance for the following Data:

Class-Interval	Frequency
0 - 10	1
10 - 20	2
20 – 30	3
30 – 40	3
40 – 50	1

MIND MAP

Coefficient of variation

$$= \frac{\sigma}{\bar{x}} \times 100, \bar{x} \neq 0$$

Measure of dispersion is the measurement of spread or scatteredness about some average value

Range

= Largest observation – smallest observation (For ungrouped data)

or

upper limit of Highest class – lower limit of lowest class (for grouped data)

Statistics

If $x_i \rightarrow Kx_i$, then $\sigma^2 \rightarrow k^2\sigma^2$

If $x_i \rightarrow x_i \pm k$, then $\sigma^2 \rightarrow \sigma^2$

Standard deviation = $\sqrt{Variance}$

Variance is mean of the squares of deviation from mean variance = σ^2

$$= \begin{cases} \dfrac{1}{n}\Sigma(x_i - \bar{x})^2, \text{For ungrouped data} \\ \dfrac{1}{N}\Sigma f_i(x_i - \bar{x})^2 = \dfrac{1}{N}[N\Sigma f_i x_i^2 - (\Sigma f_i x_i)^2] \end{cases}$$

Mean Deviation about Mean

$$= \begin{cases} \dfrac{\Sigma|x_i - \bar{x}|}{n}, \text{For ungrouped data} \\ \dfrac{\Sigma f_i|x_i - \bar{x}|}{N}, \text{For grouped data} \end{cases}$$

Where x = mean, N = ΣF_i

Mean Deviation about Median

$$= \begin{cases} \dfrac{\Sigma|x_i - M|}{n}, \text{For ungrouped data} \\ \dfrac{\Sigma f_i|x_i - M|}{N}, \text{For grouped data} \end{cases}$$

where M = Median, $N = \Sigma F_i$

LONG ANSWER TYPE – I QUESTIONS

9. In a series of '2p' observations, half of the observations are equal 'a' each and remaining half equal (–a) each. If the standard deviation of the observations is 2, then find the value of |a|.

10. In the following Distribution

x	f
A	2
2A	1
3A	1
4A	1
5A	1
6A	1

Where A is positive integer, has a variance of 160. Determine the value of A.

11. Find the mean deviation from mean of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.

12. Find the Variance and Standard Deviation of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.

13. Consider the first 10 positive integers. If we multiply each number by –1 and then add 1 to each number, find the variance of the numbers so obtained.

14. Two sets each of 20 obvservations, have the same standard deviation 5. The first set has a mean 17 and the second a mean 22. Determine the SD of the set obtained by combining the given two sets.

15. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observationsare 1, 2 and 6. Find the other two observations.

16. Calculate the possible values of 'x' if standard deviation of the numbers 2, 3, 2x and 11 is 3.5.

17. Mean and standard deviation of the data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find the correct mean and the correct standard deviation.

18. Suppose a population A has 100 observations 101, 102,……, 200. Another population B has 100 observations 151, 152,……., 250. If V_A and V_B represent the variances of the two populations respectively then find the ratio of V_A and V_B.

LONG ANSWER TYPE – II QUESTIONS

19. Calculate the mean deviation about mean for the following data.

X	2	4	6	8	10	12	14	16
f	2	2	4	5	3	2	1	1

20. If for a distribution $\sum(x-5)=3$, $\sum(x-5)^2=43$ and the total number of item is 18, find the mean and standard deviation.

21. Calculate the mean deviation about median for the following data:

X	10	15	20	25	30	35	40	45
f	7	3	8	5	6	8	4	4

22. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test :

X	0	1	2	3	4	5
f	p – 2	p	p^2	$(p + 1)^2$	2p	2p + 1

where p is positive integer. Determine the mean and standard deviation of the marks.

23. Calculate the mean deviation about mean

Class Interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
f	2	3	8	14	8	3	2

24. Mean and standard deviation of 100 observations were found to be 40 and 10respectively. If at the time of calculation two observations were wrongly taken as 30 and70 in place of 3 and 27 respectively. Find correct standard deviation.

25. Calculate the mean deviation about mean for the following data:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	5	8	15	16	6

26. Calculate the mean deviation about median for the following data:

Class Interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
f	8	10	10	16	14	2

27. The mean and standard deviation of some data taken for the time to complete a test are calculated with following results:

Number of observations = 25,
mean = 18.2 seconds
Standard deviation = 3.25 seconds

Further another set of 15 observations x_1, x_2, ………....., x_{15}, also in seconds is now available and we have $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524.$

Calculate the standard deviation based on all 40 observations.

28. Find the mean deviation about mean of the following data:

Class Interval	f
20 - 29	5
30 - 39	12
40 - 49	15
50 - 59	20
60 - 69	18
70 - 79	10
80 - 89	6
90 - 99	4

CASE STUDY TYPE QUESTIONS

29. Following data represents the salaries of 11 employees in a firm

10000, 12000, 15000, 13000, 11000, 12000, 12000, 14000, 10000, 13000, 12000.

i. Find the mean salary.

(a) 11181.82   (b) 12181.82

(c) 13181.82   (d) 10000.82

ii. What is the median salary?

(a) 12000   (b) 11000

(c) 12181.82   (d) 11181.82

iii. When arranged in ascending order, which entry gives the median salary?

(a)6th (b) 5th (c) 4th (d) 7th

iv. The mean deviation about the median salary is

(a)1190.99   (b) 1000   (c) 1100   (d) 1090.91

v. What is the range of salaries?

(a)4500   (b) 4000   (c) 5000   (d) 6000

30. Following are the prices of shares X and Y (of ten days) :

Days	X	Y
1	35	108
2	54	107
3	52	105
4	53	105
5	56	106
6	58	107
7	52	104

8	50	103
9	51	104
10	49	101

i.   What is the mean price of the share X during these 10 days?

(a) 52   (b) 51   (c) 50.3   (d) 51.5

ii. What is the mean price of the share Y during these 10 days?

(a) 105   (b) 106   (c) 104   (d) 107

iii. What is the standard deviation of the price of share X?

(a) 5.01   (b) 6.75   (c) 5.92   (d) 7.25

iv. What is the standard deviation of the price of share Y?

(a) 1.75   (b) 2.87   (c) 1.25   (d) 2

v.   If a person wants to invest in shares (X or Y) whose price remain more stable. He should invest in

(a) X

(b) Y

(c) Both are equally stable. So, he can invest in anyone

(d) Insufficient data to decide

Multiple Choice Questions

Note: **Q.31–Q.46** are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

31.   The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new mean -
(a) 12   (b) 16
(c) 24   (d) 48.

32. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new standard deviation -

(a) 4   (b) 8

(c) 16   (d) 32.

33. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new mean-

(a) 25   (b) 29

(c) 30   (d) 35.

34. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new variance -

(a) 4   (b) 14

(c) 16   (d) 25.

35. Match the following:

If the mean of $x_1, x_2, \ldots\ldots, x_{20}$ is 10.

	Column-1		Column-2
A	mean of $2x_1, 2x_2, \ldots\ldots, 2x_{20}$	P	0
B	mean of $(-3x_1 + 32), (-3x_2 + 32), \ldots, (3x_{20} +32)$	Q	2
C	mean of $(x_1 + 2), (x_2 + 2), \ldots\ldots, (x_{20} + 2)$	R	12
D	mean of $(x_1 - 10), (x_2 - 10), \ldots\ldots, (x_{20} - 10)$	S	20

(a)   A → P, B→ Q, C→ R, D → S

(b)   A → S, B→ Q, C→ R, D → P

(c)   A → Q, B→ S, C→ R, D → P

(d)   A → S, B→ Q, C→ P, D → R

36. If mean of first n natural numbers is $\dfrac{5n}{9}$, then n =

    (a) 5   (b) 4

    (c) 9   (d) 10

37. Find the mean of 6, 7, 10, 12, 13, 4, 8, 12

    (a) 9   (b) 10

    (c) 12   (d) 13

38. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is

    (a) 2.23   (b) 2.57

    (c) 3.23   (d) 3.57

39. The following information relates to a sample of size

    $60 : \Sigma x^2 = 18000, \Sigma x = 960$

    The variance is

    (a) 6.63   (b) 16

    (b) 22   (d) 44

40. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

    (a) $\sqrt{\dfrac{52}{7}}$ (b) $\dfrac{52}{7}$

    (c) $\sqrt{6}$ (d) 6

41. The variance of n observation $x_1, x_2, \ldots\ldots, x_n$ is given by

    (a) $\sigma^2 = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n}(x_i - \overline{x})$ (b) $\sigma^2 = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n}(x_1 - \overline{x})^2$

    (c) $\sigma^2 = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n}(x_i + \overline{x})$ (d) $\sigma^2 = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n}(x_i + \overline{x})^2$

42. Variance of the numbers 3, 7, 10, 18, 22 is equal to

    (a) 12   (b) 64

    (c) $\sqrt{49.2}$ (d) 49.2

43. The mean deviation from the mean of the following data:

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	5	8	15	16	6

    is

    (a) 10   (b) 10.22

    (c) 9.86   (d) 9.44

44. The mean of the numbers, a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b?

    (a) a = 0, b = 7   (b) a = 5, b = 2

    (c) a = 1, b = 6   (d) a = 3, b = 4

45. Find the mean deviation about the mean for the data: 12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5

    (a) 5.2   (b) 6.2

    (c) 7.2   (d) 8.2

46. Find the mean deviation about the mean for the data:

x_1	5	10	15	20	25
f_i	7	4	6	3	5

    (a) 6.32   (b) 7.32

    (c) 8.32   (d) 9.32

Directions: Each of these questions contains two statements. Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct;l reason is not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

47. **Assertion:** Mean of deviations = $\dfrac{\text{Prouduct of deviations}}{\text{No. of observations}}$

Reason: To find the dispersion of values of x from mean $\bar{x}$, we take absolute measure of dispersion.

48. Let x_1, x_2, ... x_n be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.

Assertion: Variance of $2x_1$, $2x_2$,...... $2x_n$ is $4\sigma^2$.

Reason: Arithmetic mean of $2x_1$, $2x_2$, ... $2x_n$ is $4\bar{x}$.

49. **Assertion:** The range is the difference between two extreme observations of the distribution.

Reason: The variance of a variate X is the arithmetic mean of the squares of all deviations of X from the arithmetic mean of the observations.

50. **Assertion:** The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is 2.57.

Reason: For individual observation.

Mean deviation $(\bar{X}) = \dfrac{\sum |x_i - \bar{x}|}{n}$

ANSWERS

1. 5

2. 50

3. 4 k

4. 8.33

5. 72

6. $\sqrt{33}$

7. $10\sqrt{6}$

8. $\sqrt{129}$

9. 2

10. A = 7

11. $\dfrac{(n-1)(d-1)}{2}$

12. Variance = $\dfrac{(n^2-1)}{12}d^2$

Standard Deviation = $d\sqrt{\dfrac{(n^2-1)}{12}}$

13. 8.25

14. 5.59 Hint: $\left[S.D. = \sqrt{\dfrac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \dfrac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{n_1 + n_2}} \right]$

15. 4, 9

16. 3, $\dfrac{7}{3}$ Hint: $\bar{x} = 4 + \dfrac{x}{2}$ S.D. $= \sqrt{\dfrac{1}{n}\Sigma(x_i - \bar{x})^2}$

17. 6.5, 2.5 Hint: [correct mean $= \dfrac{126 + 12 - 21}{18}$]

[Correct $\Sigma x_i^2 = 1170 - 21^2 + 12^2 = 873$]

18. 1 : 1 $\left[\text{Hint: } V_A = \dfrac{\dfrac{1}{2}(1^2 + 3^2 + ... + 99^2)}{100} \\ V_B = \dfrac{\dfrac{1}{2}(99^2 + 97^2 + ...1^2)}{100}\right]$

19. 2.8

20. Mean = 5.17,
     Standard Deviation = 1.53

21. 10.1

22. Mean = 2.8,
     Standard deviation = 1.12

23. 10

24. 10.24 $\left[\begin{array}{l}\text{Hint: correct mean} = 39.3 \\ \text{Correct } \Sigma x_i^2 = 164938 \\ \text{Correct } \sigma^2 = \dfrac{164938}{100} - (39.3)^2\end{array}\right]$

25. 9.44   26. 11.44

27. 3.87

29. i. (b)   ii. (a)   iii. (a)   iv. (d)   v. (c)

30. i. (b)   ii. (a)   iii. (c)   iv. (d)   v. (b)

31. (d)   32. (c)

33. (d)   34. (c)

35. (b)   36. (c)

37. (a)   38. (b)

39. (d)   40. (a)

41. (b) $\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

42. (d) Mean = 12, Variance = 49.2

43. (d) 9.44

44. (d) a = 3, b = 4 [Hint: a + b = 7, $a^2 - 7a + 12 = 0$]

45. (b) 6.2 [Hint: Mean = 10]

46. (a) 6.32 [Hint: Mean = 14]

47. (d)   48. (c)

49. (b)   50. (a)