CHAPTER - 12

LIMITS AND DERIVATIVES

KEY POINTS

- To check whether limit of f(x) as x approaches to a exists i.e., $\lim_{x \to a} f(x)$ exists, we proceed as follows.

  (i) Find L.H.L at x = a using L.H.L. = $\lim_{h \to 0} f(a-h)$.

  (ii) Find R.H.L at x = a using R.H.L. = $\lim_{h \to 0} f(a+h)$.

  (iii) If both L.H.L. and R.H.L. are finite and equal, then limit at x = a i.e., $\lim_{x \to a} f(x)$ exists and equals to the value obtained from L.H.L or R.H.L else we say "limit does not exist".

- $\lim_{x \to a} f(x) = l$, if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = l$

▶ **ALGEBRA OF LIMITS:** Let f, g be two functions such that $\lim_{x \to c} f(x) = l,$ and $\lim_{x \to c} g(x) = m.$

- $\lim_{x \to c} [\alpha f(x)] = \alpha \lim_{x \to c} f(x) = \alpha l, \text{ for all } \alpha \; \varepsilon \; R$

- $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = l \pm m$

- $\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = l \cdot m$

- $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{l}{m}, \; m \neq 0, \; g(x) \neq 0$

- $$\lim_{x \to c} \frac{1}{f(x)} = \frac{1}{\lim_{x \to c} f(x)} = \frac{1}{l},\ l \neq 0,\ f(x) \neq 0$$

- $$\lim_{x \to c} [f(x)]^n = [(\lim_{x \to c} f(x)]^n = l^n, \text{ for all } n \ \varepsilon\ N$$

▶ SOME IMPORTANT RESULTS ON LIMITS:

- $$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$$
- $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$
- $$\lim_{x \to 0} \frac{\tan x}{x} = 1$$
- $$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$
- $$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$
- $$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$$
- $$\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$$
- $$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$$
- $$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(-x)$$

▶ SOME IMPORTANT RESULTS ON DERIVATIVE:

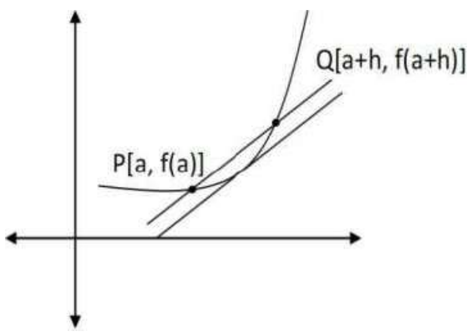
• $\frac{d}{dx}(\sin x) = \cos x$	• $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$
• $\frac{d}{dx}(\cos x) = -\sin x$	• $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$
• $\frac{d}{dx}(\tan x) = \sec^2 x$	• $\frac{d}{dx}(\text{cosecx}) = -\text{cosec} x \cdot \cot x$

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(a) = 0,\ a = \text{constant}$

- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$
- $\frac{d}{dx}(a^x) = a^x \cdot \log a$

▶ **Logarithm Properties:** $a^b = c$ (exponential form)
$\text{Log}_a c = b$, (log form) $a > 0$, $c > 0$

- $\log_e(A \cdot B) = \log_e A + \log_e B$
- $\log_e\left(\frac{A}{B}\right) = \log_e A - \log_e B$
- $\log_e(A^m) = m \cdot \log_e A$
- $\log_a(1) = 0$
- $\log_a a = 1$
- $\log_b a = \frac{1}{\log_a b}$
- $\log_b a = \frac{\log_c a}{\log_c b}$ (base change formula)
- $a^{m\log_a k} = k^m$
- $\log_B(A) = x$, then $B^x = A$
- Let y = f(x) be a function defined in some neighbourhood of the point 'a'. Let P[a, f(a)] and Q[a + h, f(a + h)] are two points on the graph of f(x) where h is very small and h ≠ 0.



$$\text{Slope of } PQ = \frac{f(a+h) - f(a)}{h}$$

- If $\lim_{h \to 0}$ point Q approaches to P and the line PQ becomes a tangent to the curve at point P.

  $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ (if exists) is called derivative of f(x) at the point 'a'. It is denoted by f'(a).

▶ ALGEBRA OF DERIVATIVES:

- $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$, where c is a constant
- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Product Rule:

- $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot \frac{d}{dx}[g(x)] + g(x) \frac{d}{dx}[f(x)]$

Quotient Rule:

- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$
- If y = f(x) is a given curve then slope of the tangent to the curve at the point (h,k) is given by $\frac{dy}{dx}\Big|_{(h,k)}$ and is denoted by 'm'

MIND MAP

Limits and Derivatives

Limits of Polynomial function

A function *f* is said to be a polynomial function of degree *n*.

$f(x) = a_0 + a_1x + a_2x^2 + - a_nx^n$

where ais are real numbers such that $a_n \neq 0$ for some natural number *n*.

Limit

Limit is used when we have to find value of a function near to some value. To express the limit of a function, we represent it as:

$\lim_{y \to c}$

Derivative at a point

He derivative of *f* at '*a*' is defined by

$$\lim_{(h \to 0)} \frac{f(a+h) - f(a)}{h}$$

Voided this limit exists. Derivative of *f*(*x*) '*a*' is denoted by *f*'(*a*)

$$f'(a) \lim_{(h \to 0)} \frac{f(a+h) - f(a)}{h}$$

Limits of Trigonometric Function

$\lim_{x \to 0} \sin x = 0$

$\lim_{x \to 0} \cos x = 1$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\lim_{x \to 0} \frac{\tan x}{x} = 1$

$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

$\lim_{x \to 0} \frac{\log_e(1+x)}{x} = 1$

$\lim_{x \to 0} \frac{\log_a(1+x)}{x} = \log_e a$

$\lim_{x \to 0} \frac{x^n - a^n}{x - a} = na^{n-1}$

$\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n$

LEFTHAND LIMIT (LHL) at '*a*'

$= \lim_{x \to 0} - f(n) = \lim_{n \to 0} f(a - h)$

RIGHT HAND LIMIT (RHL) at '*a*'

$= \lim_{x \to 0} + f(n) = \lim_{x \to 0} f(a + h)$

Computing Limits

- $\lim_{x \to a}[p(x) + g(x)] = \lim_{x \to a} p(x) + \lim_{x \to a} g(x)$
- $\lim_{x \to a}[p(x) - g(x)] = \lim_{x \to a} p(x) - \lim_{x \to a} g(x)$
- For every real number k, $\lim_{x \to a} [kp(x)] = k \lim_{x \to a} p(x)$
- $\lim_{x \to a} [p(x)q(x)] = \lim_{x \to a} p(x) \times \lim_{x \to a} q(x)$
- $\lim_{x \to a} \frac{p(x)}{q(x)} = \frac{\lim_{x \to a} p(x)}{\lim_{x \to a} q\hat{A} \times \hat{A}}$

Rules of Differentation

$\frac{d}{dx}(cu) = c.u.'$

$\frac{d}{dx}(u^n) = n.u^{n-1}.u'$

$\frac{d}{dx}(u \pm v) = u' \pm v'$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.u' - u.v'}{v^2}$

$\frac{d}{dx}(u.v)\, u.v' + v.u'$

$\frac{d}{dx}(\log_a u) \frac{1}{u \ln a}.u'$

$\frac{d}{dx}(a^u) = a^u. \text{In } a.u'$

$\frac{d}{dx}(\sin u) = \cos u.u' \frac{d}{dx}(\text{coses } u) = -\csc u \cot .u'$

$\frac{d}{dx}(\cos u) = -\sin u.u' \frac{d}{dx}(\sec u) = \sec u \tan u.u'$

$\frac{d}{dx}(\tan u) \sec^2 u.u' \frac{d}{dx}(\cot u) = -\text{coses } u \cot u.u'$

$\frac{d}{dx}(arc \sin u) = \frac{1}{\sqrt{1-u^2}}.u' \frac{d}{dx}(arc \cos u) = \frac{-1}{\sqrt{1-u^2}}.u'$

$\frac{d}{dx}(arc \tan u) = \frac{1}{1+u^2}.u' \frac{d}{dx}(arc \cot u) = \frac{1}{1+u^2}.u'$

$\frac{d}{dx}(arc \text{ coses } u) = \frac{1}{|u|\sqrt{u^2-1}}.u' \frac{d}{dx}(arc \sec u) = \frac{-1}{|u|\sqrt{u^2-1}}.u'$

VERY SHORT ANSWER TYPE QUESTIONS

1. Evaluate $\lim_{x\to 3}\frac{4x+3}{x-2}$

2. Evaluate $\lim_{x\to 2}\frac{x^2+x-2}{x-1}$

3. Evaluate $\lim_{x\to 2}\frac{x^4-16}{x-2}$

4. Evaluate $\lim_{x\to 0}\frac{(1+x)^8-1}{x}$

5. Evaluate $\lim_{x\to 0}\frac{\sin^2 3x}{x^2}$

6. Evaluate $\lim_{x\to 0}\frac{\sin^3 x/2}{x^3}$

7. Evaluate $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

8. Evaluate $\lim_{x\to 0}\frac{2^x-1}{x}$

9. Evaluate $\lim_{x\to 0}\frac{5^x-1}{3^x-1}$

10. Evaluate $\lim_{x\to 0}\frac{3^{-2x}-1}{x}$

11. Evaluate $\lim_{x\to 0}\frac{\log(1-3x)}{x}$

12. Evaluate $\lim_{x\to 0}\frac{7^x-1}{\tan x}$

13. Differentiate $f(x)=x^2+\cos x$

14. If $y = (x+1)(x-2)$, find $\frac{dy}{dx}$

15. If $y = \frac{x^5}{x-3}$, find $\frac{dy}{dx}$

Short Answer Type Questions

16. Evaluate $\lim_{x \to 0} \frac{(1+x)^m - 1}{(1+x)^n - 1}$

17. Evaluate $\lim_{x \to 0} \frac{(\sin 2x) + 3x}{2x + (\tan 3x)}$

18. Evaluate $\lim_{x \to 0} \frac{1 - \cos 2x}{1 - \cos 4x}$

19. If y = $\sin^2 x.\cos^3 x$, then $\frac{dy}{dx}$.

20. If y = sin 2x.cos3x, then $\frac{dy}{dx}$.

21. Differentiate $\frac{\sin x}{x}$ with respect to x.

22. Differentiate $x^3 + 3^3 + 3^x$ with respect to x.

23. Differentiate $\sin^2(x^3 + x - 1) + \frac{1}{\sec^2(x^3 + x - 1)}$ with respect to x.

24. Differentiate $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$ with respect to x.

25. Differentiate $\frac{1}{1 + x^{b-a} + x^{c-a}} + \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{a-c} + x^{b-c}}$ w.r.t to x.

26. Find the derivative of x using first principle method.

27. If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then find the value of k.

28. Find the derivative of $(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$ with respect to x.

29. Differentiate $\frac{x^8 - 1}{x^4 - 1}$ with respect to x.

Differentiate the following with respect to x using First principle method. (For Q. 30 – 35)

30. $\frac{1}{x}$

31. $\sqrt{x}$

32. $\cos(x + 1)$

33. $\sqrt{\sin x}$

34. $\frac{2x + 3}{x + 1}$

35. $x \cos x$

Long Answer Type Questions

Evaluate the following Limits: (For Q. 36 – 58)

36. $\lim_{x \to \infty} \frac{2x^8 - 3x^2 + 1}{x^8 + 6x^5 - 7}$

37. $\lim_{x \to 2} \frac{x^3 - 6x^2 + 11x - 6}{x^2 - 6x + 8}$

38. $\lim_{x \to 0} \frac{1 - \cos 2x}{x \cdot \tan 3x}$

39. $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

40. $\lim_{x \to \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{\frac{\pi}{6} - x}$

41. $\lim_{x \to 0} \frac{\sin x}{\tan x^0}$ (where x^0 represents x degree)

42. $\lim_{x \to 9} \frac{x^{\frac{3}{2}} - 27}{x^2 - 81}$

43. $\lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x - a}$

44. $\lim_{x \to 0} \frac{\cos ax - \cos bx}{1 - \cos x}$

45. $\lim_{x \to a} \frac{\cos x - \cos a}{\cot x - \cot a}$

46. $\lim_{x \to \pi} \frac{1 + \sec^3 x}{\tan^2 x}$

47. $\lim_{x \to 1} \frac{x - 1}{\log_e x}$

48. $\lim_{x \to e} \frac{x - e}{(\log_e x) - 1}$

49. $\lim_{x\to 2}\left[\frac{4}{x^3-2x^2}+\frac{1}{2-x}\right]$

50. $\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

51. $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$

52. $\lim_{x\to 0}\frac{1-\cos x\cdot\sqrt{\cos 2x}}{\sin^2 x}$

53. $\lim_{x\to 0}\frac{6^x-2^x-3^x+1}{\log(1+x^2)}$

54. $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}$

55. $\lim_{x\to \frac{\pi}{4}}\frac{1-\tan x}{1-\sqrt{2}\sin x}$

56. Find the values of a and b if $\lim_{x\to 2} f(x)$ and $\lim_{x\to 4} f(x)$ exists where

$$f(x)=\begin{cases} x^2+ax+b, & 0\le x<2 \\ 3x+2, & 2\le x\le 4 \\ 2ax+5b, & 4<x<8 \end{cases}$$

57. Differentiate the following w.r.t.

(a) $\frac{(x-1)(x-2)(x-3)}{x^2-5x+6}$

(b) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

(c) $\dfrac{x\sin x + \cos x}{x\sin x - \cos x}$

(d) $x \cdot \sin x \cdot e^x$

58. Prove the following statements

(a) If $y = \dfrac{x}{x+2}$, then $\dfrac{dy}{dx} = \dfrac{(1-y)y}{x}$

(b) If $y = e^x \cos x$, then $\dfrac{dy}{dx} = \sqrt{2}\, e^x \cos\left(x + \dfrac{\pi}{4}\right)$

(c) If $y = \dfrac{1-x}{1+x}$, then $\dfrac{dy}{dx} = \dfrac{-2}{(1+x)^2}$

(d) If $xy = 4$, then $x\left(\dfrac{dy}{dx} + y^2\right) = 3y$

CASE STUDY TYPE QUESTIONS

59. Mr. Pradeep has a rectangular plot, which is used for growing vegetables. Perimeter of plot is 50m. Length and width of plot are x m and y m respectively.

Based on the above information, answer the following questions:-

i. Relation between x and y is

(a) $x + y = 50$ (b) $x + y = 100$

(c) $x + y = 25$ (d) $x = y$

ii. Area function, A(x) =

(a) $x^2 - 5$ (b) $25x - x^2$ (c) $x^2 - 25x$ (d) $25 - x$

iii. Derivative of A(x) w.r.t. x [A'(x)] =

(a) 2x (b) –2x (c) 25 – 2x (d) 2x – 25

iv. Value of x for which A'(x) = 0 is

(a)25 (b) 12.5 (c) 5 (d) 0

v. Value of A(x) at x = 12.5 is

(a) 625 (b) 250 (c) 156.25 (d) 144.25

60. Consider the following functions.

$u(x) = \sqrt{x}, \quad v(x) = \cot x, \quad f(x) = u(x) \times v(x)$

$g(x) = \frac{u(x)}{v(x)}$ and $h(x) = \frac{v(x)}{u(x)}$

Based on the above information answer the following :-

i. Derivative of $u(x)$ is

ii. Derivative of $v(x)$ is

iii. Derivative of $f(x)$ is

iv. Derivative of $g(x)$ is

v. Derivative of $h(x)$ is

Multiple Choice Questions

Note: Q.61 – Q.70 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

61. $\lim_{x \to \pi} \frac{\sin x}{x - \pi}$ is -

(a) 1 (b) 2

(c) –1 (d) does not exist.

62. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80,$ then n is -

(a) 2 (b) 3

(c) 4 (d) 5.

63. If $L = \lim_{x \to 1} \frac{x^4 - 1}{x^3 - 1},$ then 3L is -

(a) 2 (b) 3

(c) 4 (d) None of these.

64. $\lim_{x \to 0} \frac{(1+x)^{16} - 1}{(1+x)^4 - 1}$ is -

(a) 0 (b) 4

(c) 8 (d) 16.

65. $\lim_{x \to 1} \frac{x + x^2 + x^3 + x^4 - 4}{x - 1}$ is -

(a) 0 (b) 4

(c) 10 (d) Does not exist.

66. $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$ is -

(a) 0 (b) 1

(c) 2 (d) 4.

67. $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) 1

68. $\lim_{x\to 0}(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x})$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) 1

69. If $y=\sin^4 x+\cos^4 x,\ \text{then}\ \frac{dy}{dx}=$

(a) $4\sin^3x + 4\cos^3x$ (b) $4\sin^3x - 4\cos^3x$

(c) $-\sin 4x$ (d) 0.

70. If $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$ then $\frac{dy}{dx}$ is

(a) y^2 (b) $1 + y^2$

(c) $y^2 - 1$ (d) $1 - y^2$

71. $\lim_{x\to\infty}\frac{\sin x}{x}$ equals.

(a) 0 (b) ∞

(c) 1 (d) does not ewists

72. $\lim_{x-\frac{\pi}{3}}\frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$ is equal to

(a) $\sqrt{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) 0

73. If f(x) = $\frac{x-4}{2\sqrt{x}}$, then f′(1) is

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$

(c) 1 (d) 0

74. If f(n) = $\frac{x^n - a^n}{x-a}$, then f′ (a) is

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) does not exist

75. If $y = \frac{\sin(x+9)}{\cos x}$ then $\frac{dy}{dx}$ at x = 0 is

(a) cos 9 (b) sin 9

(c) 0 (d) 1

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

76. **Assertion:** $\lim_{x \to 0} \frac{\sin ax}{bx} = \frac{a}{b}$

Reason: $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{b}{a} (a, b \neq 0)$

77. **Assertion:** $\lim_{x\to 0}(\cos\text{es}\, x - \cot x) = 0$

Reason: $\lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = -1$

78. **Assertion:** If a and b are non-zero constants, then the derivative of f(x) = ax + b is a.

Reason: If a, b and c are non-zero constants, then the derivative of f(x) = ax^2 + bx + c is ax + b.

79. Let a_1, a_2, a_3, ... a_n be fixed real numbers and define a function f(x) = (x – a_1) (x – a_2) .... (x – a_n), then

Assertion: $\lim_{x\to a_1} f(x) = 0$

Reason: $\lim_{x\to a}(x) = (a-a_1)(a-a_2)...(a-a_n)$, for some a = a_1, a_2, ... a_n.

80. **Assertion:** Suppose f is real valued function, the derivative of f at x is given by f'(x) is given by f'(x) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Reason: If y = f(x) is the function, then derivative of 'f' at any x is denoted by f'(x).

ANSWERS

1. 15	2. 4
3. 32	4. 8
5. 9	6. $\frac{1}{8}$
7. $\frac{1}{2}$	8. $\log 2$

9. $\log_3 5$

10. –2 log 3

11. –3

12. log 7

13. 2x – sinx

14. 2x – 1

15. $\dfrac{4x^5 - 15x^4}{(x-3)^2}$

16. m/n

17. 1 (Hint: divide by x)

18. $\dfrac{1}{4}$ (Hint: $\cos 2\theta = 1 - 2\sin^2\theta$)

19. $\cos^2 x.\sin x(2\cos^2 x - 3\sin^2 x)$

20. 2cos2x.cos3x – 3sin2x.sin3x

21. $\dfrac{x\cos x - \sin x}{x^2}$

22. $3x^2 + 3x.\log 3$

23. 0 (Hint: Use $\sin^2\theta + \cos^2\theta = 1$)

24. 0 (Hint: firstly simplify and then differentiate)

25. 0 (Hint: Simplify and then differentiate)

26. 1

27. $\dfrac{8}{3}$ (Hint: Use $\lim\limits_{x\to a}\dfrac{x^n - a^n}{x-a} = na^{n-1}$)

28. $8x^7$ (Hint: Simplify using identify $(a + b)(a - b) = a^2 - b^2$)

29. $4x^3$ (Hint: Simplify using identify $(a + b)(a - b) = a^2 - b^2$)

30. $\dfrac{-1}{x^2}$

31. $\dfrac{1}{2\sqrt{x}}$

32. –sin(x + 1)

33. $\frac{1}{2}\cot x\sqrt{\sin x}$

34. $\frac{-1}{(x+1)^2}$

35. $\cos x - x\sin x$

36. 2

37. $\frac{1}{2}$

38. $\frac{2}{3}$

39. $\sqrt{2}$

40. – 2

41. $\frac{180^0}{\pi}$

42. $\frac{1}{4}$

43. $\frac{5(a+2)^{\frac{3}{2}}}{2}$

44. $b^2 - a^2$

45. $\sin^3 a$

46. $\frac{-3}{2}$

47. 1

48. e

49. –1

50. $\frac{2}{3\sqrt{3}}$

51. 2cos2

52. $\frac{3}{2}$

53. (log 2) (log 3)

54. $\frac{1}{2}$

55. 2

56. a = –3, b = – 2

57. (a) 1

(b) $8x + 8x^{-9}$

(c) $\frac{-2(x+\sin x\cdot\cos x)}{(x\sin x-\cos x)^2}$

(d) $e^x(x\sin x + x\cos x + \sin x)$

59. i. (c) ii. (b) iii. (c) iv. (b) v. (c)

60. i. $\frac{1}{2}\sqrt{x}$ ii. $-\text{coses}^2\,x$ iii. $\frac{-2x\,\text{coses}^2 x+\cot x}{2\sqrt{x}}$

iv. $\frac{2x \operatorname{cosec}^2 x + \cot x}{2\sqrt{x} \cot^2 x}$ v. $\frac{-(2x \operatorname{cosec}^2 x + \cot x)}{2x^{3/2}}$

61.	(c)	62.	(d)	63.	(c)
64.	(b)	65.	(c)	66.	(c)
67.	(b)	68.	(a)	69.	(c)
70.	(d)	71.	(a)	72.	(c)
73.	(a)	74.	(d)	75.	(a)
76.	(c)	77.	(c)	78.	(c)
79.	(a)	80.	(b)		