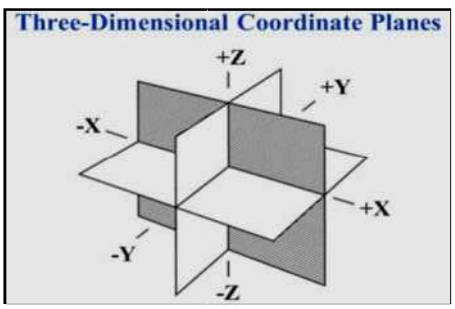
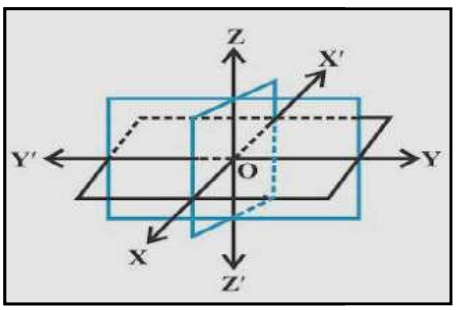
CHAPTER - 11

INTRODUCTION TO THREE-DIMENSIONAL COORDINATE GEOMETRY

KEY POINTS

- Three mutually perpendicular lines X′OX, Y′OY and Z′OZ in space **constitute rectangular coordinate system** which in turn divide the space into eight parts known as **octants** and the lines are known as **Coordinate axes.**



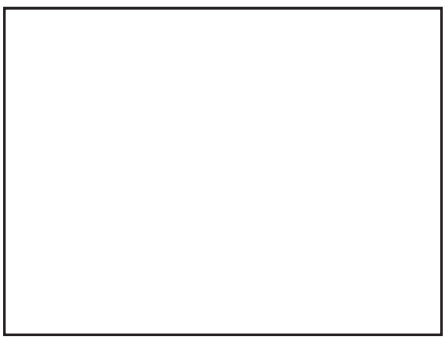


- ❖ **Coordinate axes:** XOX', YOY', ZOZ' are respectively called x-axis, y-axis and z-axis.
- ❖ **Coordinate planes:** XOY, YOZ, ZOX or XY, YX, ZX planes
- ❖ **Octants:** XOYZ, X'OYZ, X'OY′Z, XOY'Z, XOYZ', X'OYZ′, X′OY'Z' and XOY′Z' denoted as **I**, **II**, ….. **VIII** octant respectively.
- ❖ Coordinates of a points lying on x-axis, y-axis and z-axis are of the form (x,0,0), (0,y,0),(0,0,z) respectively.

- ❖ The signs of coordinates in eight octants are as follows:
  (i) (+ + +) (iii) (- - +) (v) (+ + -) (vii) (- - -)
  (ii) (- + +) (iv) (+ - +) (vi) (- + -) (viii) (+ - -)

- ❖ Coordinates of a points lying on xy-plane, yz-plane and xz-plane are of the form (x,y,0), (0,y,z),(x,0,z) respectively.
- ❖ The reflection of the point (x, y, z) in xy-plane, yz-plane and xz-plane is (x, y, –z), (–x, y,z) and (x, –y, z) respectively.

- ❖ Absolute value of the Coordinates of a point P (x, y, z) represents the perpendicular distances of point P from three coordinate planes YZ, ZX and XY respectively.



- ● The distance between the point A(x_1, y_1, z_1) and B(x_2, y_2, z_2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

VERY SHORT ANSWER TYPE QUESTIONS

1. What will be the image of (–1, 2, –3) in XZ plane.
2. What will be the image of (–1, 2, –3) in YZ plane.
3. In which octant The Point P (–5, 4, –3), lies?
4. If a < 0, b > 0 & c < 0, in which octant the Point P (a, b, –c) lies.

5. Find the perpendicular distance of the point P(- 6, 7, –8) from xy-plane.

6. Find the perpendicular distance of the point P(- 3, 5, –12) from x-axis.

7. Find the perpendicular distance of the point P(- 3, 4, –5) from z-axis.

8. Find the coordinates of foot of perpendicular from (3, 7, 9) on y-axis.

9. If the distance between the points (a, 2, 1) and (1, –1, 1) is 5, then find the sum of all possible value of a.

10. Name the axis formed by intersection of two planes xy-plane and yz-plane.

11. Find Distance of the point (3, 4, 5) from the origin (0, 0, 0).

12. If $(c - 1) > 0$, $(a + 2) < 0$ and $b > 0$ then the point P(a, –b, c) lies in which octant?

13. What are the coordinates of the vertices of a cube whose edge is 2 unit, one of whose vertices coincides with the origin and the three edges passing through the origin coincides with the positive direction of the axes through the origin?

14. Let A, B, C be the feet of perpendiculars from point P(1, –2, –3) on the xy-plane, yz-plane and xz-plane respectively. Find the coordinates of A, B, C.

15. If a parallelepiped is formed by planes drawn through the point (5,8,10) and (3,6,8) parallel to the coordinates planes, then find the length of the diagonal of the parallelepiped.

16. Find the length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 13,10 and 8 unit.

17. Show that points (4, –3, –1), (5, –7, 6) and (3, 1, –8) are collinear.

18. Find the point on y–axis which is equidistant from the point (3,1,2) and (5, 5, 2).

19. Determine the point in yz plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

20. Find the length of the medians of the triangle with vertices A(0, 0, 3), B(0, 4, 0) and C (5, 0, 0).

21. If the extremities (end points) of a diagonal of a square are (1,– 2, 3) and (2,–3, 5) then find the length of the side of square.

22. Three consecutive vertices of a parallelogram ABCD are A(6,–2,4) B(2, 4,–8), C(–2, 2, 4). Find the coordinates of the fourth vertex.

23. If the points A(1, 0, –6), B(3, p, q) and C(5, 9, 6) are collinear, find the value of p and q.

24. Show that the point A(1, 3, 0), B(–5, 5,, 2), C(–9, –1, 2) and D(–3, –3, 0) are the vertices of a parallelogram ABCD, but it is not a rectangle. (Hint: diagonals are not equal)

25. Describe the vertices and edges of the rectangular parallelepiped with one vertex (3, 5, 6) placed in the first octant with one vertex at origin and edges of parallelepiped lie along x, y and z-axis.

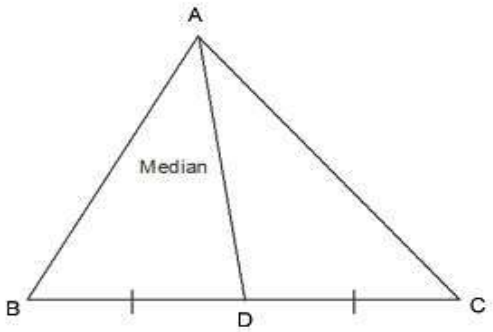
26. Find the coordinates of the point which is equidistant from the point (3, 2, 2), (–1, 2, 2), (4, 5, 6) and (2, 1, 2).

27. Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) form a right angled isosceles triangle.

28. Show that the points (5, -1, 1), (7, -4, 7), (1, -6, 10) and (-1, -3, 4) are the vertices of a rhombus.

CASE STUDY TYPE QUESTIONS

29. Consider a ΔABC with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$. AD, BE and CF are medians of ΔABC.



Based on the above information, answer the following questions:-

i. Coordinates of Point D are?

(a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ (b) $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right)$

(c) $\left(\frac{x_3+x_1}{2}, \frac{y_3+y_1}{2}, \frac{z_3+z_1}{2}\right)$ (d) None of these

ii. A point G divides AD in 2 : 1, the coordinates of G are

(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

(b) $\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}, \frac{z_1+2z_2}{3}\right)$

(c) $\left(\frac{x_2+2x_1}{3}, \frac{y_2+2y_1}{3}, \frac{z_2+2z_1}{3}\right)$

(d) None of these

iii. For ΔABC, G is

(a) Incentre (b) Circumcentre

(c) Centroid (d) Orthocentre

iv. G divides BE in ratio

(a) 1 : 2 (b) 2 : 1

(c) 3 : 1 (d) 1 : 3

v. If ΔABC is equilateral, then coordinates of circumcentre are

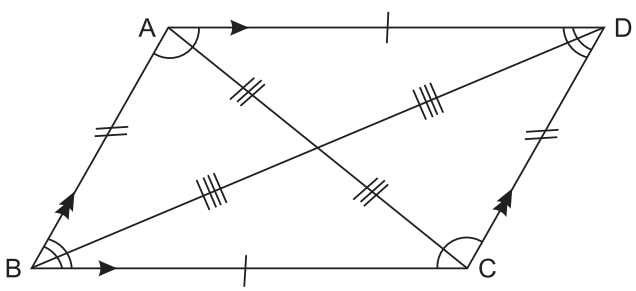
(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

(b) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}, \frac{z_1+z_2+z_3}{2}\right)$

(c) $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right)$

(d) None of these

30. ABCD is a field in shape of parallelogram coordinate of A, B and C are (3, –1, 2), (1, 2, –4) and (–1, 1, 2) resp.



Based on the above information answer the following :-

i Coordinates of mid point of AC be

ii. Coordinates of D be

iii. Length of side BC is

iv. Coordinates of centroid G of ΔABC be

v. Length of AC is

Multiple Choice Questions

31. A point on Z-plane which is equidistant from the points (1, -1, 0), (2, 1, 2), (3, 2, -1) is

(a) $\left(\frac{1}{5},0,\frac{31}{10}\right)$ (b) $\left(\frac{1}{10},0,\frac{31}{5}\right)$

(c) $\left(\frac{31}{10},0,\frac{1}{5}\right)$ (d) $\left(\frac{31}{5},0,\frac{1}{10}\right)$

32. Lengths of medians of triangle ABC with vertices

A(0, 0, 2), B(0, 4, 0) and C(8, 0, 0) are:

(a) $2\sqrt{6},\sqrt{33},\sqrt{69}$ (b) 2, 4, 8

(c) 8, 4, 2 (d) $2\sqrt{5},10,2\sqrt{17}$

33. A point on y-axis which is at a distance of $\sqrt{10}$ from the point (1, 2, 3) is

(a) (2, 0, 2) (b) (0, 2, 2)

(c) (2, 2, 2) (d) (0, 2, 0)

34. The locus of a point for which y = 0, z = 0 is

(a) x-axis (b) y-axis

(c) z-axis (d) y and z-axes

35. A line is parallel to xy-plane if all points on the line have equal

(a) x-coordinates (b) y-coordinates

(c) z-coordinates (d) x and y-coordinate

36. x-axis is the intersection of two planes

(a) xy and xz (b) yz and zx

(c) xy and yz (d) none of these

37. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of a is

(a) 5 (b) ± 5

(c) – 5 (d) None of these

38. The point (2, 3, –4) lies in the

(a) First octant (b) Second octant

(c) Fifth octant (d) Seventh octant

39. x = a. represents a plane parallel to

(a) xy – plane (b) yz-plane

(c) xz-plane (d) none of these

40. The distance between the point (a, b, c) and (0, 0, -c) is

(a) $\sqrt{a^2 + b^2}$ (b) $\sqrt{a^2 + b^2 + c^2}$

(c) $\sqrt{a^2 + b^2 + 2c^2}$ (d) $\sqrt{a^2 + b^2 + 4c^2}$

Direction: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below:

(a) Assertion is correct, reason is correct; reason is not a explanation for assertion.
(b) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reason is correct.

41. **Assertion:** If three vertices of a parallelogram ABCD are A(3, –1, 2) B(1, 2, –4) and C(–1, 1, 2), then the fourth vertex is (1, –2, 8).

    Reason: Diagonals of a parallelogram bisect each other and mid-point of AC and BD coincide.

42. Assertion: The distance of a point P(x, y, z) from the origin O(0, 0, 0) is given by OP = $\sqrt{x^2 + y^2 + z^2}$.

Reason: A point is on the x-axis. Its y-coordinate and z-coordinate are 0 and 0 respectively.

43. Assertion: Coordinates (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.

Reason: Opposite sides of a parallelograom are equal and diagonals are not equal.

44. Assertion: If P(x, y, z) is any point in the space, then x, y and z are perpendicular distance from YZ, ZX and XY-planes, respectively.

Reason: If three planes are drawn parallel to YZ, ZX and XY-planes such that they intersect X, Y and Z-axes at (x, 0, 0), (0, y, 0) and (0, 0, z), then the planes meet in space at a point P(x, y, z).

45. Assertion: The distance between the points P(1, – 3, 4) and Q(–4, 1, 2) is $\sqrt{5}$ units.

Reason: PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

ANSWERS

1. (–1, –2, –3)
2. (1, 2, –3)
3. Octant VI (OX' YZ')
4. Octant II (OX' YZ)
5. 8 units
6. 13 units
7. 5 units
8. (0, 7, 0)
9. 5 + (–3) = 2
10. Y-axis
11. $5\sqrt{2}$
12. Octant III (OX' Y'Z)
13. (2, 0, 0) , (2, 2, 0) , (0, 2, 0) , (0, 2, 2) , (2, 0, 2), (0, 0, 2) , (2, 2, 2)
14. (4, –3, 0) , (0, –3, –5) , (4, 0, –5)
15. $2\sqrt{3}$
16. $\sqrt{333}$
18. (0, 5, 0)
19. (0, 1, 3)
20. $\frac{\sqrt{77}}{2}, \frac{7\sqrt{2}}{2}, \frac{5\sqrt{5}}{2}$
21. $\sqrt{3}$
22. (2, –4, 16)

23. $p = \frac{9}{2}, q = 0$

25. (0, 0, 0), (3, 0, 0), (3, 5, 0), (0, 5, 0), (0, 5, 6)
(0, 0, 6), (3, 0, 6), (3, 5, 6), $\sqrt{61}$, $\sqrt{45}$, $\sqrt{34}$

26. (1, 3, 5)

27. [Hint: length of three side of triangle is $3\sqrt{2}, 3\sqrt{2}, 6$]

28. [Hint: each side= 7 units]

29. i. (b) ii. (a) iii. (c) iv. (b) v. (a)

30. i. (1, 0, 2) ii. (1, –2, 8) iii. $\sqrt{41}$ iv. (1, 2/3, 0) v. $2\sqrt{5}$

31. (c)	32. (a)	33. (d)
34. (a)	35. (c)	36. (a)
37. (b)	38. (c)	39. (b)
40. (d)	41. (a)	42. (b)
43. (a)	44. (d)	45. (d)