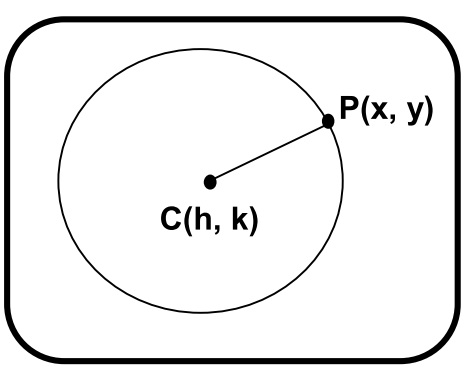
CHAPTER - 10

CONIC SECTIONS

KEY POINTS

- The curves obtained by slicing the cone with a plane not passing through the vertex are called conic sections or simply conics.
- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions.
- A conic is the locus of a point which moves in a plane, so that its distance from a fixed point bears a constant ratio to its distance from a fixed straight line.
- The fixed point is called focus, the fixed straight line is called directrix, and the constant ratio is called eccentricity, which is denoted by '*e*'.
- **Circle:** It is the set of all points in a plane that are equidistant from a fixed point in that plane

  Equation of circle: $(x - h)^2 + (y - k)^2 = r^2$ where Centre (h, k), radius = r



C (h, k)

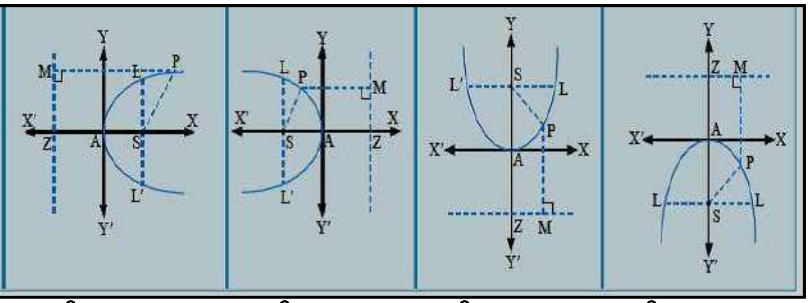
CP = CONSTANT = r

❖ **Parabola:** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line $\frac{c}{a} = 1$ $e = 1$

	$y^2 = 4ax$ Parabola towards right	$y^2 = -4ax$ Parabola towards left	$x^2 = 4ay$ Parabola opening upwards	$x^2 = -4ay$ Parabola opening downwards
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	(a, 0)	(−a, 0)	(0, a)	(0, −a)
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Equation of directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Length of latus rectum	4a	4a	4a	4a

Note: In the standard equation of parabola, a > 0.

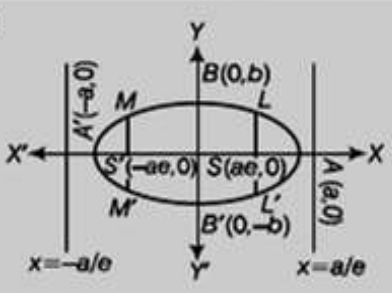
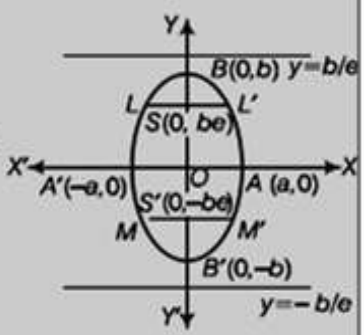
Figure



$y^2 = 4ax$ $y^2 = -4ax$ $x^2 = 4ay$ $x^2 = -4ay$

Note: In the figure above, A represents the vertex, S represents the Focus, LL' represents the Latus Rectum and Line MZ represents the Directrix to the parabola.

- **Latus Rectum:** A chord through focus perpendicular to axis of parabola is called its latus rectum.
- **Ellipse:** It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points. $\frac{c_1 + c_2}{a} > 1$ $e = 1$

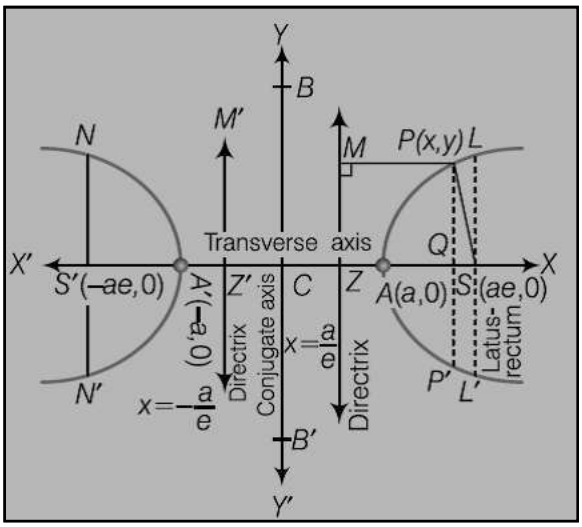
Standard equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 (a>b)$ (Horizontal form of an ellipse)	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 (a<b)$ (Vertical form of an ellipse)
Shape of the ellipse		
Centre	(0, 0)	(0, 0)
Equation of major axis	$y = 0$	$x = 0$
Equation of minor axis	$x = 0$	$y = 0$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Eccentricity	$e = \sqrt{\frac{a^2 - b^2}{a^2}}$	$e = \sqrt{\frac{b^2 - a^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

Note: If e = 0 for an ellipse then b = a and equation of ellipse will be converted in equation of the circle. Its eq. will be $x^2 + y^2 = a^2$. It is called auxiliary circle. For auxiliary circle, diameter is equal to length of major axis and e = 0.

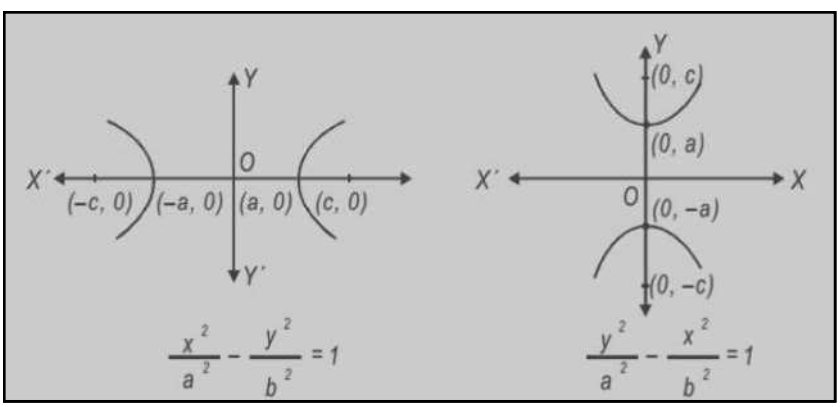
- **Latus rectum:** Chord through foci perpendicular to major axis called latus rectum.
- **Hyperbola:** It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.

	Hyperbola	**Conjugate hyperbola**
Standard equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$
Centre	$(0, 0)$	$(0, 0)$
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = 0$	$y = 0$
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Eccentricity	$e = \sqrt{\frac{a^2 + b^2}{a^2}}$	$e = \sqrt{\frac{a^2 + b^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

- **STANDARD HYPERBOLA:**



- **STANDARD HYPERBOLA (CONJUGATE HYPERBOLA):**



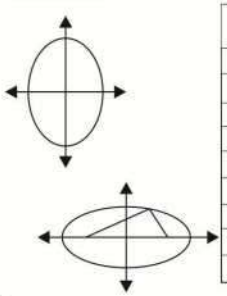
- **Latus Rectum:** Chord through foci perpendicular to transverse axis is called latus rectum.

  If $e = \sqrt{2}$ for hyperbola, then hyperbola is called rectangular hyperbola.

  For $e = \sqrt{2}$ then b = a and eq. of its hyperbola will be $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$.

MIND MAP

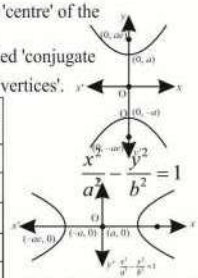
- An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is constant.
- The two fixed points are called the 'foci' of the ellipse.
- The midpoint of line segment joining foci is called the 'centre' of the ellipse.
- The line segment through the foci of the ellipse is called 'major axis'.
- The line segment through centre & perpendicular to major axis is called minor axis.



Forms of ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
	$a > b$	$a > b$
Equation of major axis	$y = 0$	$x = 0$
Length of major axis	$2a$	$2a$
Equation of Minor axis	$x = 0$	$y = 0$
Leright of Minor axis	$2b$	$2b$
Directrices	$\frac{2b^2}{a}$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
Equation of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Length of latus rectum	(0, 0)	(0, 0)
Centre		

- A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
- The two fixed points are called the 'foci' of the hyperbola.
- The mid-point of the line segment joining the foci is called the 'centre' of the hyperbola.
- The line through the foci is called 'transverse axis'.
- Line through centre and perpendicular to transverse axis is called 'conjugate axis'.
- Points at which hyperbola intersects transverse axis are called 'vertices'.

Forms of the hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = a$	$y = a$
Length of transverse axis	$2a$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm ae)$
Equation of latus rectum	$x = \pm ae$	$x = \pm ae$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Centre	(0, 0)	(0, 0)

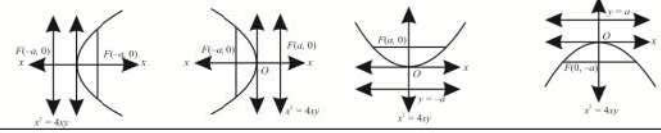


Here, $b^2 = a^2(e^2 - 1)$, $e > 1$

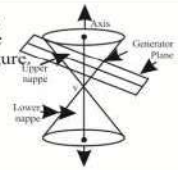
- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane. Fixed line is called 'directrix' of parabola. Fixed point F is called the 'focus'. A line through focus & perpendicular to directrix is called 'axis'. Point axis intersection of parabola with axis is called 'vertex'.

Main facts about the parabola

Forms of Parabolas	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ax$	$x^2 = -4ax$
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directix	$x = -a$	$x = a$	$y = -a$	$y = a$
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	$(a, 0)$	$(-a, 0)$	$(0, -a)$	$(0, -a)$
Lenght of latus rectum	$4a$	$4a$	$4a$	$4a$
Equations of latus rectum	$x = a$	$x = -a$	$y = a$	$y = -a$



Circles, ellipses, parabolas and hyperbolas are known as conic sections because they can be obtained as intersections of plane with a double napped right circular cone α. From the given figure,
(i) Section will represent circle, if $\beta = 90°$
(ii) Section will represent an Ellipse, if $\alpha < \beta < \pi/2$
(iii) Section will represent a parabola if if $\alpha = \beta$
(iv) Section will represent a hyperbola if $0 \le \beta < \alpha$



A circle is a set of all points in a plane that are equidistant from a fixed point in the plane. The fixed point is called the 'centre' of the circle and the distance from the centre to a point on the circle is called the 'radius' of the circle.

The equation of a circle with centre (h, k) and the radius r is

$(x-h)^2 + (y-k)^2 = r^2$

The general equation of circle in $x^2 + y^2 + 2gx + 2fy + c = 0$

its centre is $(-g, -f)$ and radius $r = \sqrt{g^2 + f^2 - c}$

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the centre of the circle $3x^2 + 3y^2 + 6x – 12y – 6 = 0$.

2. Find the radius of the circle $3x^2 + 3y^2 + 6x – 12y –15 = 0$.

3. Find the equation of circle whose end points of one of its diameter are (– 2, 3) and (0, –1).

4. If parabola $y^2 = px$ passes through point (2, –3), then, find the length of latus rectum.

5. Find the coordinates of focus of parabola $3y^2 = 8x$.

6. Find the equation of the circle which passes through the point (4, 6) and has its centre at (1, 2).

7. Find the equation of the ellipse having foci (0, 3), (0, –3) and minor axis of length 8.

8. Find the length of the latus rectum of the ellipse $3x^2 + y^2 = 12$.

9. Find the eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci.

10. If the lines $5x + 12y = 3$ and $10x + 24y – 58 = 0$ are tangents to a circle, then find the radius of the circle.

11. Find the length of major and minor axis of the following ellipse, $16x^2 + 25y^2 = 400$.

12. Find the eqn. of hyperbola satisfying given conditions foci (±5, 0) and transverse axis is of length 8.

13. Find the coordinates of points on parabola $y^2 = 8x$ whose focal distance is 4.

14. Find the distance between the directrices to the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$.

15. If the eccentricity of the ellipse is zero. Then show that ellipse will be a circle.

16. If the eccentricity of the hyperbola is $\sqrt{2}$. Then find the general equation of hyperbola.

17. A circle is circumscribed on an equilateral Triangle ABC where AB = 6 cm. The area of the Circumcircle is $K\pi$ cm^2. Find the value of K.

SHORT ANSWER TYPE QUESTIONS

18. Find equation of an ellipse having vertices (0, ± 5) and foci (0, ± 4).

19. If the distance between the foci of a hyperbola is 16 and its eccentricity is 2, then obtain the equation of a hyperbola.

20. Find the equation for the ellipse that satisfies the given condition Major axis on the *x*-axis and passes through the points (4, 3) and (6, 2).

21. If one end of a diameter of the circle $x^2 + y^2 – 4x – 6y + 11 = 0$ is (3, 4), then find the coordinates of the other end of diameter.

22. Find the equation of the ellipse with foci at (± 5, 0) and x = 18 as one of the directrices.

23. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, find the equation of the hyperbola if its eccentricity is 2.

24. Find the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which passes through the points (3, 0) and $(3\sqrt{2}, 2)$.

25. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

26. Find equation of circle concentric with circle $4x^2 + 4y^2 – 12x – 16y – 21 = 0$ and of half its area.

27. Find the equation of a circle whose centre is at (4, –2) and $3x – 4y + 5 = 0$ is tangent to circle.

28. If equation of the circle is in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$ then prove that its centre and radius will be (–g, –f) and $\sqrt{g^2+f^2-c}$ respectively. (Hint: Complete the square and compare with standard formula)

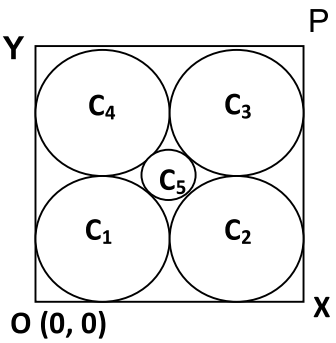
29. If the end points of a diameter of circle are (x_1, y_1) and (x_2, y_2) then show that equation of circle will be $(x – x_1)(x – x_2) + (y – y_1)(y – y_2) = 0$. (Hint: Angle in semicircle is of 90°)

30. Find the equation of the circle which touches the lines $x = 0$, $y = 0$ and $x = 2c$ and $c > 0$.

31. Find the equation of the set of all points the sum of whose distance from A(3,0) and B(9,0) is 12 unit. Identify the curve thus obtained.

32. Find the equation of the set of all points such that the difference of their distance from (4,0) and (–4,0) is always equal of 2 unit. Identify the curve thus obtained.

33. If OXPY is a square of Side 4 cm in First Quadrant, where O is the origin. (OY and OX lie on y-axis and x-axis respectively). Find the equation of the circle C_1, C_2, C_3, $C_4$and C_5.



LONG ANSWER TYPE QUESTIONS

34. Prove that the points (1, 2), (3, – 4), (5, – 6) and (11, – 8) are concyclic.

35. A circle has radius 3 units and its centre lies on the line y = x – 1. If it is passes through the point (7, 3) then find the equations of the circle.

36. Find the equation of the circle which passes through the points (20, 3), (19, 8) and (2, – 9). Find its centre and radius.

37. Find the equation of circle having centre (1, – 2) and passing through the point of intersection of the lines 3x + y = 14 and 2x + 5y = 18.

38. Show that the points A(5,5), B(6,4), C(–2,4) and D(7,1) all lies on the circle. Find the centre, radius and equation of circle.

39. Find the equation of the ellipse in which length of minor axis is equal to distance between foci. If length of latus rectum is 10 unit and major axis is along the x axis.

40. Find the equation of the hyperbolas whose axes (transverse and conjugate axis) are parallel to x axis and y axis and centre is origin such that Length of latus rectum length is 18 unit and distance between foci is 12 unit.

41. Prove that the line $3x + 4y + 7 = 0$ touches the circle $x^2 + y^2 – 4x – 6y – 12 = 0$. Also find the point of contact.

42. Find the equations of tangents to the circle

(a) $x^2 + y^2 – 2x – 4y – 4 = 0$ which are parallel to $3x – 4y – 1 = 0$

(b) $x^2 + y^2 – 4x – 6y – 12 = 0$ which are perpendicular to $4x + 3y = 7$

43. Find the equation of Circle in each of the following cases:

(a) Touches both the coordinate axes in first quadrant and having radius = 1 unit

(b) Touches both the coordinate axes in second quadrant and having radius = 2 units

(c) Touches both the coordinate axes in third quadrant and having radius = 3 units

(d) Touches both the coordinate axes in fourth quadrant and having radius = 4 units

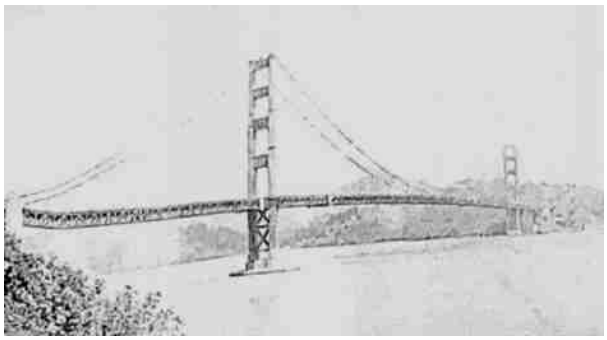
(e) Touches the x-axis at origin and having radius = 5 units

(f) Touches the y-axis at origin and having radius = 6 units

CASE STUDY TYPE QUESTIONS

44. A beam is supported at its ends by supports which are 12m apart. Since the load is concentrated at its centre, there is a

deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola.



Based on the above information answer the following :-

i. How for from the centre is deflection of 1cm?

(a) $2\sqrt{6}$ m (b) $3\sqrt{6}$ m

(c) $2\sqrt{3}$ m (d) $4\sqrt{3}$ m

ii. What will be the equation of parabola?

(a) $x^2 = 240000y$ (b) $x^2 = 120000y$

(c) $x^2 = 160000y$ (d) $x^2 = 100000y$

iii. At a distance of 2m from the centre, what will be the deflection of the beam?

(a) $\frac{3}{2}$ (b) $\frac{8}{3}$

(c) $\frac{4}{3}$ (d) $\frac{1}{5}$

iv. What is the length of latus rectum of the parabola?

(a) 100000 (b) 120000

(c) 130000 (d) 140000

v. What is the difference of deflection of beam at a distance of 1m and 2m from the centre?

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$

(c) $\frac{1}{4}$ (d) $\frac{3}{7}$

45. A window is in the shape of parabola with a triangle inscribed in it. The triangle is formed in such a way that the verticles of triangle coincides with vertex of parabola and end points of latus rectum. The equation of parabola is given by $x^2 = 24y$.

What are the vertices of triangle

i. What are the verticle of triangle

ii. Find the length of altitude of the triangle -

iii. Find the area of the triangle?

iv. Find the length of the longest side of the triangle?

v. Find the length of latus rectum of the parabola?

Multiple Choice Questions

46. The equation of the circle which passes through the points of intersection of the circles $x^2 + y^2 – 6x = 0$ and $x^2 + y^2 – 6y = 0$ and has its centre at (3/2, 3/2) is -

(a) $x^2 + y^2 + 3x + 3y + 9 = 0$

(b) $x^2 + y^2 + 3x + 3y = 0$

(c) $x^2 + y^2 – 3x – 3y = 0$

(d) $x^2 + y^2 – 3x – 3y + 9 = 0$.

47. The centre of circle inscribed in square formed by the lines $x^2 – 8x + 12 = 0$ and $y^2 – 14y + 45 = 0$ -

(a) (4,9) (b) (9,4)

(c) (7,4) (d) (4,7).

48. Value of p, for which the equation $x^2 + y^2 – 2px + 4y – 12=0$ represent a circle of radius 5 units is -

(a) 3 (b) – 3

(c) both (a) & (b) (d) Neither (a) nor (b).

49. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is ‘e’ then the value of ‘5e’ is -

(a) 3 (b) 4

(c) 2 (d) 1.

50. The centre of the circle $x^2 + y^2 – 6x + 4y – 12 = 0$ is (a, b) then (2a + 3 b) is -

(a) 0 (b) 2

(c) 3 (d) 5.

51. The radius of the circle $x^2 + y^2 – 6x + 4y – 12 = 0$ is -

(a) 1 (b) 2

(c) 3 (d) 5.

52. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 8y$ to the ends of its latus rectum is -

(a) 4 sq. units (b) 8 sq. units

(c) 12 sq. units (d) 16 sq. units.

53. Match the following:

	COLUMN 1 Conic		**COLUMN 2 Eccentricity**
A	CIRCLE	P	$e < 1$
B	PARABOLA	Q	$e > 1$
C	ELLIPSE	R	$e = 0$
D	HYPERBOLA	S	$e = 1$

Which one of the following is true?

A → P, B→ Q, C→ R, D → S

A → S, B→ Q, C→ R, D → P

A → Q, B→ S, C→ R, D → P

A → R, B→ S, C→ P, D → Q

54. At what point on the parabola $x^2 = 9y$ is the abscissa three times that of ordinate

(a) (1, 1) (b) (3, 1)

(c) (–3, 1) (d) (–3, –3)

55. The equation of parabola with vertex at origin and axis on x-axis and passing through point (2, 3) is

(a) $y^2 = 9x$ (b) $y^2 = \frac{9x}{2}$

(c) $y^2 = 2x$ (d) $y^2 = \frac{2x}{9}$

56. If the centroid of an equilateral triangle is (1, 1) and its one vertex is (–1, 2) then the equation of its circumcirle is

(a) $x^2 + y^2 – 2x – 2y – 3 = 0$

(b) $x^2 + y^2 + 2x – 2y – 3 = 0$

(c) $x^2 + y^2 + 2x + 2y – 3 = 0$

(d) none of these

57. If the circle $x^2 + y^2 = a$ and $x^2 + y^2 – 6x – 8y + 9 = 0$ touch externally, then a =

(a) 1 (b) –1

(c) 21 (d) 16

58. The area of the triangle formed by the line joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum is

(a) 12 sq units (b) 16 sq units

(c) 18 sq units (d) 24 sq units

59. The eccentricity of the ellipse, if the distance between the foci is equal to the length of the latus rectum is

(a) $\frac{\sqrt{5}-1}{2}$ (b) $\frac{\sqrt{5}+1}{2}$

(c) $\frac{\sqrt{5}-1}{4}$ (d) None of these

60. If e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ then, the relation between, e_1 and e_2 is

(a) $3e_1^2 + e_2^2 = 2$ (b) $e_1^2 + 2e_2^2 = 3$

(c) $2e_1^2 + e_2^2 = 3$ (d) $e_1^2 + 3e_2^2 + 2$

Directions: Each of these questions contains two statements. Assertion and Reason. Each of these questions also has four alternative choices. Only one of which is the correct answer. You have to select of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct, reason is a correct explanation for asserction.

(b) Assertion is correct, reason is correct, reason is not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

61. Parabola is symmetric with respect to the axis of the parabola.

Assertion: If the equation has a term y^2, then the axis of symmetry is along the x-axis.

Reason: If the equation has a term x^2, then the axis of symmetry is along the x-axis.

62. Let the centre of an ellipse is at (0, 0)

Assertion: If major axis is on the y-axis and ellipse passes thorught the points (3, 2) and (1, 6), then the equation of ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$

Reason: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ is an equation of ellipse if major axis is along y-axis. (if a > b)

63. **Assertion:** Centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is (3, – 2)

Reason: The coordinates of the centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are ($-\frac{1}{2}$ coefficient of x, $-\frac{1}{2}$ coefficient of y)

64. **Assertion:** Radius of the circle $2x^2 + 2y^2 + 3x + 4y + \frac{9}{8} = 0$ is 1.

Reason: Radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $\sqrt{\left(\frac{1}{2}\text{coeff. of } x\right)^2+\left(\frac{1}{2}\text{coeff. of } y\right)^2-\text{constant term}}$

ANSWERS

1. (–1, 2)
2. $2\sqrt{5}$ Units
3. $x^2 + y^2 + 2x - 2y - 3 = 0$ (Hint: Mid-point of diameter is center)
4. 4.5 units
5. $\left(\frac{2}{3}, 0\right)$
6. $(x - 1)^2 + (y - 2)^2 = 25$
7. $\frac{x^2}{16}+\frac{y^2}{25}=1$
8. $\frac{4\sqrt{3}}{3}$
9. e = $2\sqrt{3}$

10. 2 units (Hint: distance between two parallel tangents is the length of diameter.)

11. Length of Major Axis = 10, Length of Major Axis = 8

12. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

13. (2, ±4)

14. 18 (Hint: Distance between two directrices is $\frac{2a}{e}$)

16. $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$

17. K = 12

18. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

19. $x^2 - y^2 = 32$ or $y^2 - x^2 = 32$

20. $\frac{x^2}{52} + \frac{y^2}{13} = 1$

21. (1, 2)

22. $\frac{x^2}{90} + \frac{y^2}{65} = 1$

23. $\frac{x^2}{4} - \frac{y^2}{12} = 1$

24. $e = \frac{\sqrt{13}}{3}$

25. $e = \frac{\sqrt{3}}{2}$

26. $2x^2 + 2y^2 - 6x + 8y + 1 = 0$

27. $x^2 + y^2 - 8x + 4y - 5 = 0$

30. $x^2 + y^2 - 2cx \pm 2cy + c^2 = 0$

31. $3x^2 + 4y^2 = 36$, Ellipse

32. $15x^2 - y^2 = 15$, Hyperbola

33. C_1: $(x - 1)^2 + (y - 1)^2 = 1$

C_2: $(x - 3)^2 + (y - 1)^2 = 1$

C_3: $(x-3)^2 + (y-3)^2 = 1$

C_4: $(x-1)^2 + (y-3)^2 = 1$

C_5: $(x-2)^2 + (y-2)^2 = \left(\sqrt{2}-1\right)^2$

35. $x^2 + y^2 - 8x - 6y + 16 = 0$ or $x^2 + y^2 - 14x - 12y + 76 = 0$

36. $x^2 + y^2 - 14x - 6y - 111 = 0$ Centre (7, 3), Radius = 13 units

37. $(x-1)^2 + (y+2)^2 = 25$

38. $x^2 + y^2 - 4x - 2y - 20 = 0$ Centre(2, 1), Radius = 5 units

39. $x^2 + 2y^2 = 100$ 40. $3x^2 - y^2 = 27$

41. Point of contact = (–1, –1)

42. (a) $3x - 4y - 10 = 0$ or $3x - 4y + 20 = 0$

(b) $3x - 4y + 31 = 0$ or $3x - 4y - 19 = 0$

43. (a) $(x-1)^2 + (y-1)^2 = 1$ (b) $(x+2)^2 + (y-2)^2 = 4$

(c) $(x+3)^2 + (y+3)^2 = 9$ (d) $(x-4)^2 + (y+4)^2 = 16$

(e) $x^2 + (y \pm 5)^2 = 25$ (f) $(x \pm 6)^2 + y^2 = 36$

44. i. (a) ii. (b) iii. (b) iv. (b) v. (c)

45. i. (0, 0), (±12, 6) ii. 6 units iii. 72 sq units iv. 24 units v. 24 units

46. (c)	47. (d)	48. (c)	49. (b)
50. (a)	51. (d)	52. (b)	53. (d)
54. (b)	55. (b)	56. (a)	57. (a)
58. (c)	59. (a)	60. (c)	61. (c)
62. (b)	63. (a)	64. (a)	