

CHAPTER - 9

STRAIGHT LINES

KEY POINTS

- Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

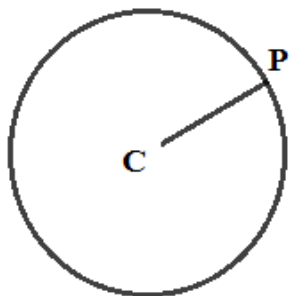
- Let the vertices of a triangle ABC are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Then area of triangle

$$\text{ar}(\triangle ABC) = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Note: Area of a triangle is always positive. If the above expression is zero, then a triangle is not possible. Thus the points are collinear.

- LOCUS:** When a variable point $P(x, y)$ moves under certain condition then the path traced out by the point P is called the locus of the point.

For example: Locus of a point P , which moves such that its distance from a fixed point C is always constant, is a circle.

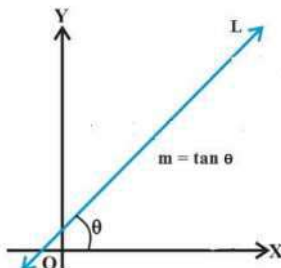


$CP = \text{constant}$

- A line is also defined as the locus of a point satisfying the condition $ax + by + c = 0$ where a, b, c are constants.

- **Slope of a straight line:**

If θ is the inclination of a line then $\tan\theta$ is defined as slope of the straight line L and denoted by m



$$m = \tan\theta, \theta \neq 90^\circ$$

If $0^\circ < \theta < 90^\circ$ then $m > 0$ and

$90^\circ < \theta < 180^\circ$ then $m < 0$

Note-1: The slope of a line whose inclination is 90° is not defined. Slope of x-axis is zero and slope of y-axis is not defined

Note-2: Slope of any horizontal line i.e. \parallel to x-axis is zero. Slope of a vertical line i.e. \parallel to y-axis is not defined.

- Three points A, B and C lying in a plane are collinear, if slope of AB = Slope of BC.
- Slope of a line through given points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- **Intercept:** There are two types of intercepts x-intercept and y-intercept. The x-intercept is the x-coordinate of the point where line cut x axis while y-intercept is the coordinate of the point where line cut y axis.
- Two lines are parallel to each other if and only if their slopes are equal.
i.e., $l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$.

- Two lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.

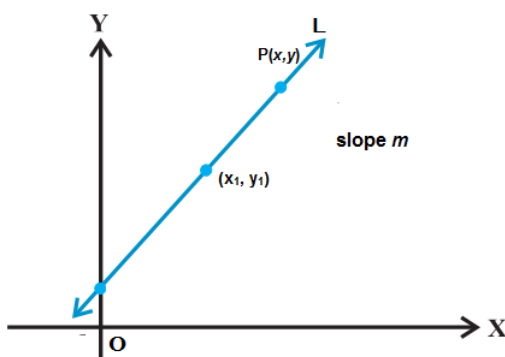
$$\text{i.e., } l_1 \perp l_2 \Leftrightarrow m_1 m_2 = -1 \Leftrightarrow m_2 = \frac{-1}{m_1}.$$

- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan \alpha = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$, $1 + m_1 m_2 \neq 0$ and obtuse angle is

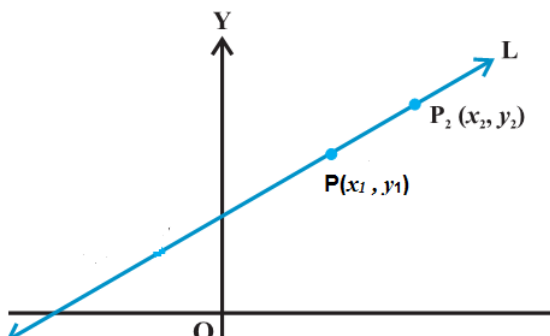
$$\phi = 180^\circ - \alpha \quad \text{or} \quad \pi - \alpha$$

- Point slope form:**

Equation of a line passing through given point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$



- Two Point Form:**



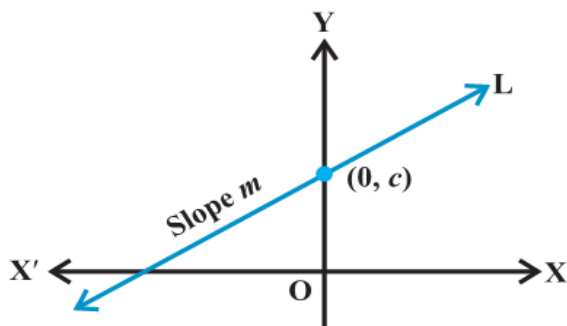
Equation of a line passing through given points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1).$$

- **Slope intercept form(y-intercept):**

Equation of a line having slope m and y-intercept ' c ' is given by

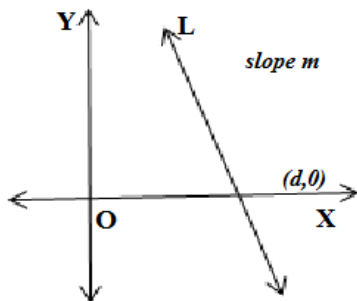
$$y = mx + c$$



- **Slope intercept form (x-intercept):**

Equation of a line having slope m and x-intercept d is given by

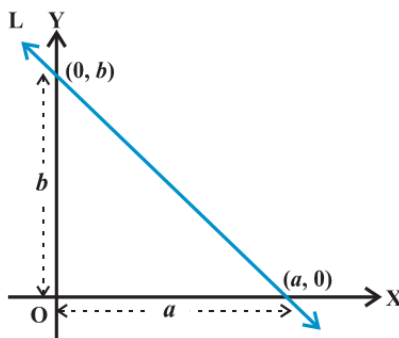
$$y = m (x - d)$$



- **Intercept Form:**

Equation of line having intercepts a and b on x -axis and y -axis respectively is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$



- **General Equation of a line:**

Equation of line in general form is given by $Ax + By + C = 0$, A , B and C are real numbers and at least one of A or B is non-zero.

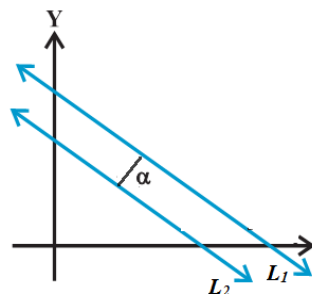
Slope = $\frac{-A}{B}$ and y -intercept = $\frac{-C}{B}$, x -intercept = $\frac{-C}{A}$.

- Distance of a point (x_1, y_1) from line $Ax + By + C = 0$ is given by

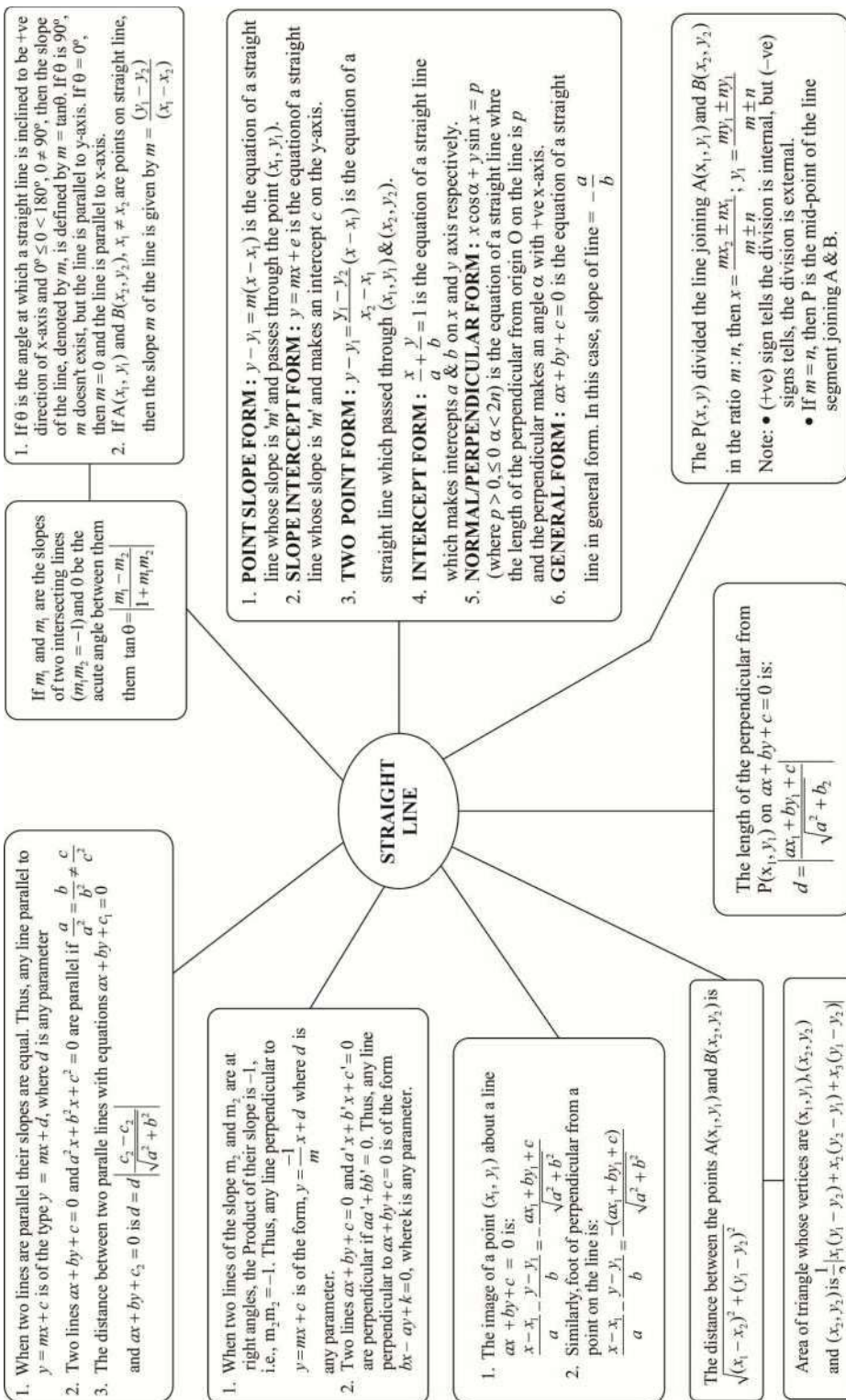
$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

- Distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$



MIND MAP



VERY SHORT ANSWER TYPE QUESTIONS

1. Three consecutive vertices of a parallelogram are $(-2, -1)$, $(1, 0)$ and $(4, 3)$, find the fourth vertex.
2. For what value of k are the points $(8, 1)$, $(k, -4)$ and $(2, -5)$ collinear?
3. Coordinates of centroid of $\triangle ABC$ are $(1, -1)$. Vertices of $\triangle ABC$ are $A(-5, 3)$, $B(p, -1)$ and $C(6, q)$. Find p and q .
4. In what ratio y -axis divides the line segment joining the points $(3, 4)$ and $(-2, 1)$?
5. Show that the points $(a, 0)$, $(0, b)$ and $(3a, -2b)$ are collinear.
6. Find the equation of straight line cutting off an intercept -1 from y axis and being equally inclined to the axes.
7. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through $(2, 5)$.
8. Find k so that the line $2x + ky - 9 = 0$ may be perpendicular to $2x + 3y - 1 = 0$
9. Find the acute angle between lines $x + y = 0$ and $y = 0$
10. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x -axis.
11. Find the equation of a line with slope $1/2$ and making an intercept 5 on y -axis.
12. Find Equation of line which is parallel to y -axis and at distance 5 units from y -axis.
13. Find the length of perpendicular from a point $(1, 2)$ to a line $3x + 4y + 5 = 0$.

SHORT ANSWER TYPE QUESTIONS

14. Determine the equation of line through a point $(-4, -3)$ and parallel to x-axis.
15. Check whether the points $\left(0, \frac{8}{3}\right)$, $(1, 3)$ and $(82, 30)$ are the vertices a triangle or not?
16. If a vertex of a triangle is $(1, 1)$ and the midpoints of two sides through this vertex are $(-1, 2)$ and $(3, 2)$. Then find the centroid of the triangle.
17. If the medians through A and B of the triangle with vertices $A(0, b)$, $B(0, 0)$ and $C(a, 0)$ are mutually perpendicular. Then show that $a^2 = 2b^2$.
18. If the image of the point $(3, 8)$ in the line $px + 3y - 7 = 0$ is the point $(-1, -4)$, then find the value of p.
19. Find the distance of the point $(3, 2)$ from the straight line whose slope is 5 and is passing through the point of intersection of lines $x + 2y = 5$ and $x - 3y + 5 = 0$
20. The line $2x - 3y = 4$ is the perpendicular bisector of the line segment AB. If coordinates of A are $(-3, 1)$ find coordinates of B.
21. The points $(1, 3)$ and $(5, 1)$ are two opposite vertices of a rectangle. The other two vertices lie on line $y = 2x + c$. Find c and remaining two vertices.

22. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.
23. Find the equation of a line with slope -1 and whose perpendicular distance from the origin is equal to 5.
24. If a vertex of a square is at $(1, -1)$ and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.
25. What is the value of y so that line through $(3, y)$ and $(2, 7)$ is parallel to the line through $(-1, 4)$ and $(0, 6)$?
26. In what ratio, the line joining $(-1, 1)$ and $(5, 7)$ is divided by the line $x + y = 4$?
27. Find the equation of the lines which cut-off intercepts on the axes whose sum and product are 1 and -6 respectively.
28. Find the area of the triangle formed by the lines $y = x$, $y = 2x$, $y = 3x + 4$.
29. Find the coordinates of the orthocentre of a triangle whose vertices are $(-1, 3)$ $(2, -1)$ and $(0, 0)$. [Orthocentre is the point of concurrency of three altitudes].
30. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.
31. If the image of the point $(2, 1)$ in a line is $(4, 3)$ then find the equation of line.
32. The vertices of a triangle are $(6,0)$, $(0,6)$ and $(6,6)$. Find the distance between its circumcenter and centroid.

LONG ANSWER TYPE QUESTIONS

33. Find the equation of a straight line which makes acute angle with positive direction of x-axis, passes through point $(-5, 0)$ and is at a perpendicular distance of 3 units from origin.
34. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are $(-3, 1)$ and $(1, 1)$. Find the equation of other three sides.
35. If $(1, 2)$ and $(3, 8)$ are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.
36. Find the equations of the straight lines which cut off intercepts on x-axis twice that on y-axis and are at a unit distance from origin.
37. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.
38. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$. Obtain its equation.
39. If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at $(1, 2)$, then find the equation of sides of the square passing through this vertex.
40. If the slope of a line passing through to point $A(3, 2)$ is $\frac{3}{4}$ then find points on the line which are 5 units away from the point A.
41. Find the equation of straight line which passes through the intersection of the straight line $3x + 2y + 4 = 0$ and $x - y - 2 = 0$ and forms a triangle with the axis whose area is 8 sq. unit.

42. Find points on the line $x + y + 3 = 0$ that are at a distance of 5 units from the line $x + 2y + 2 = 0$
43. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L.
44. Two equal sides of an isosceles triangle are given by the equation $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side pass through the point $(1, -10)$. Determine the equation of the third side.
45. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2 AC$. If the coordinates of D and M are $(1, 1)$ and $(2, -1)$ respectively. Then find the coordinates of A.
46. Find the area enclosed within the curve $|x| + |y| = 1$.
47. Find the coordinates of the circumcentre of the triangle whose vertices are $(5, 7)$, $(6, 6)$ and $(2, -2)$.
48. Find the equation of a straight line, which passes through the point $(a, 0)$ and whose \perp distance from the point $(2a, 2a)$ is a.
49. Line L has intercepts a and b on the coordinate axis when the axis are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then prove that $a^{-2} + b^{-2} = p^{-2} + q^{-2}$.

CASE STUDY TYPE QUESTIONS

50. A person is standing at a point A of a triangular park ABC whose vertices are $A(2, 0)$, $B(3, 4)$ and $C(5, 6)$.

Based on the above information answer the following :-



i. He wants to reach BC in least time. Find the equation of the path he should follow.

(a) $2x + y = 3$

(b) $2x + 3y = 4$

(c) $x + y = 2$

(d) $x + 4y = 7$

ii. Find the shortest distance travelled by him to reach BC -

(a) $\frac{5}{2}\sqrt{2}$ units

(b) $\frac{3}{2}\sqrt{2}$ units

(c) $\frac{4}{3}\sqrt{2}$ units

(d) $\frac{7}{3}\sqrt{2}$ units

iii. Suppose he meets BC at a point D. Find the coordiantes of the point D.

(a) $\left(\frac{5}{2}, \frac{7}{2}\right)$

(b) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(c) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(d) $\left(\frac{7}{2}, \frac{5}{2}\right)$

iv. Find the area of the triangular park ABC.

(a) 5 sq units

(b) 10 sq units

(c) 3 sq units

(d) None of these

- v. Find the coordinator of the centroid of the triangular park ABC?

(a) $\left(\frac{5}{3}, \frac{7}{3}\right)$

(b) $\left(\frac{10}{3}, \frac{10}{3}\right)$

(c) $\left(\frac{7}{3}, \frac{8}{3}\right)$

(d) $\left(\frac{2}{3}, \frac{8}{3}\right)$

51. If A and B are two points $(2, -3)$ and $(6, -5)$ respectively. If C is the point between A and B such that it divides the line AB in 1 : 3 ratio

Based on the above information, answer the following Questions

- (i) Find the distance between A and B
- (ii) Find eq of AB
- (iii) What are the co-ordinates of C?
- (iv) Find The Length AC
- (v) Find the slope of line BC.

Multiple Choice Questions

52. The angle between the straight lines $x - y\sqrt{3} = 5$ and $\sqrt{3}x + y = 7$ is-

(a) 90°

(b) 60°

(c) 75°

(d) 30°

53. If p is the length of the perpendicular drawn from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then which one of the following is correct?

(a) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

(b) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

(c) $\frac{1}{p} = \frac{1}{a} + \frac{1}{b}$

(d) $\frac{1}{p} = \frac{1}{a} - \frac{1}{b}$

54. What is the equation of the line passing through $(2, -3)$ and parallel to y-axis?
- (a) $y = -3$ (b) $y = 2$
(c) $x = 2$ (d) $x = -3$
55. If the lines $3x + 4y + 1 = 0$, $5x + \lambda y + 3 = 0$ and $2x + y - 1 = 0$ are concurrent, then λ is equal to -
- (a) -8 (b) 8
(c) 4 (d) -4 .
56. The x-intercept and the y-intercept of the line $5x - 7 = 6y$, respectively are -
- (a) $\frac{7}{5}$ and $\frac{7}{6}$ (b) $\frac{7}{5}$ and $-\frac{7}{6}$
(c) $\frac{5}{7}$ and $\frac{6}{7}$ (d) $-\frac{5}{7}$ and $\frac{6}{7}$.
57. If p be the length of the perpendicular from the origin on the straight line $x + 2y = 2q$, then what is the value of q ?
- (a) $1/p$ (b) p
(c) $p/2$ (d) $\frac{\sqrt{5}p}{2}$
58. A straight line through $P(1, 2)$ is such that its intercept between the axes is bisected at P . Its equation is-
- (a) $x + y = -1$ (b) $x + y = 3$
(c) $x + 2y = 5$ (d) $2x + y = 4$.
59. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b ?
- (a) 1 (b) 3
(c) 6 (d) 0 .

60. The triangle formed by the lines $x + y = 0$, $3x + y = 4$ and $x + 3y = 4$ is -
- (a) Isosceles (b) Equilateral
(c) Right angled (d) None of these.
61. What is the foot of the perpendicular from the point $(2, 3)$ on the line $x + y - 11 = 0$?
- (a) $(1, 10)$ (b) $(5, 6)$
(c) $(6, 5)$ (d) $(7, 4)$.
62. A line cutting off intercept -3 from the Y-axis and the tangent at angle to the X-axis is $\frac{3}{5}$, its
- (a) $5y - 3x + 15 = 0$ (b) $3y - 5x + 15 = 0$
(c) $5y - 3x - 15 = 0$ (d) None of the above
63. The equation of straight line passing through the point $(3, 2)$ and perpendicular to the line $y = x$ is
- (a) $x - y = 5$ (b) $x + y = 5$
(c) $x + y = 1$ (d) $x - y = 1$
64. The tangent of angle between the line whose intercepts on the axes are $a, -b$ and $b, -a$ respectively, is
- (a) $\frac{a^2 - b^2}{ab}$ (b) $\frac{b^2 - a^2}{2}$
(c) $\frac{b^2 - a^2}{2ab}$ (d) None of these
65. The equation of the lines which pass through the point $(3, -2)$ and are inclined at 60° to the line $\sqrt{3}x + y = 1$ is
- (a) $y + 2 = 0, \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$
(b) $x - 2 = 0, \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$

(c) $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

(d) None of the above

66. The coordinates of the foot of perpendiculars from the point (2, 3) on the line $y = 3x + 4$ given by

(a) $\left(\frac{37}{10}, \frac{-1}{10}\right)$

(b) $\left(\frac{-1}{10}, \frac{37}{10}\right)$

(c) $\left(\frac{10}{37}, -10\right)$

(d) $\left(\frac{2}{3}, -\frac{1}{3}\right)$

Directions: Each of these questions contains two statements. Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

- (a) Assertion is correct, reason is correct: reason is a correct explanation for assertion.
- (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.
- (c) Assertion is correct, reason is incorrect.
- (d) Assertion is incorrect, reason is correct.

67. **Assertion:** If θ is the inclination of a line l , then the slope or gradient of the line l is $\tan \theta$.

Reason: The slope of a line whose inclination is 90° , is not defined.

68. **Assertion:** The inclination of the line l may be acute or obtuse.

Reason: Slope of x-axis is zero and slope of y-axis is not defined.

69. **Assertion:** Slope of the line passing through the points (3, -2) and (3, 4) is 0.

Reason: If two lines having the same slope pass through a common point, then these lines will coincide.

70. **Assertion:** If A $(-2, -1)$, B $(4, 0)$, C $(3, 3)$ and D $(-3, 2)$ are the vertices of a parallelogram, then mid-point of AC = Mid-point of BD

Reason: The points A, B and C are collinear \Leftrightarrow Area of $\triangle ABC = 0$.

71. **Assertion:** Pair of lines $x + 2y - 3 = 0$ and $-3x - 6y + 9 = 0$ are coincident.

Reason: Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

ANSWERS

1. $(1, 2)$
2. $k = 3$ (Use Slope formula)
3. $p = 2, q = -5$
4. $3 : 2$ (internally)
6. $y = x - 1$ and $y = -x - 1$.
7. $x + y = 7$
8. $-\frac{4}{3}$
9. $\frac{\pi}{4}$
10. $\frac{2\pi}{3}$
11. $y = \frac{x}{2} + 5$
12. $x = 5$
13. $16/5$
14. $y + 3 = 0$
15. No (Use slope formula)
16. $\left(1, \frac{7}{3}\right)$

35. $x - 2y + 3 = 0$, $2x + y - 14 = 0$,
 $x - 2y + 13 = 0$, $2x + y - 4 = 0$
 $3x - y - 1 = 0$, $x + 3y - 17 = 0$ (Hints: angle between side and diagonal is 45°)
36. $x + 2y + \sqrt{5} = 0$, $x + 2y - \sqrt{5} = 0$
37. $x = y$ (Hint: Given diagonal does not pass through the point of intersection of given sides)
38. $107x - 3y - 92 = 0$
39. $23x - 7y - 9 = 0$ and $7x + 23y - 53 = 0$
40. $(-1, -1)$ or $(7, 5)$
41. $x - 4y - 8 = 0$ or $x + 4y + 8 = 0$
42. $(1, -4)$, $(-9, 6)$
43. $x + 5y = \pm 5\sqrt{2}$
44. $x - 3y - 31 = 0$, $3x + y + 7 = 0$
45. $\left(1, \frac{-3}{2}\right)$ or $\left(3, \frac{-1}{2}\right)$
46. $\sqrt{3}$ (Hint: Use modulus functions property)
47. $(2, 3)$ (Hint: Circumcentre is equidistant from the vertices of triangle)
48. $3x - 4y - 3a = 0$ and $x - a = 0$
50. i. (c) ii. (b) iii. (b) iv. (c) v. (b)

51. i. $2\sqrt{5}$ ii. $2y + x + 4 = 0$ iii. $\left(3, \frac{-7}{2}\right)$

iv. $\frac{\sqrt{5}}{2}$ v. $\frac{-1}{2}$

52. (a)

53. (a)

54. (c)

55. (b)

56. (b)

57. (b)

58. (d)

59. (c)

60. (a)

61. (b)

62. (a) $5y - 3x + 15 = 0$

63. (b) $x + y = 5$

64. (c) $\left(\frac{b^2 - a^2}{2ab}\right)$

65. (a) $y + 2 = 0$, $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$

66. (b) $\left(-\frac{1}{10}, \frac{37}{10}\right)$

67. (b) 68. (b)

69. (d)

70. (b)

71. (a)