CHAPTER - 8

SEQUENCES AND SERIES

KEY POINTS

- In general, listing of any collection of objects in certain order is **sequence.**
- A sequence is a function whose domain is the set N of natural numbers or some subset of it.
- Let a_1, a_2, a_3, …….. be a sequence, then the expression $a_1 + a_2 + a_3 +$ …….. is called series associated with given sequence.
- A sequence containing finite number of terms is called finite sequence.
- A sequence is infinite, if it is not finite sequence.
- A sequence is said to be a progression if all the terms of the sequence can be expressed by same formula
- **Arithmetic Progression:** A sequence is called an arithmetic progression if the difference between of a term and its previous term is always same, i.e., $a_{n+1} - a_n$ = constant (=d) for all $n \in N$.
- General A.P. isa, a + d and a + 2d, ..., where a = first term and d = common difference.
- $a_n = a + (n - 1)d$ = n^{th} term of A.P. = l
- S_n = Sum of first n terms of A.P. = $\frac{n}{2}[a + l]$, where l = last term N.

$$= \frac{n}{2}[2a + (n-1)d]$$

- If a, b, c are in A.P. then a ± k, b ±k, c ± k are in A.P.

  ak, bk, ck also in A.P., k ≠ 0

  $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P. where k ≠ 0.

- If a, A, b are in A.P., then A is called **arithmetic** mean of a and b.

- Arithmetic mean between a and b is = $\frac{a+b}{2}$.

- If A_1, A_2, A_3, ...A_n are n numbers inserted between a and b, such that the resulting sequence is A.P.

  then, $A_n = a + nd$ where $d = \frac{b-a}{n+1}$

- $S_k - S_{k-1} = a_k$

- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term.

- If a, b, c are in A.P. then 2b = a + c.

- Three terms of A.P. can be chosen as a – d, a, a + d

- Four terms of A.P. can be chosen as a – 3d, a – d, a + d, a + 3d.

- **G.P. (Geometrical Progression)**

  (i) a, ar, ar^2, ...(General G.P.)

  Where a = First term

  And r = common ratio

(ii) $a_n = ar^{n-1}$

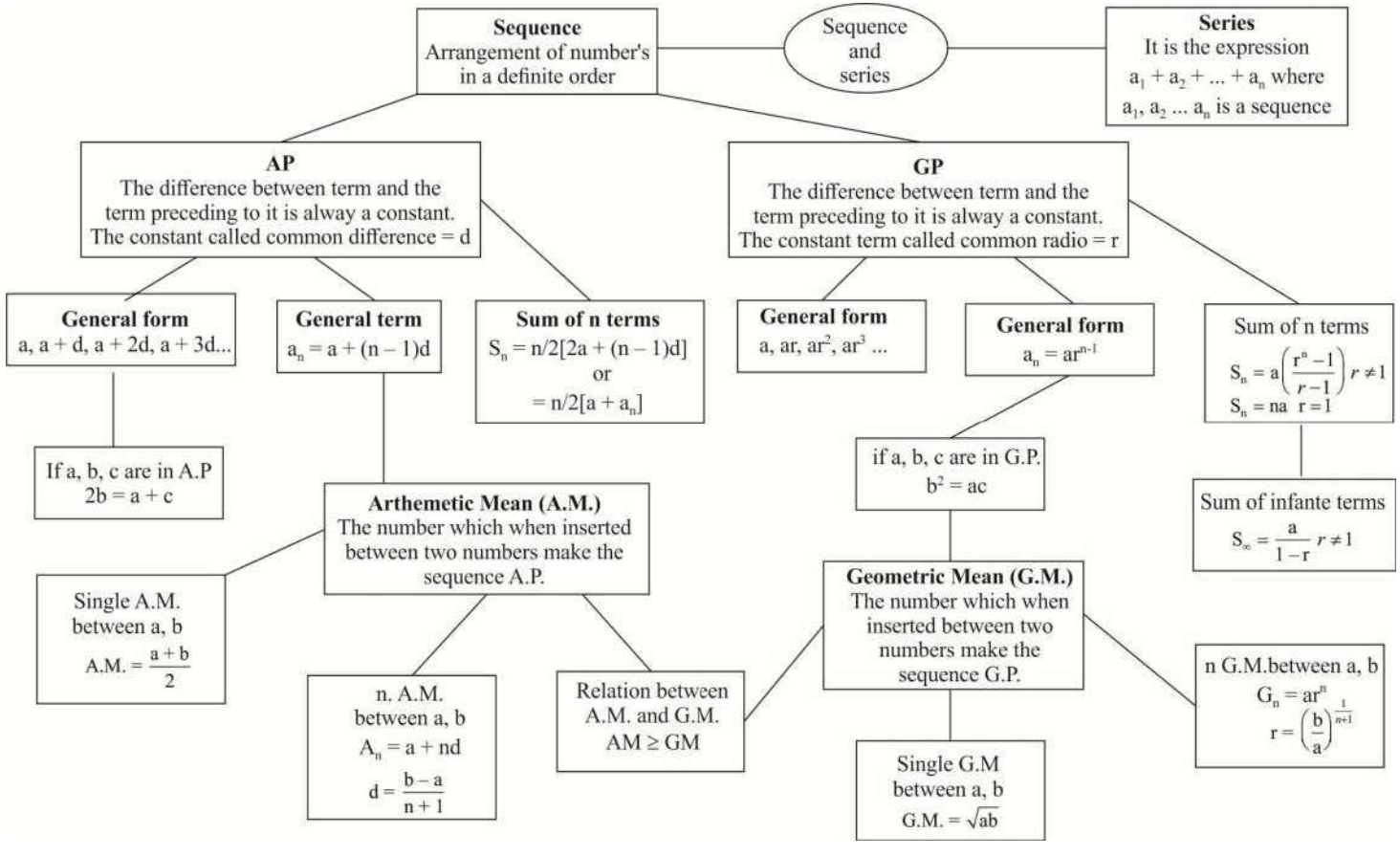
(iii) $S_n = \frac{a(r^n - 1)}{r - 1}$ or $S_n = \frac{a(1 - r^n)}{1 - r}$, $r \neq 1$

- If a, b, c are in G.P., then $b^2 = ac$.
- If a, G, b are in GP, then G is called **geometric** mean of a and b
- Geometric mean of two positive numbers a and b is $\sqrt{ab}$.
- If $G_1, G_2, G_3, \ldots\ldots G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $G_n = ar^n$ where $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

- Three terms of G.P. are chosen as $\frac{a}{r}, a, ar$.
- Four terms of G.P. are chosen as $\frac{a}{r^3}, \frac{a}{r}, a, ar^3$.
- If a, b, c are in G.P. then (i) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in GP, (ii) ak, bk, ck are also in G.P., where $k \neq 0$ (iii) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in G.P. where $k \neq 0$, a^n, b^n, c^n are also in GP.
- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.
- Sum of infinite G.P. is possible if $|r| < 1$ and sum is given by $\frac{a}{1 - r}$.

MIND MAP


Sequence
Arrangement of number's in a definite order
Sequence and series
Series
It is the expression
$a_1 + a_2 + ... + a_n$ where
$a_1, a_2 ... a_n$ is a sequence
AP
The difference between term and the term preceding to it is alway a constant. The constant called common difference = d
GP
The difference between term and the term preceding to it is alway a constant. The constant term called common radio = r
General form
a, a + d, a + 2d, a + 3d...
General term
$a_n = a + (n - 1)d$
Sum of n terms
$S_n = n/2[2a + (n - 1)d]$
or
$= n/2[a + a_n]$
General form
a, ar, ar^2, ar^3 ...
General form
$a_n = ar^{n-1}$
Sum of n terms
$S_n = a\left(\frac{r^n - 1}{r - 1}\right) r \neq 1$
$S_n = na$ r = 1
If a, b, c are in A.P
$2b = a + c$
Arthemetic Mean (A.M.)
The number which when inserted between two numbers make the sequence A.P.
if a, b, c are in G.P.
$b^2 = ac$
Sum of infante terms
$S_\infty = \frac{a}{1 - r}$ $r \neq 1$
Single A.M. between a, b
$A.M. = \frac{a + b}{2}$
n. A.M. between a, b
$A_n = a + nd$
$d = \frac{b - a}{n + 1}$
Relation between A.M. and G.M.
$AM \geq GM$
Geometric Mean (G.M.)
The number which when inserted between two numbers make the sequence G.P.
n G.M.between a, b
$G_n = ar^n$
$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$
Single G.M between a, b
$G.M. = \sqrt{ab}$

VERY SHORT ANSWER TYPE QUESTIONS

1. If n^{th} term of an A.P. is 6n – 7 then write its 50^{th} term.

2. If $S_n = 3n^2 + 2n$, then write a_2

3. Which term of the sequence 3, 10, 17, ... is 136?

4. If in an A.P. 7^{th} term is 9 and 9^{th} term is 7, then find 16^{th} term.

5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its n^{th} term.

6. Which term of the G.P.2, 1, $\frac{1}{2}, \frac{1}{4}$ ..... is $\frac{1}{1024}$?

7. If in a G.P., $a_3 + a_5 = 90$ and if r = 2 find the first term of the G.P.

8. In G.P. $2\sqrt{2}, 4, ..., 128\sqrt{2},$ find the 4^{th} term from the end.

9. If the product of 3 consecutive terms of G.P. is 27, find the middle term.

10. Find the sum of first 8 terms of the G.P. $10, 5, \frac{5}{2}$, ...

11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ ..... upto infinity.

12. Write the value of $0.\overline{3}$

    [Hint: $0.\overline{3} = 0.3 + 0.03 + 0.003 + ... = \frac{0.3}{1-0.1}$]

13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

14. If 7^{th} and 13^{th} terms of an A.P. be 34 and 64 respectively, find 18^{th} term.

15. Find geometric mean of 4 and 9.

16. Find If the sum of first p terms of an A.P. is q and sum of first q terms is p, then the sum of first p + q terms.

17. Find sum to infinity of sequence $5, \frac{5}{3}, \frac{5}{9}, \ldots\ldots$.

18. If a, b, c are in A.P. and x, y, z are in G.P., then find the value of $x^{b-c} \times y^{c-a} \times z^{c-a}$.

19. Find two geometric means between numbers 1 and 64.

20. Write third term of sequence whose general term is $a_n = \frac{2n-3}{4}$.

SHORT ANSWER TYPE QUESTIONS

21. Write the n^{th} term of the series, $\frac{3}{7.11^2} + \frac{5}{8.12^2} + \frac{7}{9.13^2} + \ldots\ldots$

22. Find the number of terms in the A.P. 7, 10, 13, ..., 31.

23. In an A.P., 8, 11, 14, ... find $S_n - S_{n-1}$

24. Find the sum of given terms:-

    (a) 81 + 82 + 83 ..... + 89 + 90

    (b) 251 + 252 + 253 + ... + 259 + 260

25. (a) If a, b, c are in A.P. then show that 2b = a+c.

    (b) If a, b, c are in G.P. then show that $b^2 = a \cdot c$.

26. If a, b, c are in G.P. then show that $a^2 + b^2$, ab + bc, $b^2 + c^2$ are also in G.P.

27. Find the least value of n for which

    $1 + 3 + 3^2 + \ldots + 3^{n-1} > 1000$

28. Write the first negative term of the sequence $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, ... $[a_n < 0]$

29. Determine the number of terms in A.P. 3, 7, 11, ... 407. Also, find its 11^{th} term from the end.

30. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

31. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

32. Find the sum of the sequence,

    72 + 70 + 68 + ……. + 40

33. If in an A.P $\frac{a_7}{a_{10}} = \frac{5}{7}$, find $\frac{a_4}{a_7}$.

34. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

35. Solve: 1 + 6 + 11 + 16 + ... + x = 148

36. The ratio of the sum of n terms of two A.P.'s is (7n – 1): (3n + 11), find the ratio of their 10^{th} terms.

37. If the I^{st}, 2^{nd} and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

38. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{a}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [**Hint.** : Add 3 to each term].

39. The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find the G.P.

40. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

41. Find the sum to infinity of the series:

$$1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \ldots\ldots \infty$$

42. If $A = 1 + r^a + r^{2a} + \ldots\ldots\ldots$ up to infinity, then express r in terms of 'a' and 'A'.

43. Find the sum of first terms of the series $0.7 + 0.77 + 0.777 + \ldots\ldots$

44. If $x = a + \frac{a}{r} + \frac{a}{r^2} + \ldots\ldots\ldots + \infty$; $y = b - \frac{b}{r} + \frac{b}{r^2} - \ldots\ldots\ldots \infty$ and $z = c + \frac{c}{r^2} + \frac{c}{r^4} + \ldots\ldots\ldots \infty$ Prove that $\frac{xy}{z} = \frac{ab}{c}$.

45. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

46. Prove that $0.003\overline{1} = \frac{7}{225}$.

[**Hint:** 0.031 = 0.03 + 0.001 + 0.0001 +..... Now use infinite G.P.]

47. If a, b, c are in G.P. that the following are also in G.P.

(i) a^2, b^2, c^2

(ii) a^3, b^3, c^3

(iii) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in G.P.

48. If a, b, c are in A.P. that the following are also in A.P:

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

(ii) b+c, c+a, a+b

(iii) $\frac{1}{a}\left(\frac{1}{b}+\frac{1}{c}\right), \frac{1}{b}\left(\frac{1}{c}+\frac{1}{a}\right), \frac{1}{c}\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

49. If the numbers a^2, b^2 and c^2 are given to be in A.P., show that $\frac{1}{b+c}, \frac{1}{c+a}$ and $\frac{1}{a+b}$ are in A.P.

50. Show that: $0.3\overline{56} = \frac{353}{990}$

51. The n^{th} term of a G.P. is 128 and the sum of its n term is 255. If its common ratio is 2, find the first term.

52. The fourth term of a G.P. is 4. Find product of its first seven terms.

53. If A_1, A_2, A_3, A_4 are four A.M's between $\frac{1}{2}$ and 3, then prove $A_1 + A_2 + A_3 + A_4 = 7$.

54. If S_n denotes the sum of first n terms of an A.P. If $S_{2n} = 5S_n$, then prove $\frac{S_{6n}}{S_{3n}} = \frac{17}{4}$.

LONG ANSWER TYPE QUESTIONS

55. Prove that the sum of n numbers between a and b such that the resulting series becomes A.P. is $\frac{n(a+b)}{2}$.

56. If a, b, c are in G.P., then prove that $\frac{1}{a^2-b^2} - \frac{1}{b^2-c^2} = -\frac{1}{b^2}$.

[**Hint :** Put b = ar, c = ar^2]

57. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

58. If a is A.M. of b and c and c, G_1, G_2, b are in G.P., then prove that $G_1^3 + G_2^3 = 2abc$

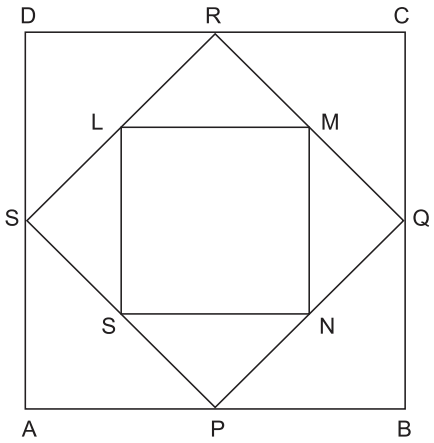
59. The sum of an infinite G.P. is 57 and the sum of the cubes of its term is 9747, find the G.P.

60. Find the sum of first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....n$ terms. Hint: $\left[1-\frac{1}{2}+1-\frac{1}{4}+1-\frac{1}{8}+1-\frac{1}{16}+....\right]$

61. Three positive numbers form an increasing G.P. If the middle term in the G.P. is doubled, then new numbers are in A.P. then find the common ratio of the G.P.

62. Find three numbers in G.P. whose sum is 13 and the sum of whose squares is 91.

63. The side of given space is 10 cm. The mid points at its, sides are joined to form a new square. Again the mid point of the sides of this new square are joined to form another square. This process is contined indefinitely. Based on the information answer the following questions.



(i) The side of first square is 10 cm what is the side of IInd square formed.

(ii) What is the sum of area's of all the square formed?

(iii) What is the sum of perimeters of all the square formal?

CASE STUDY TYPE QUESTIONS

64. Abhishek buys Kisan Vikas Patra (KVP) from post office every year. Each year he exceeds the value of KVP by ₹1000 from last year's purchase. After 5 years he finds that the total value of KVP purchased by him is ₹40,000.00.

Based on the above information answer the following :-

i. The sequence of amount of KVP forms a/an

(a) Arithmetic Progression (b) Geometric Progression

(c) Harmonic Progression (d) None of these

ii. Find the amount of KVP purchased by him initially.

(a) ₹7000 (b) ₹8000

(c) ₹6000 (d) ₹7500

iii. What will be the total amount of KVP purchased by him after 10 years?

(a) ₹1,20,000 (b) ₹1,05,000

(c) ₹1,40,000 (d) ₹1,35,000

iv. What is the amount of KVP purchased by him in the 8^{th} year?

(a) ₹14,000 (b) ₹15,000

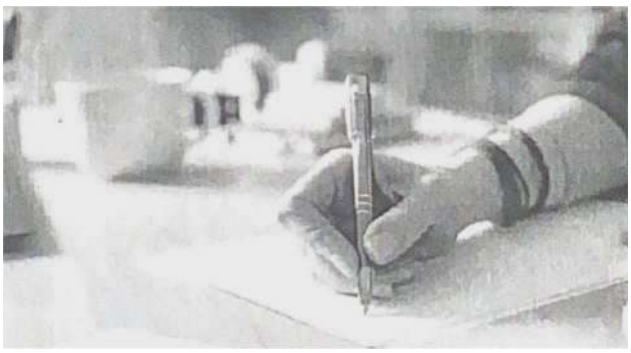
(c) ₹13,000 (d) ₹12,000

v. If he buys KVP every year for 10 years, how much will he spend in the purchase of last 4 KVP?

(a) ₹65,000 (b) ₹54,000

(c) ₹75,000 (d) None of these

65. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail it to four different persons with the instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paisa to mail one letter, anwer the following questions.



i. The sequence of letters mailed in each set forms a/an.

(a) Arithmetic Progression (b) Geometric Progression

(c) Harmonic Progression (d) None of these

ii. Find the number of letters mailed in the 4^{th} set.

(a) 64 (b) 16

(c) 256 (d) 1024

iii. Find the total number of letters mailed in the first 5 sets.

(a) 1364 (b) 1650 (c) 1236 (d) 1368

iv. Find the amount spent on the postage when 8^{th} set of letters is mailed?

(a) ₹46,930 (b) ₹54,930 (c) ₹87,380 (d) ₹43,690

v. Find the amount spent on the mailing of 9^{th} set?

(a) ₹1,74,762 (b) ₹1,31,072

(c) ₹1,54,536 (d) None of these

Multiple Choice Questions

66. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5, then the number of side is -

(a) 8 (b) 10

(c) 9 (d) 6.

67. α and β are the roots of the equation $x^2 - 3x + a = 0$ and γ and δ are the roots of the equation $x^2 - 12x + b = 0$. If α, β, γ and δ form an increasing G.P., then (a, b)-

(a) (3, 12) (b) (12, 3)

(c) (2, 32) (d) (4, 16).

68. If A be the arithmetic mean between two numbers and S be the sum of n arithmetic means between the same numbers, then -

(a) S = nA (b) A = nS

(c) A = S (d) None of these.

69. If n geometric means be inserted between a and b, then the n^{th} geometric mean will be-

(a) $a\left[\frac{b}{a}\right]^{\frac{n}{n-1}}$ (b) $a\left[\frac{b}{a}\right]^{\frac{n-1}{n}}$

(c) $a\left[\frac{b}{a}\right]^{\frac{n}{n+1}}$ (d) $a\left[\frac{b}{a}\right]^{\frac{1}{n}}$.

70. If the arithmetic and geometric means of two numbers are 10 and 8 respectively, then one number exceeds the other number by-

(a) 8 (b) 10

(c) 12 (d) 16.

71. The first and last terms of A.P. are 1 and 11. If the sum of its term is 36, then the number of terms will be-

(a) 5 (b) 6

(c) 7 (d) 8.

72. If the first, second and last term of an A.P. are a, b and 2a respectively, then its sum is -

(a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$

(c) $\frac{3ab}{2(b-a)}$ (d) None of these.

73. If p^{th}, q^{th} and r^{th} terms of an A.P. are in G.P., then the common ratio of this G.P.is -

(a) $\frac{p-q}{q-r}$ (b) $\frac{q-r}{p-q}$

(c) pqr (d) None of these.

74. If A be one A.M. and p, q be two GM's between two numbers, then 2A is equal to-

(a) $\frac{p^3+q^3}{pq}$ (b) $\frac{p^3-q^3}{pq}$

(c) $\frac{p^2+q^2}{2}$ (d) $\frac{pq}{2}$.

75. In a G.P. if the $(m + n)^{th}$ term is p and $(m – n)^{th}$ term is q, then its m^{th} term is -

(a) O (b) pq

(c) $\sqrt{pq}$ (d) $\frac{1}{2}(p+q)$.

76. If S be the sum, P the product, R be the sum of reciprocals of n terms of G.P. then P^2 is equal to

(a) $\frac{S}{R}$ (b) $\frac{R}{S}$

(c) $\left(\frac{R}{S}\right)^n$ (d) $\left(\frac{S}{R}\right)^n$

77. The n^{th} term of a G.P. is 128 and the sum of its n terms is 225. If its common ratio is 2, then its first term is

(a) 1 (b) 3

(c) 8 (d) none of these

78. If second term of a G.P. is 2 and the sum of its infinite term is 8, then its first term is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 2 (d) 4

79. The two geometric means between the numbers 1 and 64 are

(a) 1 and 64 (b) 4 and 16

(c) 2 and 16 (d) 8 and 16

80. The product (32), $(32)^{1/6}$, $(32)^{1/36}$ .... to ∞ is equal to

(a) 64 (b) 16

(c) 32 (d) 0

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct: reason is not a correct explanation for assertion.
(b) Assertion is correct, reason is correct: reason is not a correct explanation for assertion.
(c) Assertion is correct , reason is incorrect.
(d) Assertion is incorrect, reason is correct.

81. **Assertion:** Value of a_{17}. Whose n^{th} term is $a_n = 4n - 3$, is 65.

Reason: Value of a_9, whose nth term is $a_n = (-1)^{n-1} . n^3$.

82. **Assertion:** If the third term of a G.P. is 4, then the product of its first five terms is 4^5.

Reason: Product of first five terms of a G.P. is given as a (ar) (ar^2) (ar^3) (ar^4).

83. **Assertion:** If a, b, c are in A.P. then b + c, c + a, a + b are in A.P.

Reason: If a, b, c are in A.P., then 10^a, 10^b, 10^c are in G.P.

84. **Assertion:** If $\frac{2}{3}$, k, $\frac{5}{8}$ are in A.P., then the value of k is $\frac{31}{48}$.

Reason: Three numbers a, b, c are in A.P. iff 2b = a + c

85. **Assertion:** For x = ± 1, the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.

Reason: Three numbers a, b, c are in G.P. if b^2 = ac.

ANSWERS

1.	293	2.	11
3.	20^{th}	4.	0
5.	4n + 5	6.	12th
7.	$\frac{9}{2}$	8.	64
9.	3	10.	$20\left(1-\frac{1}{2^8}\right)$
11.	5	12.	$\frac{1}{3}$
13.	$\frac{2}{3}$	14.	89
15.	6	16.	–(p + q)
17.	15/2	18.	1

19. 4 and 16

20. 3/4

21. $\frac{2n+1}{(n+6)(n+10)^2}$

22. 9

23. 3n + 5

27. n = 7

24. 855, 2555

28. $-\frac{1}{4}$

29. 102, 367

30. 33

31. 7799

32. 952

33. $\frac{3}{5}$

34. 11

35. 36

36. 33:17

37. $\frac{(b+c-2a)(a+c)}{2(b-a)}$

39. 5, 10, 20, ...; or 20, 10, 5,

40. 18, 6, 2; or 2, 6, 18

41. 6

42. $\left(\frac{A-1}{A}\right)^{\frac{1}{a}}$

43. $\frac{7}{81}\left[9n-1+10^{-n}\right]$

45. $\frac{15}{7}\left(2^n-1\right)$

51. 1

52. 16384

57. 16, 4

59. 19, $\frac{38}{3}$, $\frac{76}{9}$, ...

60. $n + 2^{-n} - 1$

61. $r = 2+\sqrt{3}$

62. 1, 3, 9

63. i. $5\sqrt{2}$ cm ii. 200 cm^2 iii. $(80+40\sqrt{2})$ cm

64. i. (a) ii. (c) iii. (b) iv. (c) v. (b)

65. i. (b) ii. (c) iii. (a) iv. (d) v. (b)

66. (c) 67. (c) 68. (a)

69. (c) 70. (c) 71. (b)

72. (c) 73. (b) 74. (a)

75. (c) 76. (d) 77. (a)

78. (d) 79. (b) 80. (a)

81. (b) 82. (a) 83. (b)

84. (a) 85. (a)