CHAPTER - 7

BINOMIAL THEOREM

KEY POINTS

▶ **Binomial Theorem for Positive Integers :**

- $(x + y)^n = {}^nC_0x^ny^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots\ldots\ldots$

  $\ldots\ldots\ldots\ldots + {}^nC_r x^{n-r} y^r + \ldots\ldots\ldots\ldots {}^nC_n x^0 y^n,$

  Where n is any positive integer.

- It is written as $(x+y)^n = \sum_{r=0}^{n} {}^nc_r x^{n-r} y^r$
- Total number of terms in expansion $(x + y)^n$ is $(n + 1)$
- General Term = $T_{r+1} = {}^nC_r x^{n-r} y^r$, where $0 \le r \le n$.

▶ **Middle Term :**

- If n is even, then there is only one middle term

  M.T. = $\left(\frac{n}{2}+1\right)^{th}$ term

- If n is odd, then there are two middle terms

  First M.T. = $\left(\frac{n+1}{2}\right)^{th}$ term

  Second M.T. = $\left(\frac{n+1}{2}+1\right)^{th}$ term

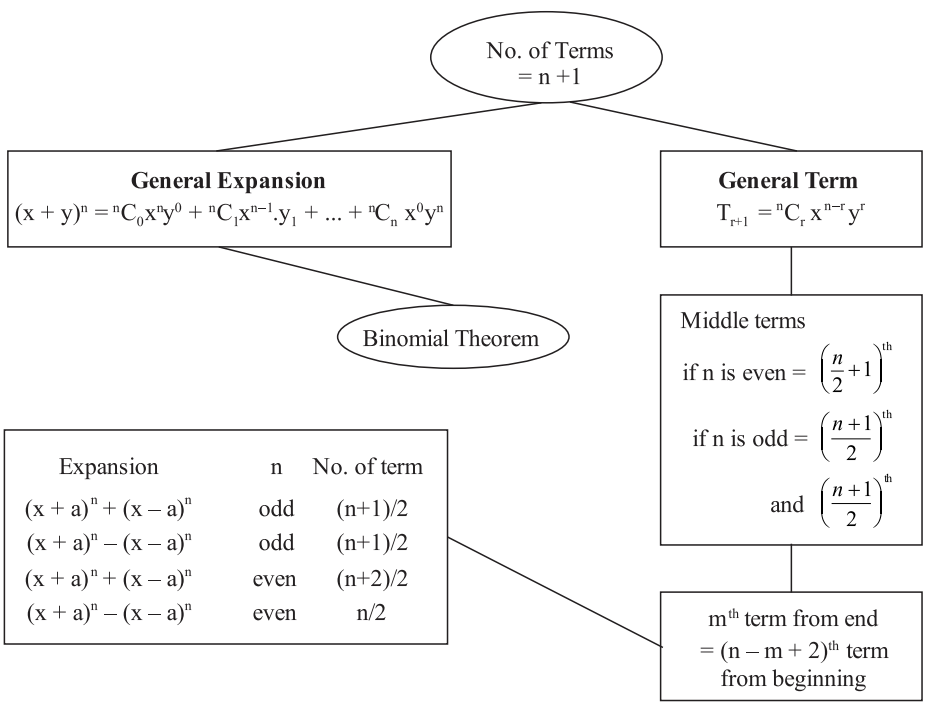
▶ Some important observations :

- In expansion $(x + y)^n$

  ${}^nC_r$, ${}^nC_{r-1}$,... ${}^nC_0$ are called **binomial coefficients**

- Sum of indices of x and y is n in each of the expansion.
- T_{r+1} = [$(r + 1)^{th}$ term from beginning] = ${}^nC_r\, x^{n-r}\, y^r$
- T'_{r+1} = [$(r + 1)^{th}$ term from end] = ${}^nC_{n-r}\, x^r\, y^{n-r}$
- $(x-y)^n = \Sigma(-1)^r\ {}^nC_r\ x^{n-r}y^r$
- $(1+x)^n = \sum_{r=0}^{n} {}^nc_r x^r$
- $(1-x)^n = \sum_{r=0}^{n} (-1)^r\ {}^nc_r x^r$

MIND MAP



VERY SHORT ANSWER TYPE QUESTIONS

1. Write number of terms in the expansion of $\left\{\left(2x+y^3\right)^4\right\}^7$.

2. Expand $\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6$ using binomial theorem.

3. Write value of

   ${}^{2n-1}C_5 + {}^{2n-1}C_6 + {}^{2n}C_7$ use $\left[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right]$

4. Which term is greater $(1.2)^{4000}$ or 800?

5. Find the coefficient of x^{-17},in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$.

6. Find the sum of the coefficients in $(x + y)^8$

   [**Hint :** Put x = 1, y = 1]

7. If ${}^{n}C_{n-3} = 120$, find n.

8. Find number of terms in expansion of $(-2x + 3y)^{17}$.

9. Find term independent of x in expansion of $\left(x-\frac{1}{3x^2}\right)^9$

10. Find the middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$

11. If the coefficient of x in $\left(x^2+\frac{\lambda}{x}\right)^5$ is 270, then find λ.

12. Find the coefficient of x^5 in $(x + 3)^8$

13. Find 4^{th} term from the end in expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$

14. Find number of terms in $(x + y)^5 + (x - y)^5$

15. Find coefficient f x^5 in $(1 + x)^{10}$

16. Find number of terms in

(i) $(3x - y)^{10} + (3x + y)^{10}$

(ii) $(2x + y)^8 - (2x - y)^8$

(iii) $(x + 5y)^{15} - (x - 5y)^{15}$

(iv) $(5x - y)^9 + (5x - y)^9$

17. Using Binomial Theorem, Evaluate.

(i) 96^3 [Hint $(100 - 4)^3$] (ii) $(101)^4$ (iii) $(10.1)^5$

SHORT ANSWER TYPE QUESTIONS

18. How many term are free from radical signs in the expansion of $\left(x^{\frac{1}{5}} + y^{\frac{1}{10}}\right)^{55}$.

19. Find the constant term in expansion of $\left(x - \frac{1}{x}\right)^{10}$.

20. Find 4th term from end in the expansion of $\left(\frac{x^3}{2} + \frac{2}{x^2}\right)^9$.

21. Find middle term in the expansion of $(x - 2y)^8$.

22. Which term is independent of x in the expansion of $\left(3x^3 - \frac{1}{2x^3}\right)^{10}$.

23. Find the 11th term from end in expansion of $\left(2x-\frac{1}{x^2}\right)^{25}$.

24. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

25. In $\left(3x^2-\frac{1}{x}\right)^{18}$ which term contains x^{12}.

26. In $\left(\frac{\sqrt{x}}{\sqrt{3}}+\frac{\sqrt{3}}{\sqrt{2}x^2}\right)^{10}$ find the term independent of x.

27. Evaluate $\left(\sqrt{2}+1\right)^5-\left(\sqrt{2}-1\right)^5$ using binomial theorem.

28. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4.

29. Find the coefficients of x^4 in $(1 – x)^2 (2 + x)^5$ using binomial theorem.

30. Show that $3^{2n+2} – 8n – 9$ is divisible by 8. [$3^{2n+2} = 9.9^n = 9(1 + 8)^n$]

31. If the term free from x in the expansion of $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ is 405. Find the value of k.

32. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1024}$.

33. If a, b, c and d in any binomial expansion be the 6^{th}, 7^{th}, 8^{th} and 9^{th} terms respectively, then prove that $\frac{b^2-ac}{c^2-bd}=\frac{4a}{3c}$.

34. If in the expansion of $(1 + x)^n$, the coefficients of three consecutive of three consecutive terms are 56, 70 and 56. Then find n and the position of terms of these coefficients.

35. Show that $2^{4n+4} - 15n - 16$ where $n \in N$ is divisible by 225. [$2^{4n+4} = 2^4.2^{4x} = 16(1 + 15)^n$]

36. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:3:5, then show that n = 7.

37. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$.

38. Find the value of r, if the coefficient of $(2r + 4)^{th}$ term and $(r - 2)^{th}$ term in the expansion of $(1 + x)^{18}$ are equal.

39. Prove that there is no term involving x^6 in the expansion of $\left(2x^2 - \frac{3}{x}\right)^{11}$.

40. The coefficient of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 6 : 30. Find n.

LONG ANSWER TYPE QUESTIONS

41. Show that the coefficient of x^5 in the expansion of product $(1 + 2x)^6 (1 - x)^7$ is 171.

42. If the 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n.

43. If the coefficients of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax - \frac{1}{bx^2}\right]^{11}$ are equal, then show that ab = 1.

44. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is 1:6 , find n.

45. If p is a real number and if middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, find p.

46. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms inthe expansion of $(1 + x)^n$

Prove that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$.

47. Find the remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9.

48. Find the value of $\left(\sqrt{2}+1\right)^6-\left(\sqrt{2}-1\right)^6$ and show that $\left(\sqrt{2}+1\right)^6$ lies between 197 and 198.

49. Find the term independent of x in the expansion of : $(1 + x + 2x^3)\left(\frac{3}{2}x^2-\frac{1}{3}x\right)^9$.

50. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1 + x)^4$are in A.P find the value of r.

51. If the expansion of $(1 - x)^{2n-1}$, the coefficients of x^r is denoted by a_r, then prove $a_{(r-1)} + a_{(2n-r)} = 0$.

52. If the coefficient of 5^{th}, 6^{th} and 7^{th} terms in the expansion of $(1 + x)^n$ are in A.P., then find the value of n.

53. The coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^{2n}$ are in A.P. Prove that $2n^2 - 9n + 7 = 0$.

54. Show that the middle term in the expansion of $\left[x-\frac{1}{x}\right]^{2n}$ is $\frac{1\cdot 3\cdot 5.....(2n-1)}{n!}(-2n)^n$.

55. If *n* is a positive integer, find the coefficient of x^{-1} in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$

Multiple Choice Questions

56. The middle term of $\left[2x-\frac{1}{3x}\right]^{10}$ is -

(a) ${}^{10}C_4\ \frac{2^4}{3^4}$ (b) $-{}^{10}C_5\ \frac{2^5}{3^5}$

(c) $-{}^{10}C_4\ \frac{2^4}{3^5}$ (d) ${}^{10}C_5\ \frac{2^5}{3^5}$.

57. For all $n \in N$, $2^{4n} - 15n - 1$ is divisible by -

(a) 125 (b) 225

(c) 450 (d) 625.

58. What is the coefficient of x^n in $(x^2 + 2x)^{n-1}$?

(a) $(n-1)\,2^{(n-2)}$ (b) $(n-1)\times 2^{(n-1)}$

(c) $(n-1)\,2^n$ (d) $n\,.\,2^{(n-1)}$.

59. The coefficient of x^{-3} in the expansion of $\left[x-\frac{m}{x}\right]^{11}$ is –

(a) $-924\ m^7$ (b) $-792\ m^5$

(c) $-792\ m^6$ (d) $-330\ m^7$.

60. In the expansion of $\left[x^2 - \frac{1}{3x}\right]^9$, the term without x is equal to -

(a) $\frac{28}{81}$ (b) $\frac{-28}{243}$

(c) $\frac{28}{243}$ (d) None of these.

61. If in the expansion of $(1 + x)^{20}$, the coefficients of r^{th} and $(r + 4)^{th}$ term are equal, then r is equal to -

(a) 7 (b) 8

(c) 9 (d) 10.

62. If in the expansion of $(1 + x)^5$, the coefficients of $(r - 1)^{th}$ and $(2r + 3)^{th}$ terms are equal, then the value of r -

(a) 5 (b) 6

(c) 4 (d) 3.

63. The total number of terms in expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is -

(a) 202 (b) 51

(c) 50 (d) None of these.

64. The middle term in the expansion of $\left[\frac{2x}{3} - \frac{3}{2x^2}\right]^{2n}$ is -

(a) ${}^{2n}C_n$ (b) $(-1)^n {}^{2n}C_n x^{-n}$

(c) ${}^{2n}C_n x^{-n}$ (d) None of these.

65. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the some, then the value of a is -

(a) $\frac{-7}{9}$ (b) $\frac{-9}{7}$

(c) $\frac{7}{9}$ (d) $\frac{9}{7}$.

66. The total number of term in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is

(a) 102 (b) 25

(c) 26 (d) 28

67. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then

(a) Re(z) = 0 (b) Im(z) = 0

(c) Re(z) > 0, In (z) > 0 (d) Re(z) > 0, In (z) < 0

68. The two successine terms in the expansion of $(1 + x)^{24}$ whose coefficients are in the ratio 1 : 4 are

(a) 3^{rd} and 4^{th} (b) 4^{th} and 5^{th}

(c) 5^{th} and 6^{th} (d) 6^{th} and 7^{th}

69. Constant term in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is

(a) 152 (b) – 152

(c) – 252 (d) 252

70. The coelficent of x^4 in $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is

(a) $\frac{405}{256}$ (b) $\frac{504}{259}$

(c) $\frac{450}{263}$ (d) none of these

Directions: Each of these questions contains two statements. Assertion and Reason. Each of these questions also has four alternative choices. Only one of which is the correct answer. You have to select one of the codes. (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct. Reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct: reason in not a correct explanation for assertion.

(c) Assertion is correct, reason is incorrect.

(d) Assertion is incorrect, reason is correct.

71. **Assertion:** The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$.

Reason: The coefficient of x^6 in the expansion of $(1 + x)^n$ is ${}^nC_6$.

72. **Assertion:** The r^{th} term from the end in the expansion of $(x + a)^n$ is ${}^nC_{n-r+1}\, x^{r-1}\, a^{n-r+1}$.

Reason: The r^{th} term from the end in the expansion of $(x + a)^n$ is $(n - r + 2)^{th}$ term.

73. **Assertion:** In the expansion of $(x + 2y)^8$, the middle term is 4^{th} term.

Reason: If n is even in the expansion of $(a + b)^n$, then $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term.

74. **Assertion:** General term of the expansion $(x + 2y)^9$ is ${}^9C_r 2^r . x^{9-r} y^r$.

Reason: General term of the expansion $(x + a)^n$ is given by $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

75. **Assertion:** In the binomial expansion $(a + b)^n$, r^{th} term is ${}^nC_r a^{n-r} . b^r$.

Reason: If n is odd, then there are two middle terms.

ANSWERS

1. 29

2. $\frac{x^3}{a^3}-\frac{6x^2}{a^2}+15\frac{x}{a}-20+15\frac{a}{x}-\frac{6a^2}{x^2}+\frac{a^3}{x^3}$

3. ${}^{2n+1}C_7$

4. $(1.2)^{4000}$

5. –1365

6. 256

7. n = 10

8. 18

9. ${}^{9}C_3 \times \left(\frac{-1}{3}\right)^3$

10. ${}^{10}C_5$

11. 3

12. 152

13. $\frac{672}{x^3}$

14. 3

15. ${}^{10}C_5$

16. (i) 6 (ii) 4 (iii) 8 (iv) 5

17. (i) 884736 (ii) 104060401 (iii) 105101.00501

18. 6 terms (0, 10, 20, 30, 40, 40, 50)

19. $-252 = -{}^{10}C_5$

20. $\frac{672}{x^3}$

21. $1120\, x^4\, y^4$

22. $\frac{-15309}{8}$

23. ${}^{25}C_{15} \times \frac{2^{10}}{x^{20}}$

24. a = 3, b = 2, n = 3

25. 9^{th} term

26. $T_3 = \frac{5}{6}$

27. 82

28. 28

29. 10

31. $k = \pm 3$

32. 129 integral terms

34. $n = 8, 4^{th}, 5^{th}$ and 6^{th}

38. $r = 6$

40. $n = 41$

42. $a = 2, x = 1, n = 7$

44. 9

45. $p = \pm 2$

47. 2

48. 198

49. $\frac{17}{54}$

50. 5

52. $n = 7$ or 14

55. ${}^{2n}C_{n-1}$

56. (b)

57. (b)

58. (a)

59. (d)

60. (c)

61. (c)

62. (a)

63. (b)

64. (b)

65. (d)

66. (c)

67. (b)

68. (c)

69. (c)

70. (a)

71. (d)

72. (a)

73. (d)

74. (a)

75. (d)