CHAPTER - 6

PERMUTATIONS AND COMBINATIONS

KEY POINTS

- **Fundamental principle of counting**

- **Multiplication Principle:** If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of simultaneous occurrence of the two events in order is m × n.

- **Fundamental Principle of Addition:** If there are two events such that they can occur independently in m and n different ways respectively, then either of the two events can occur in (m + n) ways.

- **Factorial:** Factorial of a natural number n, denoted by n! or n is the continued product of first n natural numbers.

  n! = n × (n – 1) × (n – 2) × ……... × 3 × 2 × 1

  = n × ((n – 1)!)

- **Permutation:**A permutation is an arrangement of a number of objects in a definite order taken some or all at a time.

- The number of permutation of n different objects taken r at a time where0 $\leq$ r $\leq$ n and the objects do not repeat is denoted by ${}^{n}P_{r}$ or P(n, r) where,

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

- The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.
- The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, …….. p_k are of k^{th} kind and the rest if any, are of different kinds, is $\frac{n!}{(p_1!)(p_2!)\ldots\ldots(p_k!)}$

▶ **Combination:** Each of the different selections made by choosing some or all of a number of objects, without considering their order is called a combination. The number of combination of n distinct objects taken r at a time where,

$0 \le r \le n$, is denoted by ${}^nC_r$ or C(n, r) where ${}^nC_r = \frac{n!}{r!(n-r)!}$

▶ **Some important result:**

(i) $0! = 1$

(ii) ${}^nC_0 = {}^nC_n = 1$

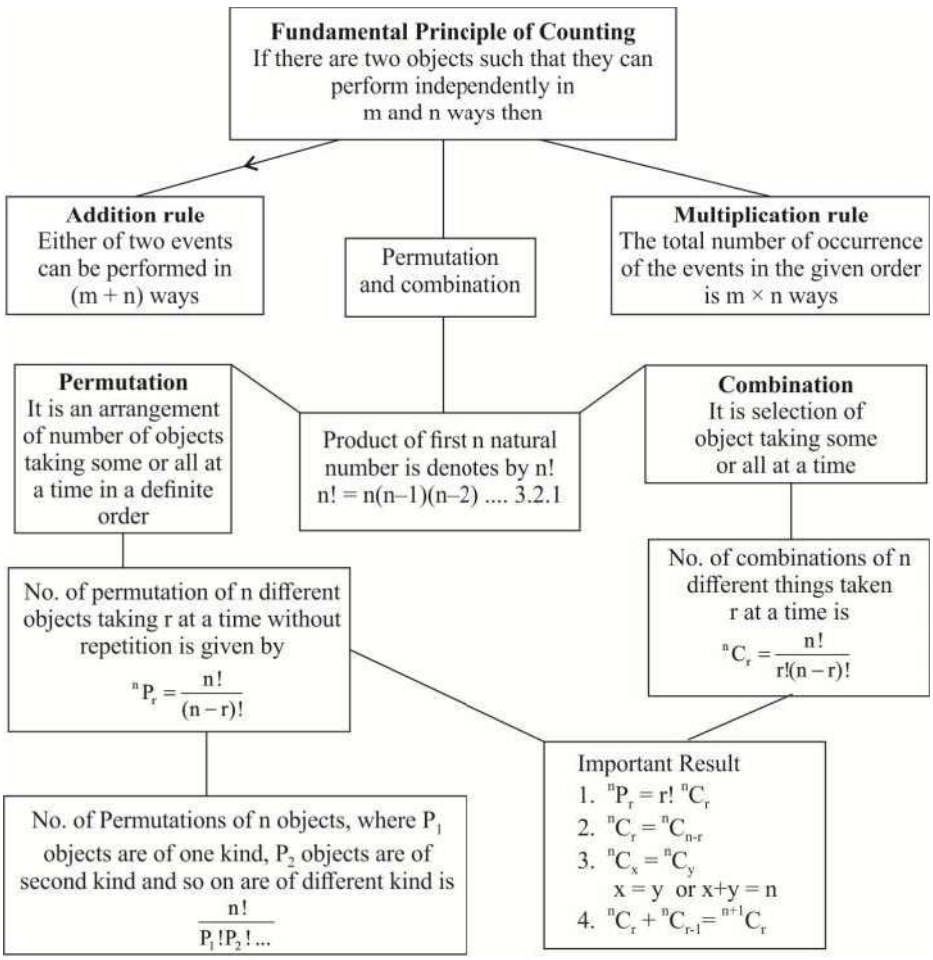
(iii) ${}^nC_r = {}^nC_{n-r}$ where $0 \le r \le n$, and r are positive integers

(iv) ${}^nP_r = \lfloor n \, {}^nC_r$ where $0 \le r \le n$, r and n are positive integers.

(v) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$ where $0 \le r \le n$ and r and N are positive integers.

(vi) If ${}^nC_a = {}^nC_b$ if either a = b or a + b = n

MIND MAP



VERY SHORT ANSWER TYPE QUESTIONS

1. How many ways are there to arrange the letters of the word "GARDEN" with the vowels in alphabetical order?

2. In how many ways 7 pictures can be hanged on 9 pegs?

3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?

4. There are 10 points on a circle. By joining them how many chords can be drawn?

5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?

6. If ${}^{n}P_4 : {}^{n}P_2 = 12$, find n.

7. How many different words (with or without meaning) can be made using all the vowels at a time?

8. In how many ways 4 boys can be chosen from 7 boys to make a committee?

9. How many different words can be formed by using all the letters of word “SCHOOL”?

10. In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

11. A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them if he has three servants to carry the cards?

12. If there are 12 persons in a party, and if each two of them Shake hands with each other, how many handshakes happen in the party?

13. If ${}^{20}C_r = {}^{20}C_{r-10}$ then find the value of ${}^{18}C_r$

SHORT ANSWER TYPE QUESTIONS

14. Find n, ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$.

15. If ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$, find r.

16. If ${}^{n}P_{r} = 336$, ${}^{n}C_{r} = 56$, find n and r. Hence find ${}^{n-1}C_{r-1}$.

17. A convex polygon has 65 diagonals. Find number of sides of polygon. (Hint: No. Of diagonals = ${}^{n}C_{2} - n$)

18. In how many ways can a cricket team of 11 players be selected out of 16 players, if two particular players are always to be selected?

19. From a class of 40 students, in how many ways can five students be chosen for an excursion party.

20. In how many ways can the letters of the word "ABACUS" be arranged such that the vowels always appear together?

21. If ${}^{n}C_{12} = {}^{n}C_{13}$ then find the value of the ${}^{25}C_{n}$.

22. In how many ways can the letters of the word "PENCIL" be arranged so that I is always next to L.

23. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seats of their choice.

24. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

25. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

26. A student has to answer 10 questions, choosing at least 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questions?

27. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

28. If the letters of the word 'PRANAV' are arranged as in dictionary in all possible ways, then what will be 182^{nd} word.

29. From a class of 15 students, 10 are to chosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

30. Using the letters of the word, 'ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

31. A polygon has 35 diagonal. Find the number of its sides.

32. Determine the number of 5 cards combinations out of a pack of 52 cards if at least 3 out of 5 cards are ace cards?

33. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

34. Find the number of all possible arrangements of the letters of the word "MATHEMATICS" taken four at a time.

35. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if a team has:

    (i) no girl

    (ii) at least 3 girls

    (iii) at least one girl and one boy?

36. In an election, these are ten candidates and four are to be elected. A voter may vote for any number of candidates, not greater than the number to be elected. If a voter vote for at least one candidate, then find the number of ways in which he can vote.

37. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in

how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

LONG ANSWER TYPE QUESTIONS

38. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

39. There are 15 points in a plane out of which only 6 are in a straight line, then
    (i) How many different straight lines can be made?
    (ii) How many triangles can be made?

40. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that
    (i) No two girls sit together?
    (ii) All the girls never sit together?

41. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

42. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,
    (i) 3 are red and 1 is black.
    (ii) All 4 cards are from different suits.
    (iii) Atleast 3 are face cards.
    (iv) All 4 cards are of the same colour.

43. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?

44. How many different four letter words can be formed (with or without meaning) using the letters of the word “MEDITERRANEAN” such that the first letter is E and the last letter is R.

45. If all letters of word ‘MOTHER’ are written in all possible orders and the word so formed are arranged in a dictionary order, then find the rank of word ‘MOTHER’?

46. From 6 different novels and 3 different dictionaries, 4 novels and a dictionary is to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then find the number of such arrangements.

47. The set S= {1,2,3,…….12} is to be partitioned into three sets A, B, and C of equal sizes. $A \cup B \cup C = S$, $A \cap B = B \cap C = C \cap A = \phi$. Find the number of ways to partition S.

48. If ${}^{15}C_{3r} : {}^{15}C_{r+1} = 11 : 3$, find r

CASE STUDY TYPE QUESTIONS

49. Anita is doing an experiment in which she has to arrange the alphabets of the word “HARYANA” in all possible orders and notes the observations. Help her to find the answers of the following:-

i. Number of words starting with A

(a) 360 (b) 720

(c) 1440 (d) 2880

ii. Number of words having H at end

(a) 72 (b) 120

(c) 240 (d) 480

iii. Number of words having Hand N together

(a) 120 (b) 60

(c) 280 (d) 240

iv. Number of words having begin with H and end with N

(a) 20 (b) 24

(c) 60 d) 48

v. Number of words having vowels together

(a) 240 (b) 120

(c) 240 (d) 720

50. A Company wants to appoint 5 persons, 3 for post A and 2 for post B for its upcoming office in Delhi. They have invited the applications for the same. 14 candidates have applied for the postA and 13 have applied for the post B

i. Find the total number of ways in which the company can make a selection for all the posts.

(a) 5! (b) C(14,3).C(13,2)

(c) P(13,2)P(14,3) (d) none of these

ii. Find the number of ways of selecting one woman for each post, if 3 women have applied for post A and 7 women have applied for post B

(a) 6 (b) 21

(c) 6930 (d) 182

iii. On the day of interview, the candidates were seated in a hall having two chambers. The chairs in both the chambers are placed in line. If the candidates for the two posts are to be seated in two different chambers. Find the total number of ways in which all the candidate can be seated.

(a) 3!2! (b) 11!11!

(c) 14!13! (d) 14! × 13! × 2

iv. During appointment procedure they came to know about a candidate whose resume is excellent and should be selected for the post B. In how many ways can the total selections now be made?

(a) 12 × C(14,3) (b) 4

(c) 168 (d) 13 × C(13,3)

v. While checking the applications the management observed that one candidate each who have applied for post A and B are not fit for the job. So they cannot be appointed. In how many ways can now the post is filled?

(a) 2184 (b) 24024

(c) 18876 (d) 1716

Multiple Answer Type Questions

51. What is the number of ways of arrangement of letters of word 'BANANA' so that no two N's are together -

(a) 40 (b) 60

(c) 80 (d) 100.

52. What is the value of n, if P(15, n – 1) : P(16, n – 2) = 3 : 4 ?

(a) 10 (b) 12

(c) 14 (d) 15.

53. The number of words which can be formed from the letters of the word MAXIMUM, if two consonants can't occur together is -

(a) 4! (b) 3! × 4!

(c) 7! (d) None of these.

54. If 7 points out of 12 are in the same straight line, then what is the number of triangles formed?

(a) 84 (b) 175

(c) 185 (d) 201

55. In how many ways can be bowler take four wickets in a single 6 balls over?

(a) 6 (b) 15

(c) 20 (d) 30.

56. What is the number of signals that can be sent by 6 flags of different colours taking one or more at a time?

(a) 45 (b) 63

(c) 720 (d) 1956.

57. There are 6 letters and 3 post boxes. The number of ways in which these letters can be posted is –

(a) 6^3 (b) 3^6

(c) ${}^6P_3$ (d) ${}^6C_3$.

58. If ${}^mC_1 = {}^nC_2$, then –

(a) 2m = n (b) 2m = n(n + 1)

(c) $2m = n(n-1)$ (d) $2n = m(m-1)$.

59. ${}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{x}$, then x = ?

(a) r (b) r – 1

(c) n (d) r + 1.

60. ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$, then value of r is –

(a) 12 (b) 8

(c) 6 (d) 10.

61. If ${}^{n}P_{s} = 60\ {}^{n-1}P_{3}$ the value of *n* is

(a) 6 (b) 10

(c) 1 (d) 16

62. The number of ways 10 digit numbers can be written using the digits 1 and 2 is

(a) 2^{10} (b) ${}^{10}C_{2}$

(c) 10! (d) ${}^{10}C_{1} + {}^{9}C_{2}$

63. The number of ways in which 8 students can be seated in a line is

(a) 5040 (b) 50400

(c) 40230 (d) 40320

64. There are 10 true-false questions in an examination. These questions can be answered in

(a) 20 ways (b) 100 ways

(c) 512 ways (d) 1024 ways

65. In how many ways can we paint the six faces of a cube with six different colours?

(a) 30 (b) 6

(c) 6! (d) None of these

Directions: Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) givne below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion in incorrect, reason is correct.

66. **Assertion:** If the letters W.I.F.E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24^{th} position.

Reason: The number of ways of arranging four distinct objects taken all at a time is C(4, 4).

67. **Assertion:** A number of four different digits is formed with the help of the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways. Then, number of numbers which are exactly divisible by 4 is 200.

Reason: A number divisible by 4, if unit place digit is divisible by 4.

68. **Assertion:** Product of five consecutive natural numbers is divisible by 4!

Reason: Product of n consecutive natural number is divisible by (n + 1)!

69. **Assertion:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^{9}C_{3}$.

Reason: The number of ways of choosing any 3 places, from 9 different places is $^{9}C_{3}$.

70. **Assertion:** A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 with repetition. The total number formed are 216.

Reason: If sum of digits of any number is divisible by 3 then the number must be divisible by 3.

ANSWERS

1. $\frac{6!}{2} = 360$
2. $\frac{9!}{2!}$
3. 100
4. 45
5. 120
6. n = 6
7. 120
8. 35
9. 360
10. 63
11. 3^6=729
12. 66
13. 816
14. n = 9
15. r = 7
16. n = 8, r = 3 and 21
17. 13
18. 2002
19. 40_{C_5}
20. $\frac{3!}{2!} \times 4!$
21. 1
22. 120
23. $90 \times {}^{10}P_8$
24. 56
25. 350
26. 266
27. 36
28. PAANVR
29. ${}^{13}C_{10} + {}^{13}C_8$
30. 5040
31. 10
32. 4560
33. 576
34. 2454
35. (i) 21; (ii) 91; (iii) 441

36. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

37. 48,144

38. 420

39. (i)91

    (ii)435

40. (i)$7! \times {}^{8}P_5$

    (ii)$12! - 8! \times 5!$

41. 24480

42. ${}^{52}C_4$

    (i) ${}^{26}C_1 \times {}^{26}C_3$

    (ii) $(13)^4$

    (iii) 9295 (Hint : Face cards : 4J + 4K + 4Q)

    (iv) $2 \times {}^{26}C_4$

43. 265 (*Hint* : make 3 cases i.e.

    (i) All 3 letters are different

    (ii) 2 are identical 1 different

    (iii) All are identical, then form the words.)

44. 59

45. 309

46. $4! {}^{6}C_4 \, {}^{3}C_1$

47. ${}^{12}C_4 \, {}^{8}C_4 \, {}^{4}C_4$

48. r = 3

49. i. (a) ii. (b) iii. (d) iv. (a) v. (b)

50. i. (b) ii. (c) iii. (d) iv. (a) v. (c)

51. (a) 52. (c) 53. (a)

54. (c) 55. (b) 56. (b)

57. (b) 58. (c) 58. (d)

60. (a) 61. (b) 62. (a)

63. (d) 64. (d) 65. (a)

66. (c) 67. (c) 68. (c)

69. (a) 70. (d)