CHAPTER - 5

LINEAR INEQUALITIES

KEY POINTS

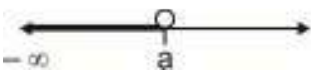
- **Inequalities:** A statement involving '<', '>','≥'or '≤'is called inequality.
  - Inequalities which do not involve variables are called numerical inequalities.
  - Inequalities which involve variables are called literal inequalities.
    Eg., $3x - 4 \leq 15$ and $4x - 3y \geq 5$
  - Inequalities involving the symbols '>' or '<' are called strict inequalities.
  - Inequalities involving the symbols '≥' or '≤' are called slack inequalities.
- **Linear inequalities in one variable:** The inequalities of form $ax + b > 0$, $ax + b < 0$, $ax + b \geq 0$ or $ax + b \leq 0$; $a \neq 0$ are called linear inequalities in one variable.

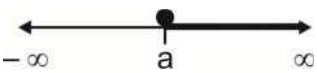
  The set of real numbers which satisfy a given linear in equality is called the solution set of the inequality.
- **Algebraic solutions of linear inequalities in one variables:**
  - **Rule-1**
    Equal numbers may be added (or subtracted from) to both sides without affecting sign of inequalities.

- **Rule-2**
  (i) If both sides of inequality are multiplied (or divided) by same positive number, then sign of inequality remains unchanged.
  (ii) If both sides are multiplied (or divided) by any negative number, then sign of inequality is reversed.

▶ **Graphical representation of solutions on number line:**

(i) $x > a \Leftrightarrow a < x < \infty \Leftrightarrow x \in (a, \infty) \Leftrightarrow$ 

(ii) $x < a \Leftrightarrow -\infty < x < a \Leftrightarrow x \in (-\infty, a) \Leftrightarrow$ 

(iii) $x \geq a \Leftrightarrow a \leq x < \infty \Leftrightarrow x \in [a, \infty) \Leftrightarrow$ 

(iv) $x \leq a \Leftrightarrow -\infty < x \leq a \Leftrightarrow x \in (-\infty, a] \Leftrightarrow$ 

(v) $a < x < b \Leftrightarrow x \in (a, b) \Leftrightarrow$ 

(vi) $a \leq x \leq b \Leftrightarrow x \in [a, b] \Leftrightarrow$ 

▶ **Linear inequalities in two variables:** The inequalities of form $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c \geq 0$ or $ax + by + c \leq 0$ are linear inequalities in two variables. $(a, b \neq 0)$

Eg., $4x - 3y < 15$ and $-4x + 15y + 3 \geq 4$

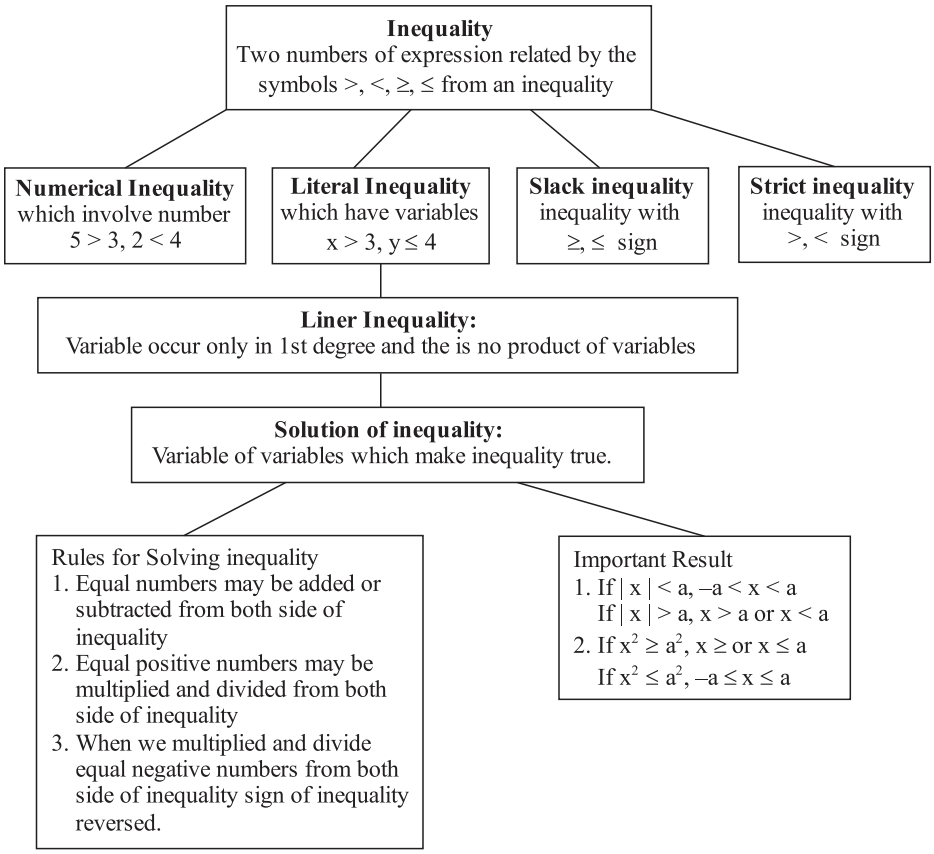
▶ **Graphical solution of linear inequalities in two variables**

- A line divides the Cartesian plane into two parts. Each part is known as a half plane.

- The region containing all the solutions of the inequality is called solution region.
- In order to identify the half plane represented by an inequality (solution region), it is just sufficient to take any point (a, b) not on the line and check whether it satisfy the inequality or not.
  (i) If it satisfies, then the regions containing that point (a, b) is solution region.
  (ii) If it does not satisfy, then the other region is solution region.
- If inequality contains '$\geq$' or '$\leq$', then points on line ax + by = c are also included in solution region. In this case we draw dark line while sketching graph of ax + by = c.
- If inequality contains '>' or '<', then points on line ax + by = c are not included in solution region. In this case we draw dotted line while sketching graph of ax + by = c.

Note: While solving system of linear inequalities in two variables, the common of solution regions of each inequality is solution region of system.

MIND MAP



VERY SHORT ANSWER TYPE QUESTIONS

1. Solve $5x < 24$ when $x \in N$

2. Solve $3 - 2x < 9$ when $x \in R$. Express the solution in the form of interval.

3. Show the graph of the solution of $2x - 3 > x - 5$ on number line.

4. Solve $0 < \frac{-x}{3} < 1$, $x \in R$

5. Solve $-3 \leq -3x + 2 < 4$, $x \in R$.

6. Draw the graph of the solution set of $x + y \geq 4$.

7. Draw the graph of the solution set of $x < y$.

8. Solve the inequality for real x: $\frac{x^2}{x-2} > 0$.

SHORT ANSWER TYPE QUESTIONS

9. Solve $\frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0, \quad x \in R.$

10. Solve $\frac{x+3}{x-1} > 0, \quad x \in R.$

Solve the inequalities for real x and represent solution on number line

11. $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}, \quad x \in R.$

12. $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2, \quad x \in R.$

13. $-5 \leq \frac{2-3x}{4} \leq 9, \quad x \in R.$

14. $\frac{x+3}{x-2} > 0, \quad x \in R$

15. $\frac{x-3}{x-5} > 2$

16. $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$

17. $\frac{2x+3}{x-3} \leq 4$

18. Find the pair of consecutive even positive integers which are greater than 5 and are such that their sum is less than 20.

19. A company manufactures cassettes and its cost and revenue functions are C(x) = 26000 + 30x and R(x) = 43x respectively, where x is number of cassettes produced and sold in a week. How many cassettes must be sold per week to realise some profit. [Profit = R(x) – C(x)]

20. While drilling a hole in the earth, it was found that the temperature (T°C) at x km below the surface of the earth was given by T = 30 + 25(x – 3), when$3 \leq x \leq 15$.

    Between which depths will the temperature be between 200°C and 300°C?

21. The water acidity in a pool is considered normal when the average PH reading of their daily measurements is between 7.2 and 7.8. If the first two PH reading are 7.48 and 7.85. Find the range of PH value for the 3rd reading that will result in acidity level being normal.

Solve the following systems of inequalities for all $x \in R$

22. $2(2x+3)-10<6(x-2), \quad \frac{2x-3}{4}+6 \geq 4+\frac{4x}{3}$

23. $|2x-3| \leq 11, \quad |x-2| \geq 3$

24. $\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4}, \quad \frac{7x-1}{3}-\frac{7x+2}{6}>x$

25. Solve $\frac{|x|-1}{|x|-2} \geq 0 \quad x \in R, \quad x \neq \pm 2$

26. In the first four papers each of 100 marks, Rishi got 95, 72, 73, 83 marks. If he wants an average of greater than or equal to 75 marks be should score in fifth paper.

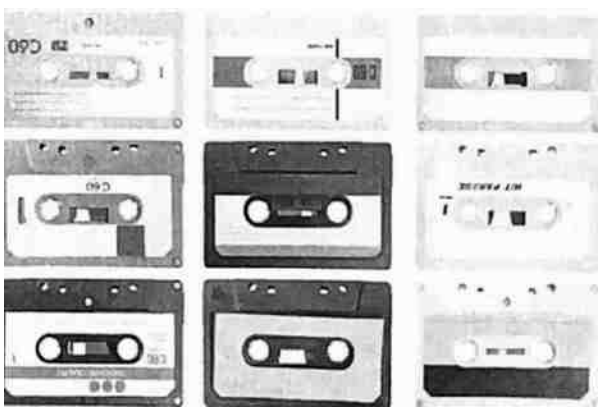
27. A milkman has 80% milk in this stock of 800 litres of adultered milk. How much 100% pure milk is to be added to it so that purity is between 90% and 95%?

28. $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}, \quad \frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$

29. $\frac{x}{2x+1} \geq \frac{1}{4}, \quad \frac{6x}{4x-1} < \frac{1}{2}$

30. $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 45$.

31. A company produced cassettes, one cassette Cost Company Rs. 30 and also an additional fixed cost 26000 per week. The company sold each Cassette at Rs. 43. If x is number of cassettes produced and sold by the company in a week. From the following information find



i. The cost function of the company

(a) 26000 + 30x (b) 26000+43x

(c) 30 +26000x (d) 43 + 26000x

ii. The revenue function of the company

(a) 30x (b) 26000x

(c) 43x (d) 13x

iii. The profit function of the company

(a) –26000 + 73x (b) –26000+ 13x

(c) 26000+43x (d) 26000+ 30x

iv. How many cassettes must be produced by the company in a week to realize some profit?

(a) more than 2000 (b) less than 2000

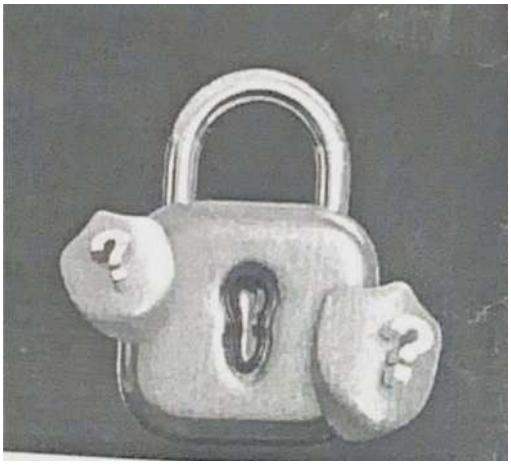
(c) more than 5000 (d) less than 5000

v. If company incurred an additional cost of Rs. 3 on each cassette per week. How many cassettes must be produced by the company in a week so that there is no profit no loss?

(a) 2000 (b) 5000

(c) 2600 (d) 1000

32. A and B tried to find the solution of the inequality $|x - 1| + |x - 2| \geq 4$. Help them to find the solution of the inequality



i. When $x < 1$

(a)$(-\infty, -1/2)$ (b)$(-\infty, -1)$

(c)$(-\infty, -1/2]$ (d)$(-\infty, 1/2)$

ii. When $1 \leq x < 2$

(a)$(-\infty, \infty)$ (b)$(-\infty, -1)$

(c) Infinite solution (d) no solution

iii. When $2 \leq x < \infty$

(a)$(-\infty, \infty)$ (b) $(7/2, \infty)$

(c) $[7/2, \infty)$ (d) no solution

iv. When $x \in R$

(a)$(-\infty, -1/2] \cup [7/2, \infty)$ (b) $(-\infty, -1/2) \cup [7/2, \infty)$

(c)$(-\infty, -1/2] \cup (7/2, \infty)$ (d) $(-\infty, -7/2] \cup [1/2, \infty)$

v. When $x > 4$

(a)$(-\infty, 4)$ (b) $(-\infty, 4]$

(c) $(4, \infty)$ (d) $[4, \infty)$

33. A student have solution of 640 litres of 8% boric acid. He wants to dilute it by using 2% solution of boric acid.

(a) How many minimum litres of 2% boric acid he must add so that resulting solution have more than 4% boric acid?

(b) How many minimum litres 2% boric acid he must add so that resulting soltution have less than 6% boric acid?

(c) How many litres of water he must add so that resulting solution have more 4% but less than 6% of boric acid?

34. If $|x + 3| \geq 10$, then

(a) $x \in (-13, 7]$ (b) $x \in (-13, 7]$

(c) $x \in (-\infty, -13] \cup [7, \infty)$ (d) $x \in [-\infty, -13] \cup [7, \infty)$

35. If $\frac{|x-7|}{(x-7)} \geq 0$, then

(a) $x \in [7, \infty)$ (b) $x \in (7, \infty)$

(c) $x \in (-\infty, 7)$ (d) $x \in (-\infty, 7]$

Multiple Choice Questions

36. If $-4x > 20$ and $x \in z^+$ then x belongs to -

(a) {–6, –7, –8, ……..} (b) ϕ

(c) {–4, –3, –2, –1} (d) {1, 2, 3, 4, ……..}.

37. If $\frac{x-3}{x-2} > 0$ then x belongs to -

(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, -3) \cup (-5, \infty)$

(c) $(-\infty, 3] \cup [5, \infty)$ (d) (3, 5)

38. Solution set for inequality $|x - 1| \leq 5$ is -

(a) [–6, 4] (b) [–4, 0]

(c) [–4, 6] (d) [0, 6].

39. Solution set for inequality $\frac{1}{x-2} < 0$ is -

(a) $(2, \infty)$ (b) ϕ

(c) $(0, 2)$ (d) $(-\infty, 2)$.

40. Solution set for inequality $5x - 3 < 3x + 1$, $x \in N$ is -

(a) $(-\infty, 2)$ (b) $\{0, 1, 2\}$

(c) $\{1\}$ (d) ϕ.

41. Which of the following point lies in solution region of inequality $3x - y \leq 5$?

(a) (5, 1) (b) (1, 5)

(c) (2, 0) (d) (2, –1).

42. If $x > 0$ and $y < 0$ then (x, y) lies in -

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant.

43. If $x^2 > 9$ then x belongs to -

(a) (–3, 3) (b) (0, 3)

(c) $(3, \infty)$ (d) $(-\infty, -3) \cup (3, \infty)$.

44. Solution set for inequality $-8x \leq 5x - 3 < 7$ is -

(a) (–1, 2) (b) (2, 3)

(c) [–1, 2) (d) [2, 3].

45. The graph of the inequalities

$x \leq 0$, $y \geq 0$, $2x + y + 6 \leq 0$ is

(a) a triangle (b) a square

(c) unbounded (d) none of these

Directions: Each of these questions contains two statements Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is correct explanation for assertion.
(b) Assertion is correct, reason is correct, reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reason is correct.

45. **Assertion:** The inequality ax + by < 0 is strict inequality.

Reason: The inequality ax + b $\leq$ 0 is slack inequality.

46. **Assertion:** The inequality ax + by < 0 is strict inequality.
Reason: The inequality ax + b $\leq$ 0 is slack inequality.

47. **Assertion:** If a < b, c < 0, then $\frac{a}{c} < \frac{b}{c}$.

Reason: If both sides are divided by the same negative quantity, then the inequality is reversed.

48. **Assertion:** $|3x - 5| > 9 \Rightarrow x \in \left(-\infty, \frac{-4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$

Reason: The region containing all the solutions of an inequality is called the solution region.

49. **Assertion:** A line divides the Cartesian plane in two part(s).

Reason: If a point P(α, β) on the line ax + by = c, then aα + bβ = c.

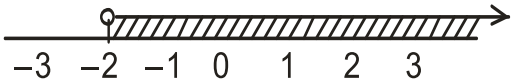
50. **Assertion:** Each part in which a line divides the Cartesian plane, is known as half plane.

Reason: A point in the Cartesian palne will either lie on a line or will lie in either of half plane **I** or **II**.

ANSWERS

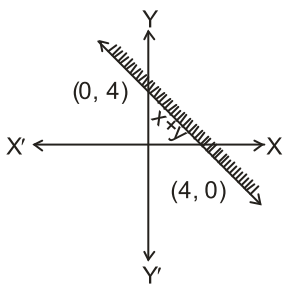
1. {1,2,3,4}

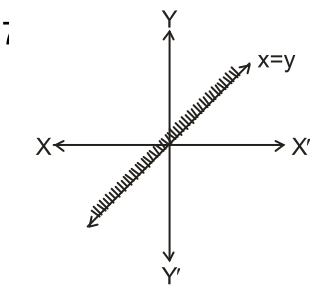
2. $(-3, \infty)$

3. 

4. $-3 < x < 0$

5. $\left(\frac{-2}{3}, \frac{5}{3}\right]$

6. 



8. $[2, -\infty]$

9. $[-\infty, -1] \cup [2, 3] \cup (4, \infty)$

10. $(-\infty, -3) \cup (1, \infty)$

11. $\left(-\infty, \frac{63}{10}\right]$

12. $\left(-\infty, \frac{-13}{2}\right)$

13. $\left[\frac{-34}{3}, \frac{22}{3}\right]$

14. $(-\infty, -3) \cup (2, \infty)$

15. (5, 7)

16. $(-\infty, 2]$

17. $(-\infty, -3) \cup \left[\frac{15}{2}, \infty\right)$

18. (6, 8) and (8, 10)

19. More than 2000 cassettes

20. Between 9.8 m and 13.8 m

21. Between 6.27 and 8.07.

22. Solution set = ϕ

23. $[-4, -1] \cup [5, 7]$

24. (4, 9)

25. $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

26. He must score greater than or equal to 52 and less than 77.

27. Between 800 litre and 2400 litre

28. $(3, \infty)$ 

29. No solution

30. $[-7, 11]$ 

31. i. (a) ii. (c) iii. (b) iv. (a) v. (c)

32. i. (c) ii. (d) iii. (c) iv. (a) v. (c)

33. i. 1280 L ii. 320 L iii. $\frac{640}{3} < x < 640$

34. (c)

35. (b)

36. (b) 37. (a) 38. (c)

39. (d) 40. (c) 41. (b)

42. (d) 43. (d) 44. (c)

45. (c) 46. (b) 47. (d)

48. (b) 49. (b) 50. (b)