

Unit - 10-A

Mechanical Waves Oscillation & Waves

10.1 Periodic Motion

A motion, which repeats itself over and over again after a regular interval of time is called a periodic motion and the fixed interval of time after which the motion is repeated is called period of the motion. *Examples* : Revolution of earth around the sun (period one year).

10.2 Oscillatory or Vibratory Motion.

The motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time. Oscillatory motion is also called as harmonic motion. *Example* : The motion of the pendulum of a wall clock.

10.3 Harmonic and Non-harmonic Oscillation.

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (*i.e.* sine or cosine function). *Example* : $y = a \sin \omega t$ or $y = a \cos \omega t$.

Non-harmonic oscillation is that oscillation which can not be expressed in terms of single harmonic function. *Example* : $y = a \sin \omega t + b \sin 2 \omega t$.

10.4 Some Important Definitions.

- (1) **Time period** : It is the least interval of time after which the periodic motion of a body repeats itself. S.I. units of time period is second.
- (2) **Frequency** : It is defined as the number of periodic motions executed by body per second. S.I unit of frequency is hertz (Hz).
- (3) **Angular Frequency** : $\omega = 2\pi n$
- (4) **Displacement**: Its deviation from the mean position.

- (5) **Phase :** It is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

$y = a \sin \theta = a \sin (\omega t + \phi_0)$ here $\theta = \omega t + \phi_0$ = phase of vibrating particle.

- (i) **Initial phase or epoch :** It is the phase of a vibrating particle at $t = 0$.
- (ii) **Same phase:** Two vibrating particle are said to be in same phase, if the phase difference between them is an even multiple of n or path difference is an even multiple of $(\lambda/2)$ or time interval is an even multiple of $(T/2)$.
- (iii) **Opposite phase :** Opposite phase means the phase difference between the particle is an odd multiple of π or the path difference is an odd multiple of λ or the time interval is an odd multiple of $(T/2)$.
- (iv) **Phase difference :** If two particles performs S.H.M and their equation are $y_1 = a \sin (\omega t + \phi_1)$ and $y_2 = a \sin (\omega t + \phi_2)$ then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

10.5 Simple Harmonic Motion.

Simple harmonic motion is a special type of periodic motion, in which Restoring force \propto Displacement of the particle from mean position.

$$F = -kx$$

Where k is known as force constant. Its S.I. unit is Newton/meter and dimension is $[MT^{-2}]$.

10.6 Displacement in S.H.M.

Simple harmonic motion is defined as the projection of uniform circular motion on any diameter of circle of reference

- (i) $y = a \sin \omega t$ when at $t = 0$ the vibrating particle is at mean position.
- (ii) $y = a \cos \omega t$ when at $t = 0$ the vibrating particle is at extreme position.
- (iii) $y = a \sin (\omega t \pm \phi)$ when the vibrating particle is ϕ phase leading or lagging from the mean position.

10.7 Comparative Study of Displacement, Velocity and Acceleration.

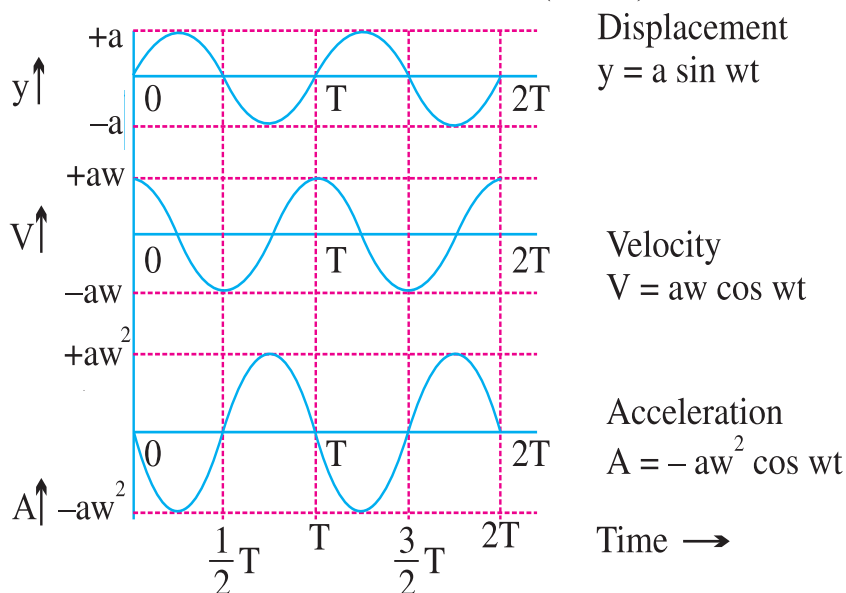
Displacement $y = a \sin \omega t$

Velocity $v = a\omega \cos \omega t$

$$\omega t = a\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$

Acceleration $A = -a\omega^2 \sin \omega t$

$$\omega t = a\omega^2 \sin(\omega t + \pi)$$



- (i) All the three quantities displacement, velocity and acceleration show harmonic variation with time having same period.
- (ii) The velocity amplitude is ω times the displacement amplitude
- (iii) The acceleration amplitude is ω^2 times the displacement amplitude
- (iv) In S.H.M. the velocity is ahead of displacement by a phase angle $\pi/2$.
- (v) In S.H.M. the acceleration is ahead of velocity by a phase angle $\pi/2$.
- (vi) The acceleration is ahead of displacement by a phase angle of π .
- (vii) Various physical quantities in S.H.M. at different position :

Physical quantities	Equilibrium position ($y = 0$)	Extreme Position ($y = \pm a$)
Displacement $y = a \sin \omega t$	Minimum (Zero)	Maximum (a)
Velocity $v = \omega\sqrt{a^2 - y^2}$	Maximum ($a\omega$)	Minimum (Zero)
Acceleration $A = -\omega^2 y$	Minimum (Zero)	Maximum ($\omega^2 a$)

10.8 Energy in S.H.M.

A particle executing S.H.M. possesses two types of energy : Potential energy and Kinetic energy

(1) **Potential energy :** $U = \frac{1}{2}m\omega^2 a^2 \sin^2 \omega t$

(i) $U_{\max} = \frac{1}{2}ka^2 = \frac{1}{2}m\omega^2 a^2$ when $y = \pm a$; $\omega t = \pi/2$; $t = T/4$

(ii) $U_{\min} = 0$ when $y = 0$; $\omega t = 0$; $t = 0$

(2) **Kinetic energy :**

$$K = \frac{1}{2}ma^2\omega^2 \cos^2 \omega t \text{ or } K = \frac{1}{2}m\omega^2(a^2 - y^2)$$

(i) $K_{\max} = \frac{1}{2}m\omega^2 a^2$ when $y = 0$; $t = 0$; $\omega t = 0$

(ii) $K_{\min} = 0$ when $y = a$; $t = T/4$, $\omega t = \pi/2$

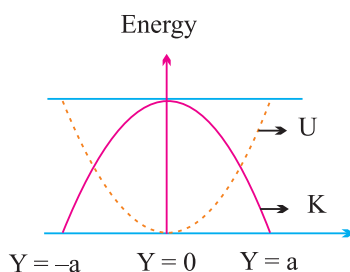
(3) **Total energy :** Total mechanical energy

$$= \text{Kinetic energy} + \text{Potential energy}$$

$$E = \frac{1}{2}m\omega^2 a^2$$

Total energy is not a position function *i.e.* it always remains constant.

(4) **Energy position graph :**



- (5) Kinetic energy and potential energy vary periodically double the frequency of S.H.M.

10.9 Time Period and Frequency of S.H.M.

$$\text{Time period (T)} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{as } \omega = \sqrt{\frac{k}{m}}$$

$$\text{Frequency (n)} = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

In general m is called inertia factor and k is called spring factor.

$$\text{Thus} \quad T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

10.10 Differential Equation of S.H.M.

$$\text{For S.H.M. (linear)} \quad m\frac{d^2y}{dt^2} + ky = 0 \quad [\text{As } \omega = \sqrt{\frac{k}{m}}]$$

$$\text{For angular S.H.M.} \quad \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \quad [\omega^2 = \frac{k}{m}]$$

10.11 Simple Pendulum

Mass of the bob = m

$$\text{Effective length of simple pendulum} = l ; T = 2\pi\sqrt{\frac{l}{g}}$$

- (i) Time period of simple pendulum is independent of amplitude as long as its motion is simple harmonic.
- (ii) Time period of simple pendulum is also independent of mass of the bob.
- (iii) If the length of the pendulum is comparable to the radius of earth

$$\text{then} \quad T = 2\pi\sqrt{\frac{1}{g\left[\frac{1}{l} + \frac{1}{R}\right]}}$$

$$\text{If } l \gg R (\rightarrow \infty) \quad 1/l < 1/R \quad \text{so } T = 2\pi\sqrt{\frac{R}{g}} \cong 84.6 \text{ minutes}$$

- (iv) The time period of simple pendulum whose point of suspension moving horizontally with acceleration,

$$T = 2\pi \sqrt{\frac{l}{(g^2 + a^2)^{1/2}}} \quad \text{and} \quad \theta = \tan^{-1} (a/g)$$

- (v) Second's Pendulum : It is that simple pendulum whose time period of vibrations is two seconds.
- (vi) Work done in giving an angular displacement θ to the pendulum from its mean position.

$$W = U = mgl (1 - \cos \theta)$$

- (vii) Kinetic energy of the bob at mean position = work done or potential energy at extreme.

10.12 Spring Pendulum

A point mass suspended from a mass less spring or placed on a frictionless horizontal plane attached with spring constitutes a linear harmonic spring pendulum

$$\text{Time period } T = 2\pi \sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{and} \quad \text{Frequency } n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

- (i) Time of a spring pendulum is independent of acceleration due to gravity.
- (ii) If the spring has a mass M and mass m is suspended from it, effective

$$\text{mass is given by } m_{\text{eff}} = m + \frac{M}{3}$$

$$\text{So that } T = 2\pi \sqrt{\frac{m_{\text{eff}}}{k}}$$

- (iii) If two masses of mass m_1 and m_2 are connected by a spring and made to oscillate on horizontal surface, the reduced mass m_r is given by



$$\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}$$

So that $T = 2\pi\sqrt{\frac{m_r}{k}}$

- (iv) If a spring pendulum, oscillating in a vertical plane is made to oscillate on a horizontal surface, (or on inclined plane) time period will remain unchanged.

- (v) If the stretch in a vertically loaded spring is y_0 then

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$$

Time period does not depends on 'g' because along with g, y_0 will also change in such a way that $\frac{y_0}{g} = \frac{m}{k}$ remains constant.

- (vi) Series combination : If n springs of different force constant are connected in series having force constant k_1, k_2, k_3, \dots respectively

then

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots$$

- (vii) Parallel combination: If the springs are connected in parallel then $k_{eff} = k_1 + k_2 + k_3 + \dots$

- (viii) If the spring of force constant k is divided in to n equal parts then spring constant of each part will become nk .

- (ix) The spring constant k is inversely proportional to the spring length.

$$\text{As } k \propto \frac{1}{\text{Extension}} \propto \frac{1}{\text{Length of spring}}$$

- (x) When a spring of length l is cut in two pieces of length l_1 and l_2 such that $l_1 = nl_2$.

If the constant of a spring is k then spring constant of first part $k_1 = \frac{k(n+1)}{n}$

Spring constant of second part $k_2 = (n+1)k$ and ratio of spring constant

$$\frac{k_1}{k_2} = \frac{1}{n}.$$

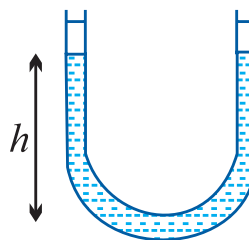
10.13 Various Formulae of S.H.M. .

S.H.M. of a liquid in U tube :

If a liquid of density ρ contained in a vertical U tube performs S.H.M. in its two limbs. Then time period

$$T = 2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{h}{g}}$$

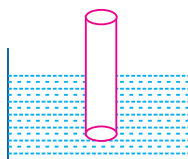
Where L = Total length of liquid column, H = Height of undisturbed liquid in each limb ($L = 2h$)



S.H.M. of a floating cylinder

If l is the length of cylinder dipping in liquid then time period

$$T = 2\pi\sqrt{\frac{l}{g}}$$



S.H.M. of ball in the neck of an air chamber

Image

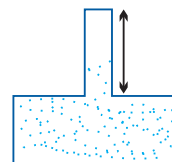
$$T = \frac{2\pi}{A} \sqrt{\frac{mV}{E}}$$

M = mass of the ball

V = volume of air chamber

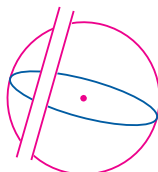
A = area of cross section of neck

E = Bulk modulus for Air



S.H.M. of a body in a tunnel dug along any chord of earth

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ minutes}$$



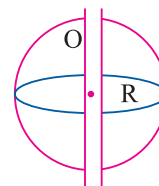
S.H.M. of body in the tunnel dug along the diameter of earth

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$T = 84.6$ minutes

R = radius of the earth = 6400 km

g = acceleration due to gravity = 9.8 m/s^2 at earth's surface



10.14 Free, Damped, Forced and Maintained Oscillation.

(1) Free oscillation

- (i) The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations
- (ii) The amplitude, frequency and energy of oscillation remains constant
- (iii) Frequency of free oscillation is called natural frequency.

(2) Damped oscillation

- (i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation.
- (ii) Amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hysteresis etc.

(3) Forced oscillation

- (i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation.
- (ii) Resonance: When the frequency of external force is equal to the natural frequency of the oscillator. Then this state is known as the state of resonance. And this frequency is known as resonant frequency.

(4) Maintained oscillation : The oscillation in which the loss of oscillator is compensated by the supplying energy from an external source are known as maintained oscillation.

10.15 Wave

A wave is a disturbance which propagates energy and momentum from one place to the other without the transport of matter.

(1) Necessary properties of the medium for wave propagation :

- (i) *Elasticity* : So that particles can return to their mean position, after having been disturbed.
- (ii) *Inertia* : So that particles can store energy and overshoot their mean position.

(iii) Minimum friction amongst the particles of the medium.

(iv) Uniform density of the medium.

(2) Mechanical waves : The waves which require medium for their propagation are called mechanical waves.

Example : Waves on string and spring, waves on water surface, sound waves, seismic waves.

(3) Non-mechanical waves : The waves which do not require medium for their propagation are called non-mechanical or electromagnetic waves.

Examples : Light, heat (Infrared), radio waves, γ -rays. X-rays etc.

(4) Transverse waves : Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.

(i) It travels in the form of crests and troughs.

(ii) A crest is a portion of the medium which is raised temporarily.

(iii) A trough is a portion of the medium which is depressed temporarily.

(iv) Examples of transverse wave motion: Movement of string of a sitar, waves on the surface of water.

(v) Transverse waves can not be transmitted into liquids and gases.

(5) Longitudinal waves: If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.

(i) It travels in the form of compression and rarefaction.

(ii) A compression (c) is a region of the medium in which particles are compressed.

(iii) A rarefaction (R) is a region of the medium in which particles are rarefied.

(iv) Examples sound waves travel through air in the form of longitudinal waves.

(v) These waves can be transmitted through solids, liquids and gases.

10.16 Important Terms

(1) Wavelength :

- (i) It is the length of one wave.
- (ii) Distance travelled by the wave in one time period is known as wavelength.

λ = Distance between two consecutive crests or troughs.

(2) Frequency : Number of vibrations completed in one second.

(3) Time period : Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

(4) Relation between frequency and time period :

Time period = 1 / Frequency

$$\Rightarrow T = 1/n$$

(5) Relation between velocity, frequency and wavelength : $v = n\lambda$.

10.17 Velocity of Sound (Wave motion)

(1) Speed of transverse wave motion :

- (i) On a stretched string : $v = \sqrt{\frac{T}{m}}$, T = Tension in the string;
 m = Linear density of string (mass per unit length).
- (ii) In a solid body : $v = \sqrt{\frac{\eta}{\rho}}$ (η = Modulus of rigidity; ρ = Density of the material.)

(2) Speed of longitudinal wave motion :

- (i) In a solid long bar $v = \sqrt{\frac{Y}{\rho}}$ (Y = Young's modulus; ρ = Density)
- (ii) In a liquid medium $v = \sqrt{\frac{k}{\rho}}$ (k = Bulk modulus)
- (iii) In gases $v = \sqrt{\frac{k}{\rho}}$

10.18 Velocity of Sound in Elastic Medium

Velocity of sound in any medium is

$$v = \sqrt{\frac{E}{\rho}} \quad (E = \text{Elasticity of the medium; } \rho = \text{Density of the medium})$$

$$(1) v_{\text{steel}} > v_{\text{water}} > v_{\text{air}} \Rightarrow 5000 \text{ m/s} > 1500 \text{ m/s} > 330 \text{ m/s}$$

(2) **Newton's formula** : He assumed that propagation of sound is isothermal

$$v_{\text{air}} = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{P}{\rho}} \quad \text{As } K = E_{\theta} = P; E_{\theta} = \text{Isothermal elasticity; } P = \text{Pressure.}$$

By calculation $v_{\text{air}} = 279 \text{ m/sec}$.

However the experimental value of sound in air is 332 m/sec

(3) **Laplace correction** : He modified that propagation of sound in air is adiabatic process.

$$v = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{E_{\phi}}{\rho}} \quad (\text{As } k = E_{\phi} = \gamma P = \text{Adiabatic elasticity})$$

$$v = 331.3 \text{ m/s } (\gamma_{\text{Air}} = 1.41)$$

$$(4) \text{ Effect of density : } v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v \propto \frac{1}{\sqrt{\rho}}$$

(5) **Effect of pressure** : Velocity of sound is independent of the pressure (when $T = \text{constant}$)

$$(6) \text{ Effect of temperature : } v \propto \sqrt{T(\text{in K})}$$

When the temperature change is small then $v_t = v_0 (1 + \alpha t)$

$$\text{Value of } \alpha = 0.608 \frac{\text{m/s}}{^{\circ}\text{C}} = 0.61 \text{ (Approx.)}$$

(7) **Effect of humidity** : With rise in humidity velocity of sound increases.

(8) Sound of any frequency or wavelength travels through a given medium with the same velocity.

(9) Sound of any frequency or wavelength travels through a given medium with the same velocity.

10.19 Reflection of Mechanical

Medium	Longitudinal wave	Transverse wave	Change in direction	Phase change	Time change	Path change
Reflection from rigid end/denser medium	Compression as rarefaction and vice-versa	Crest as crest and Trough as trough	Reversed	π	$\frac{T}{2}$	$\frac{\lambda}{2}$
Reflection from free end/rarer medium	Compression as compression and rarefaction as rarefaction	Crest as trough and trough as crest	No change	Zero	Zero	Zero

10.20 Progressive Wave

- (1) These waves propagate in the forward direction of medium with a finite velocity.
- (2) Energy and momentum are transmitted in the direction of propagation of waves.
- (3) In progressive waves, equal changes in pressure and density occurs at all points of medium.
- (4) Various forms of progressive wave function.

(i) $y = A \sin (\omega t - kx)$ Where $y = \text{displacement}$

$A = \text{amplitude}$

$\omega = \text{angular frequency}$

$n = \text{frequency}$

$k = \text{propagation constant}$

$T = \text{time period}$

$\lambda = \text{wave length}$

$v = \text{wave velocity}$

$t = \text{instantaneous time}$

(ii) $y = A \sin \left(\omega t - \frac{2\pi}{\lambda} x \right)$

$$(iii) y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda} \right]$$

$$(iv) y = A \sin \frac{2\pi}{\lambda} (vt - x)$$

$$(v) y = A \sin \omega \left(t - \frac{x}{v} \right)$$

(a) If the sign between t and x terms is negative the wave is propagating along positive X-axis and if the sign is positive then the wave moves in negative X-axis direction.

(b) The Argument of sin or cos function *i.e.* $(\omega t - kx) = \text{Phase}$.

(c) The coefficient of t gives angular frequency

$$\omega = 2\pi n = \frac{2\pi}{T}$$

(d) The coefficient of x gives propagation constant or wave number

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$

(e) The ratio of coefficient of t to that of x gives wave or phase velocity,

$$i.e. \quad v = \frac{\omega}{k}$$

(f) When a given wave passes from one medium to another its frequency does not change.

(g) From $v = n\lambda \Rightarrow v \propto \lambda \therefore n = \text{constant} \Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$.

(5) Some terms related to progressive waves

(i) **Wave number** (\vec{n}): The number of waves present in unit length.

$$(\vec{n}) = \frac{1}{\lambda}$$

(ii) Propagation constant (k): $k = \frac{\phi}{x}$

$$k = \frac{\omega}{v} = \frac{\text{Angular velocity}}{\text{Wave velocity}} \text{ and } k = \frac{2\pi}{\lambda} = 2\pi\vec{n}$$

(iii) Wave velocity (v) : $v = \frac{\omega}{k} = n\lambda = \frac{\omega\lambda}{2\pi} = \frac{\lambda}{T}$.

(iv) Phase and phase difference $\phi(x, t) = \frac{2\pi}{\lambda}(vt - x)$.

(v) Phase difference $= \frac{2\pi}{T} \times \text{Time difference}$.

(vi) Phase difference $= \frac{2\pi}{\lambda} \times \text{Path difference}$

$\Rightarrow \text{Time difference} = \frac{T}{\lambda} \times \text{Path difference}$.

10.21 Principle of Superposition

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, \dots$ are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement,

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \dots$$

Important applications of superposition principle: (a) Stationary waves, (b) Beats.

10.22 Standing Waves or Stationary Waves

When two sets of progressive wave trains of same type (both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

Characteristics of standing waves :

- (1) The disturbance confined to a particular region
- (2) There is no forward motion of the disturbance beyond this particular region.
- (3) The total energy is twice the energy of each wave.
- (4) Points of zero amplitude are known as nodes.

The distance between two consecutive nodes is $\frac{\lambda}{2}$.

- (5) Points of maximum amplitude is known as antinodes. The distance between two consecutive antinodes is also $\lambda/2$. The distance between a node and adjoining antinode is $\lambda/4$.
- (6) The medium splits up into a number of segments.
- (7) All the particles in one segment vibrate in the same phase. Particles in two consecutive segments differ in phase by 180° .
- (8) Twice during each vibration, all the particles of the medium pass simultaneously through their mean position.

10.23 Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S. No.	Parameter	Stretched string	Open organ Pipe	Closed organ Pipe
(1)	Fundamental frequency or 1 st harmonic (1 st mode of vibration)	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$
(2)	Frequency of 1 st overtone or 2 nd harmonic (2 nd mode of vibration)	$n_2 = 2n_1$	$n_2 = 2n_1$	Missing
(3)	Frequency of 2 nd overtone or 3 rd harmonic (3 rd mode of vibration)	$n_3 = 3n_1$	$n_3 = 3n_1$	$n_3 = 3n_1$
(4)	Frequency ratio of overtones	2:3:4: ...	2:3:4: ...	3:5:7: ...
(5)	Frequency ratio of harmonics	1:2:3:4: ...	1:2:3:4: ...	1:3:5:7: ...

(6) Nature of waves	Transverse stationary	Longitudinal stationary	Longitudinal stationary
(7) General formula for wavelength	$\lambda = \frac{2L}{n}, n = 1, 2, 3, \dots$	$\lambda = \frac{2L}{n}, n = 1, 2, 3, \dots$	$\lambda = \frac{4L}{(2n-1)}$
(8) Position of nodes	$x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n}, \dots$	$x = \frac{L}{(2n-1)}, \frac{3L}{(2n-1)}, \frac{5L}{(2n-1)}, \dots, \frac{(2n-1)L}{2n}$	$x = 0, \frac{2L}{(2n-1)}, \frac{4L}{(2n-1)}, \frac{6L}{(2n-1)}, \dots, \frac{2nL}{(2n-1)}$
(9) Position of antinodes	$x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \dots, \frac{(2n-1)L}{2n}$	$x = \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n}, \dots, \frac{(2n-1)L}{n}$	$x = \frac{L}{2n-1}, \frac{3L}{2n-1}, \frac{5L}{2n-1}, \dots, L$

- (i) Harmonics are the notes/sounds of frequency equal to or an integral multiple of fundamental frequency (n).
- (ii) Overtones are the notes/sounds of frequency twice/thrice/ four times the fundamental frequency (n).
- (iii) In organ pipe an antinode is not formed exactly at the open end rather it is formed a little distance away from the open end outside it. The distance of antinode from the open end of the pipe is $= 0.6r$ (where r is radius of organ pipe). This is known as end correction.

10.24 Vibration of a String

General formula of frequency $n_p = \frac{p}{2L} \sqrt{\frac{T}{m}}$

L = Length of string, T = Tension in the string

m = Mass per unit length (linear density), p = mode of vibration

- (1) The string will be in resonance with the given body if any of its natural frequencies coincides with the body.

- (2) If M is the mass of the string of length L , $m = \frac{M}{L}$.

So $n = \frac{1}{2Lr} \sqrt{\frac{T}{\pi\rho}}$ (r = Radius, ρ = Density)

10.25 Beats

When two sound waves of slightly different frequencies, travelling in a medium along the same direction, superimpose on each other, the intensity of the resultant sound at a particular position rises and falls regularly with time. This phenomenon is called beats.

- (1) **Beat period :** The time interval between two successive beats (*i.e.* two successive maxima of sound) is called beat period.
- (2) **Beat frequency :** The number of beats produced per second is called beat frequency.
- (3) **Persistence of hearing :** The impression of sound heard by our ears persist in our mind for $1/10^{\text{th}}$ of a second.

So for the formation of distinct beats, frequencies of two sources of sound should be nearly equal (difference of frequencies less than 10)

- (4) **Equation of beats :** If two waves of equal amplitudes ' a ' and slightly different frequencies n_1 and n_2 travelling in a medium in the same direction then equation of beats is given by

$y = A \sin \pi (n_1 - n_2)t$ where $A = 2a \cos \pi (n_1 - n_2)t$ = Amplitude of resultant wave.

Amplitude of resultant wave.

- (5) **Beat frequency :** $n = n_1 - n_2$.

- (6) **Beat period:** $\frac{1}{\text{Beat frequency}} = \frac{1}{n_1 - n_2}$

10.26 Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

$$\text{Apparent frequency } n' = \frac{[(v + v_m) - v_L]n}{[(v + v_m) - v_s]}$$

Here n = Actual frequency; v_L = Velocity of listener; v_s = Velocity of source

v_m = Velocity of medium and v = Velocity of sound wave

Sign convention: All velocities along the direction S to L are taken as positive and all velocities along the direction L to S are taken as negative. If the

medium is stationary $v_m = 0$ then $n' = \left(\frac{v - v_L}{v - v_s} \right) n$.

- (1) No Doppler effect takes place ($n' = n$) when relative motion between source and listener is zero.
- (2) Source and listener moves at right angle to the direction of wave propagation. ($n' = n$)
 - (i) If the velocity of source and listener is equal to or greater than the sound velocity then Doppler effect is not observed.
 - (ii) Doppler effect does not say about intensity of sound.
 - (iii) Doppler effect in sound is asymmetric but in light it is symmetric.

QUESTIONS

ONE MARK QUESTIONS

1. How is the time period effected, if the amplitude of a simple pendulum is increased?
2. Define force constant of a spring.
3. At what distance from the mean position, is the kinetic energy in simple harmonic oscillator equal to potential energy ?
4. How is the frequency of oscillation related with the frequency of change in the K.E. and P.E. of the body in S.H.M.?
5. What is the frequency of total energy of a particle in S.H.M. ?
6. How is the length of seconds pendulum related with acceleration due to gravity of any planet ?
7. If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob > density of the fluid), then time period of the pendulum increased or decrease.
8. How is the time period of the pendulum effected when pendulum is taken to hills or in mines ?
9. A transverse wave travels along x-axis. The particles of the medium must move in which direction ?
10. Define angular frequency. Give its S.I. unit.