

## Unit - 9

# Kinetic Theory Of Gases

### 9.1 Kinetic Theory of Gases : Assumption

- (1) The molecules of a gas are identical, spherical and perfectly elastic point masses.
- (2) The volume of molecules is negligible in comparison to the volume of gas.
- (3) Molecules of a gas moves randomly in all direction.
- (4) The speed of gas molecules lie between zero and infinity.
- (5) Their collisions are perfectly elastic.
- (6) The number of collisions per unit volume in a gas remains constant.
- (7) No attractive or repulsive force acts between gas molecules.

### 9.2 Pressure of an ideal Gas

$$P = \frac{1}{3} \rho V_{\text{rms}}^2 \text{ or } P = \frac{1}{3} \frac{mN}{V} V_{\text{rms}}^2$$

$$\left[ \text{rms velocity of the gas molecule } V_{\text{rms}} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + \dots}{N}} \right]$$

**Relation between pressure and kinetic energy**

$$\therefore \text{K.E. per unit volume (E)} = \frac{1}{2} \left( \frac{M}{V} \right) V_{\text{rms}}^2 = \frac{1}{2} \rho V_{\text{rms}}^2 \quad P = \frac{2}{3} E$$

### 9.3 Ideal Gas Equation

The equation which relates the pressure (P), volume (V) and temperature (T) of the given state of an ideal gas is known as gas equation.

For 1 mole or $N_A$ molecule or $M$ gram or 22.4 litres of gas	$PV = RT$
For $\mu$ mole of gas	$PV = \mu RT$
For 1 molecule of gas	$PV = \left( \frac{R}{N_A} \right) T = kT$
For $N$ molecules of gas	$PV = NkT$
For 1 gm of gas	$PV = \left( \frac{R}{M} \right) T = rT$
for $n$ gm of gas	$Pv = nrT$

**(1) Universal gas constant (R) :** Dimension  $[ML^2T^{-2}\theta^{-1}]$

Thus universal gas constant signifies the work done by (or on) a gas per mole per kelvin.

$$\text{S.T.P value : } 8.31 \frac{\text{Joule}}{\text{Mole} \times \text{kelvin}} = 1.98 \frac{\text{cal}}{\text{mole} \times \text{kelvin}}$$

**(2) Boltzman's constant (k) :** Dimension  $[ML^2T^{-2}\theta^{-1}]$

$$k = 1.38 \times 10^{-23} \text{ Joule/kelvin}$$

## 9.4 Various Speeds of Gas Molecules

$$(1) \text{ Root mean square speed } V_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

$$(2) \text{ Most probable speed } V_{\text{mp}} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$

$$(3) \text{ Average speed } V_{\text{av}} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8kT}{\pi m}}$$

- $V_{\text{rms}} > V_{\text{av}} > V_{\text{mp}}$  (remembering trick) (RAM)

## 9.5 Kinetic Energy of Ideal Gas

Molecules of ideal gases possess only translational motion. So they possess only translational kinetic energy.

Quantity of gas	Kinetic energy
Kinetic energy of a gas molecule (Emolecule)	$= \frac{1}{2} m v_{ms}^2 = \frac{1}{2} m \left( \frac{3kT}{m} \right) = \frac{3}{2} kT$ As $v_{ms} = \sqrt{\frac{3kT}{m}}$
Kinetic energy of 1 mole (M gram) gas (Emole)	$= \frac{1}{2} M v_{ms}^2 = \frac{1}{2} M \frac{3RT}{m} = \frac{3}{2} RT$ As $v_{ms} = \sqrt{\frac{3RT}{M}}$
Kinetic energy of 1 gm gas (Egram)	$= \frac{3}{2} \frac{R}{M} T = \frac{3}{2} \frac{N_A}{m N_A} T = \frac{3}{2} \frac{k}{m} T = \frac{3}{2} \epsilon T$

Here  $m$  = mass of each molecule,  $M$  = Molecular weight of gas and  $N_A$  – Avogadro number =  $6.023 \times 10^{23}$ .

## 9.6 Degree of Freedom

The total number of independent modes (ways) in which a system can possess energy is called the degree of freedom ( $f$ ).

The degree of freedom are of three types :


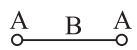
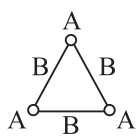
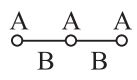
- (i) Translational degree of freedom
- (ii) Rotational degree of freedom
- (iii) Vibrational degree of freedom

General expression for degree of freedom

$f = 3N - R$ , where  $N$  = Number of independent particles,  $R$  = Number of independent restriction

- (1) **Monoatomic gas** : It can have 3 degrees of freedom (all translational).
- (2) **Diatomic gas** : A diatomic molecule has 5 degree of freedom : 3 translational and 2 rotational.
- (3) **Triatomic gas (Non-linear)** : It has 6 degrees of freedom : 3 translational and 3 rotational.

(4) Tabular display of degree of freedom of different gases

Atomicity of gas	Example	N	R	$f = 3N - R$	Figure
Monoatomic	He, Ne, Ar	1	0	$f = 3$	
Diatomic	H <sub>2</sub> , O <sub>2</sub>	2	1	$f = 5$	
Triatomic non linear	H <sub>2</sub> O	3	3	$f = 6$	
Triatomic linear	CO <sub>2</sub> , BeCl <sub>2</sub>	3	2	$f = 7$	

- The above degrees of freedom are shown at room temperature. Further at high temperature the molecule will have an additional degrees of freedom, due to vibrational motion.

## 9.7 Law of Equipartition of Energy

For any system in thermal equilibrium, the total energy is equally distributed among its various degree of freedom. And the energy associated with each

molecule of the system per degree of freedom of the system is  $\frac{1}{2}kT$ .

## 9.8 Mean Free Path

The average distance travelled by a gas molecule is known as mean free path.

Let  $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$  be the distance travelled by a gas molecule during  $n$  collisions respectively, then the mean free path of a gas molecule is given by

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n}{n}$$

$\lambda_1 = \frac{1}{\sqrt{2}n\pi d^2}$ ; where  $d$  = Diameter of the molecule,  $n$  = Number of molecules per unit volume.



## 9.9 Specific heat or Specific Heat Capacity

- (1) **Gram specific heat** : It is defined as the amount of heat required to raise the temperature of unit gram mass of the substance by unit degree. Gram

$$\text{specific heat } c = \frac{\Delta Q}{m\Delta T}.$$

- (2) **Molar specific heat** : It is defined as the amount of heat required to raise the temperature of one gram mole of the substance by a unit degree, it is represented by capital (C)

$$C = \frac{Q}{\mu\Delta T}$$

$$C = Mc = \frac{1}{\mu} \frac{\Delta Q}{\Delta T} \quad \left[ \text{As } \mu = \frac{m}{M} \right]$$

## 9.10 Specific Heat of Gases

- (i) In adiabatic process *i.e.*,  $\Delta Q = 0$ ,

$$\therefore C = \frac{\Delta Q}{m(\Delta T)} = 0 \text{ i.e., } C = 0$$

- (ii) In isothermal process *i.e.*,  $\Delta T = 0$

$$\therefore C = \frac{\Delta Q}{m(\Delta T)} = \frac{\Delta Q}{0} = \infty \text{ i.e., } C = \infty$$

Specific heat of gas can have any positive value ranging from zero to infinity. Further it can even be negative. Out of many values of specific heat of a gas, two are of special significance.

- (1) **Specific heat of a gas at constant volume ( $C_v$ )** : It is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1 K when its volume is kept constant.
- (2) **Specific heat of a gas at constant pressure ( $C_p$ )** : It is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1 K when its pressure is kept constant.

## 9.11 Mayer's Formula

$$C_p - C_v = R$$

This relation is called Mayer's formula and shows that  $C_p > C_v$  *i.e.*, molar specific heat at constant pressure is greater than that at constant volume.

## 9.12 Specific Heat in Terms of Degree of Freedom

Specific heat and kinetic energy for different gases

		Monoatomic	Diatomic	Triatomic non-linear	Triatomic linear
Atomicity	A	1	2	3	3
Restriction	B	0	1	3	2
Degree of freedom	$f = 3A - B$	3	5	6	7
Molar specific heat at constant volume	$C_v = \frac{f}{2}R = \frac{R}{\gamma - 1}$	$\frac{3}{2}R$	$\frac{5}{2}R$	$3R$	$\frac{7}{2}R$
Molar specific heat at constant pressure	$C_p = \left(\frac{f}{2} + 1\right)R = \left(\frac{\gamma}{\gamma - 1}\right)R$	$\frac{5}{2}R$	$\frac{7}{2}R$	$4R$	$\frac{9}{2}R$
Ratio of $C_p$ and $C_v$	$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$	$\frac{5}{3} = 1.66$	$\frac{7}{5} = 1.4$	$\frac{4}{3} = 1.33$	$\frac{9}{7} = 1.28$
Kinetic energy of 1 mole	$E_{\text{mole}} = \frac{f}{2}RT$	$\frac{3}{2}RT$	$\frac{5}{2}RT$	$3RT$	$\frac{7}{2}RT$
Kinetic energy of 1 molecule	$E_{\text{molecule}} = \frac{f}{2}kT$	$\frac{3}{2}kT$	$\frac{5}{2}kT$	$3kT$	$\frac{7}{2}kT$
Kinetic energy of 1 gm	$E_{\text{gram}} = \frac{f}{2}rT$	$\frac{3}{2}rT$	$\frac{5}{2}rT$	$3rT$	$\frac{7}{2}rT$

### QUESTIONS

#### VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

- Write two conditions when real gases obey the ideal gas equation ( $PV = nRT$ ).  $n \rightarrow$  number of mole.

2. If the number of molecule in a container is doubled. What will be the effect on the rms speed of the molecules ?
3. Draw the graph between  $P$  and  $1/V$  (reciprocal of volume) for a perfect gas at constant temperature.
4. Name the factors on which the degree of freedom of gas depends.
5. What is the volume of a gas at absolute zero of temperature ?
6. How much volume does one mole of a gas occupy at STP ?
7. What is an ideal gas ?
8. The absolute temperature of a gas is increased 3 times what is the effect on the root mean square velocity of the molecules ?
9. What is the Kinetic energy per unit volume of a gas whose pressure is  $P$  ?
10. A container has equal number of molecules of hydrogen and carbon dioxide. If a fine hole is made in the container, then which of the two gases shall leak out rapidly ?
11. What is the mean translational Kinetic energy of a perfect gas molecule at  $T$  temperature ?
12. Why it is not possible to increase the temperature of a gas while keeping its volume and pressure constant.

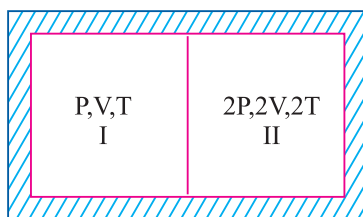
### SHORT ANSWER TYPE QUESTIONS (2 MARKS)

13. When an automobile travels for a long distance the air pressure in the tyres increases. Why ?
14. A gas storage tank has a small leak. The pressure in the tank drop more quickly if the gas is hydrogen than if it is oxygen. Why ?
15. Why the land has a higher temperature than the ocean during the day but a lower temperature at night.
16. Helium is a mixture of two isotopes having atomic masses  $3\text{g/mol}$  and  $4\text{g/mol}$ . In a sample of helium gas, which atoms move faster on average ?
17. State Avogadro's law. Deduce it on the basis of Kinetic theory of gases.
18. Although the velocity of air molecules is nearly  $0.5\text{ km/s}$  yet the smell of scent spreads at a much slower rate why.
19. The root mean square (rms) speed of oxygen molecule at certain temperature ' $T$ ' is ' $V$ '. If temperature is doubled and oxygen gas dissociates into atomic oxygen what is the speed of atomic oxygen ?

20. Two vessels of the same volume are filled with the same gas at the same temperature. If the pressure of the gas in these vessels be in the ratio 1 : 2 then state
- The ratio of the rms speeds of the molecules.
  - The ratio of the number of molecules.
21. Why gases at high pressure and low temperature show large deviation from ideal gas behaviour ?
22. A gas is filled in a cylinder fitted with a piston at a definite temperature and pressure. Why the pressure of the gas decreases when the piston is pulled out.

### SHORT ANSWER TYPE QUESTIONS (3 MARKS)

23. On what parameters does the  $\lambda$  (mean free path) depends.
24. Equal masses of oxygen and helium gases are supplied equal amount of heat. Which gas will undergo a greater temperature rise and why ?
25. Why evaporation causes cooling ?
26. Two thermally insulated vessels 1 and 2 are filled, with air at temperatures ( $T_1, T_2$ ), volume ( $V_1, V_2$ ) at pressure ( $P_1, P_2$ ) respectively. If the valve joining the two vessels is opened what is temperature of the vessel at equilibrium ?
27. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartment. What is the ratio of the number of molecules in compartments I and II ?



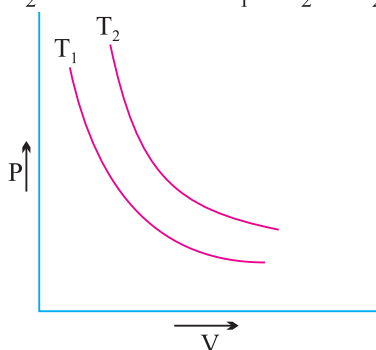
28. Prove that for a perfect gas having  $n$  degree of freedom

$$\frac{C_p}{C_v} = 1 + \frac{2}{n}$$

where  $C_p$  and  $C_v$  have their usual meaning.

29. The ratio of specific heat capacity at constant pressure to the specific heat capacity at constant volume of a diatomic gas decreases with increase in temperature. Explain.

30. Isothermal curves for a given mass of gas are shown at two different temperatures  $T_1$  and  $T_2$  state whether  $T_1 > T_2$  or  $T_2 > T_1$  justify your answer.



31. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic) the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules ? Is the root mean square speed of molecules the same in the three cases ? If not in which case is  $V_{\text{rms}}$  the largest ?
32. State Graham's law of diffusion. How do you obtain this from Kinetic Theory of gases.

### LONG ANSWER TYPE QUESTIONS (5 MARKS)

33. What are the basic assumptions of kinetic theory of gases ? On their basis derive an expression for the pressure exerted by an ideal gas.
34. What is meant by mean free path of a gas molecule ? Derive an expression for it.
35. Given that  $P = \frac{1}{3} \rho c^2$  where  $P$  is the pressure,  $\rho$  is the density and  $c$  is the rms. Velocity of gas molecules. Deduce Boyle's law and Charles law of gases from it.
36. What do you understand by mean speed, root mean square speed and most probable speed of a gas. The velocities of ten particles in m/s are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9 calculate.
- Average speed
  - r.m.s. speed
37. What is law of equipartition of energy ? Find the value of  $\gamma = C_p/C_v$  for diatomic and monatomic gas. Where symbol have usual meaning.

## NUMERICALS

38. An air bubble of volume  $1.0 \text{ cm}^3$  rises from the bottom of a lake 40 m deep at a temperature of  $12^\circ\text{C}$ . To what volume does it grow when it reaches the surface which is at a temperature of  $35^\circ\text{C}$  ?
39. A vessel is filled with a gas at a pressure of 76 cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. Find out the resultant pressure of the gas.
40. One mole of a monoatomic gas is mixed with three moles of a diatomic gas. What is the molecular specific heat of the mixture at constant volume ?  
Take  $R = 8.31/\text{mol}^{-1} \text{ K}^{-1}$ .
41. An oxygen cylinder of volume 30 litre has an initial gauge pressure of 15 atmosphere and a temperature of  $27^\circ\text{C}$ . After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atmosphere and its temperature drop to  $17^\circ\text{C}$ . Estimate the mass of oxygen taken out of the cylinder.  
( $R = 8.31/\text{mol}^{-1} \text{ K}^{-1}$ )  
(molecular mass of  $\text{O}_2 = 32$ )
42. At what temperature the rms speed of oxygen atom equal to r.m.s. speed of helium gas atom at  $-10^\circ\text{C}$  ?  
Atomic mass of helium = 4  
Atomic mass of oxygen = 32
43. Estimate the total number of molecules inclusive of oxygen, nitrogen, water vapour and other constituents in a room of capacity  $25.0 \text{ m}^3$  at a temperature of  $27^\circ\text{C}$  and 1 atmospheric pressure.
44. 0.014 kg of nitrogen is enclosed in a vessel at a temperature of  $27^\circ\text{C}$ . How much heat has to be transferred to the gas to double the rms speed of its molecules.

## OBJECTIVE QUESTIONS

45. At what temperature is the rms velocity of hydrogen molecule equal to that of an oxygen molecule at  $47^\circ\text{C}$ .
- (a)  $-73\text{K}$  (b)  $3\text{K}$   
(c)  $20 \text{ K}$  (d)  $80 \text{ K}$

46. The average kinetic energy of a gas molecule at  $27^{\circ}\text{C}$  is  $621 \times 10^{-21}$  J. The average kinetic energy of gas molecule at  $227^{\circ}\text{C}$  will be
- (a)  $52.2 \times 10^{-21}$  J                      (b)  $5.22 \times 10^{-21}$  J  
 (c)  $10.35 \times 10^{-21}$  J                      (d)  $11.35 \times 10^{-21}$  J
47. The equation of state 5 g of oxygen at a pressure P and temperature T, when occupying a volume V, will be
- (a)  $PV = \frac{5RT}{32}$      $PV =$                        $PV =$     (b)  $PV = \frac{5RT}{16}$   
 (c)  $PV = \frac{5RT}{2}$                                       (d)  $PV = 5RT$
48. A gas is found to obey the law  $P^2 V = \text{constant}$ . The initial temperature and volume are  $T_0$  and  $V_0$ . If the gas expands to a volume  $3V_0$ . Its final temperature becomes :
- (a)  $\frac{T_0}{3}$     (b)  $\frac{T_0}{\sqrt{3}}$   
 (c)  $3T_0$     (d)  $\sqrt{3}T_0$
49. A gas behaves as an ideal gas at
- (a) low pressure and high temperature  
 (b) low pressure and low temperature  
 (c) high pressure and low temperature  
 (d) high pressure and high temperature
50. If  $\gamma$  is the ratio of specific heats of a perfect gas, the no. of degrees of freedom of a molecule of the gas is
- (a)  $25 \frac{(\gamma-1)}{2}$     (b)  $9 \frac{(\gamma-1)}{2}$   
 (c)  $\frac{3\gamma-1}{2\gamma-1}$     (d)  $\frac{2}{\gamma-1}$
51. A gas is filled in a container at pressure  $P_0$ . If the mass of molecules is halved and their rms speed is doubled. The resultant pressure would be
- (a)  $2P_0$     (b)  $4P_0$   
 (c)  $\frac{P_0}{4}$     (d)  $\frac{P_0}{2}$

52. The translational kinetic energy of gas molecules for 1 mol of gas is equal to
- (a)  $\frac{3}{2}RT$  (b)  $\frac{2KT}{3}$   
(c)  $\frac{RT}{2}$  (d)  $\frac{3KT}{2}$
53. Molecular motion shows itself as
- (a) Temperature (b) Internal energy  
(c) Friction (d) Viscosity
54. A sample of gas is at  $0^{\circ}\text{C}$ . To what temperature it must be raised in order to double the rms speed of the molecule
- (a)  $270^{\circ}\text{C}$  (b)  $719^{\circ}\text{C}$   
(c)  $1090^{\circ}\text{C}$  (d)  $100^{\circ}\text{C}$
55. The work done by (or on) a gas per mole per kelvin is called
- (a) Universal gas constant (b) Boltzmann's constant  
(c) Gravitational constant (d) Entropy
56. A gas filled in a closed vessel is heated through 1 K and its pressure increases by 0.4%. What was the initial temperature of the gas?
- (a) 250 K (b) 350 K  
(c) 450 K (d) 500 K
57. The root mean square speed of the molecules of a gas is
- (a) independent of its pressure but directly proportional to its Kelvin temperature  
(b) directly proportional to two square root of both its pressure and its Kelvin temperature  
(c) independent of its pressure but directly proportional to the square root of its Kelvin temperature.  
(d) directly proportional to its pressure and its Kelvin temperature.



58. The root mean square velocity of gas molecules is 10 km/s. The gas is heated till its pressure becomes four times. The velocity of gas molecules will be
- (a) 10 Km/s (b) 20 Km/s  
(c) 40 Km/s (d) 80 Km/s
59. According to kinetic theory of gases at absolute zero
- (a) Water freezes (b) Liquid helium freezes  
(c) Molecular motion stops (d) All of the above are correct
60. The quantity of heat required to raise one mole through 1 K for a monoatomic gas at constant volume is
- (a)  $\frac{3}{2} R$  (b)  $\frac{5}{2} R$   
(c)  $\frac{7}{2} R$  (d) 4 R
61. Dimensional formula for universal gas constant R is given by
- (a)  $[ML^2T^{-2}K^{-2}]$  (b)  $[ML^2T^{-3}K^{-1}]$   
(c)  $[M^{\circ}L^2T^{-3}K^{-1}]$  (d)  $[ML^2T^{-2}K^{-4}]$
62. An ant is walking on the horizontal surface. The number of degrees of freedom of ant will be
- (a) 1 (b) 2 (c) 3 (d) 6
63. The specific heat of a gas
- (a) has only two values  $C_p$  &  $C_v$   
(b) has a unique value of given temperature  
(c) can have any values from 0 to  $\infty$   
(d) depends upon the mass of the gas
64. 250 L of an ideal gas is heated at constant pressure from 27°C such that its volume becomes 500 L. The final temperature is
- (a) 54°C (b) 300°C  
(c) 327°C (d) 600°C

## ASSERTION - REASON BASED QUESTIONS

Direction:- Read the assertion and reason carefully to mark the correct option out of the options given below :

- (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
- (b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
- (c) If assertion is true but reason is false.
- (d) If the assertion and reason both are false.
- (e) If assertion is false but reason is true.

1. Assertion: If a gas container in motion is suddenly stopped, the temperature of the gas rises.

Reason: The kinetic energy of ordered mechanical motion is converted into the kinetic energy of random motion of gas molecules.

2. Assertion: The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

Reason: The molecules of a gas collide with each other and the velocities of the molecules change due to collision.

3. Assertion: Gases do not settle to the bottom of a container.

Reason: Gases have high kinetic energy.

4. Assertion: A gas have a unique value of specific heat.

Reason: Specific heat is defined as the amount of heat required to raise the temperature of unit mass of the substance through unit degree.

5. Assertion: A gas can be liquified at any temperature by increase of pressure alone.

Reason: On increasing pressure the temperature of gas decreases.

6. Assertion: Equal masses of helium and oxygen gases are given equal quantities of heat. There will be a greater rise in the temperature of helium compared to that of oxygen .

Reason: The molecular weight of oxygen is more than the molecular weight of helium.

7. Assertion: Absolute zero is the temperature corresponding to zero energy.

Reason: The temperature at which no molecular motion cease is called absolute zero temperature.

8. Assertion: The ratio of specific heat gas at constant pressure and specific heat at constant volume for a diatomic gas is more than that for a monatomic gas.

Reason: The molecules of a monatomic gas have more degree of freedom than those of a diatomic gas.

9. Assertion: At room temperature, water does not sublime from water to steam.

Reason: The critical point of water is much above the room temperature.

10. Assertion: Specific heat of a gas at constant pressure ( $C_p$ ) is greater than its specific heat at constant volume ( $C_v$ ).

Reason: At constant pressure, some heat is spent in expansion of the gas.

11. Assertion: The internal energy of a real gas is function of both, temperature and volume.

Reason: Internal kinetic energy depends on temperature and internal potential energy depends on volume.

### CASE STUDY BASED QUESTIONS

The equipartition of kinetic energy was proposed initially in 1843 and more correctly in 1845, by John James Waterston. In 1859, James Clerk Maxwell argued that the kinetic heat energy of a gas is equally divided between linear and rotational energy. In 1876, Ludwig Boltzmann expanded on this principle by showing that the average energy was divided equally among all the independent components of motion in a system. Boltzmann applied the equipartition theorem to provide a theoretical explanation of the Dulong-Petit law for the specific heat capacities of solids.

#### Law of Equipartition of Energy

According to this law, for any system in thermal equilibrium, the total energy is equally distributed among its various degree of freedom. And each degree of freedom is associated with energy  $\frac{1}{2}kT$  (where  $k = 1.3 \times 10^{-23} \text{ J/K}$ ,  $T$  = absolute temperature of the system).

At a given temperature  $T$ ; all ideal gas molecules no matter what their mass have the same average translational kinetic energy; namely,  $\frac{3}{2}kT$ . When measure the temperature of a gas, we are also measuring the average

translational kinetic energy of its molecules.

At same temperature gases with different degrees of freedom (e.g., He and H) will have different average energy or internal energy namely  $\frac{f}{2}kT$ . (F is different for different gases)

**Answer the following questions**

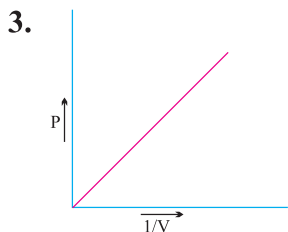
1. Relation between pressure P and average kinetic energy E per unit volume of a gas is  
(a)  $P = \frac{2E}{3}$       (b)  $P = \frac{E}{3}$       (c)  $P = \frac{3E}{2}$       (d)  $P = 3E$
2. At 0 K, which of the following properties of a gas will be zero?  
(a) kinetic energy      (b) potential energy  
(c) vibrational energy      (d) density
3. The root mean square velocity of a gas molecule of mass m at a given temperature is proportional to  
(a)  $m^0$       (b) m      (c)  $\sqrt{m}$       (d)  $m^{-1/2}$
4. An ant is walking on the horizontal surface. The number of degrees of freedom of ant will be  
(a) 1      (b) 2      (c) 3      (d) 6
5. The number of degrees of freedom for a diatomic gas molecule is  
(a) 2      (b) 3      (c) 5      (d) 6

**ANSWERS**

**VERY SHORT ANSWERS (1 MARK)**

1. (i) Low pressure    (ii) High temperature.

2. No effect



4. Atomicity and temperature.

5. 0

6. 22.4 litre
7. Gas in which intermolecular forces are absent.
8. increases  $\sqrt{3}$  times
9.  $3P/2$
10. Hydrogen (rms speed is greater)
11.  $\frac{3}{2}RT$
12.  $P = \frac{1}{3} \frac{M}{V} KT$ ,  $T \propto (PV)$

P and V are constant then T is also constant.

### SHORT ANSWERS (2 MARKS)

13. Work is done against friction. This work done is converted into heat. Temperature rises.  $PV = nRT$ , As volume of tyre is const.  $P \propto T$ .
14. Rate of diffusion of a gas is inversely proportional to the square root of the density. So hydrogen leaked out more rapidly.
15. Specific Heat of water is more than land (earth). Therefore for given heat change in temp. of land is more than ocean (water).
19.  $c = \sqrt{\frac{3RT}{M}} = v, c' = \sqrt{\frac{3R(2T)}{M/2}} = 2\sqrt{\frac{3RT}{M}}$   
 $c' = 2v$
20. (i)  $C \propto \sqrt{T}$   
as the temperature is same rms speeds are same.  
(ii)  $P = \frac{1}{3} \frac{mnc^2}{V} \Rightarrow P_1 = \frac{1}{3} \frac{mn_1c^2}{V}, P_2 = \frac{1}{3} \frac{mn_2c^2}{V}$   
*i.e.*,  $\frac{P_1}{P_2} = \frac{n_1}{n_2} = \frac{1}{2}$
21. When temperature is low and pressure is high the intermolecular forces become appreciable thus the volume occupied by the molecular is not negligibly small as composed to volume of gas.
22. When piston is pulled out the volume of the gas increases, Now losses number

of molecules colliding against the wall of container per unit area decreases.  
Hence pressure decreases.

### ANSWERS (3 MARKS)

23. (i) diameter of molecule as  $\lambda \propto \frac{1}{d^2}$

(ii) Pressure of gas as  $\lambda \propto \frac{1}{P}$

24. Heat supplied to oxygen = Heat supplied to Helium

$$mc_1\Delta T_1 = mc_2\Delta T_2$$

$$\frac{\Delta T_1}{\Delta T_2} = \frac{c_2}{c_1}, \text{ As } c \propto \frac{1}{m}, m = \text{molecular mass}$$

$$\frac{\Delta T_1}{\Delta T_2} = \frac{c_2}{c_1} = \frac{m_1}{m_2}, \text{ As } m_1 > m_2$$

$$\Delta T_1 > \Delta T_2$$

25. During evaporation fast moving molecules escape a liquid surface so the average kinetic energy of the molecules left behind is decreased thus the temperature of the liquid is lowered.

26. number of mole = Constant

$$\mu_1 + \mu_2 = \mu$$

$$\frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2} = \frac{P(V_1 + V_2)}{RT}$$

From Boyle's law,  $P(V_1 + V_2) = P_1 V_1 + P_2 V_2$

27.  $n = \frac{pV}{kT}, n' = \frac{2p2V}{kT}$

$$n/n' = \frac{1}{4}$$

30.  $T = \frac{PV}{\mu R}, T \propto PV [\mu R = \text{constant}]$

Since PV is greater for the curve at  $T_2$  than for the curve  $T_1$  therefore  $T_2 > T_1$ .

31. Three vessels at the same pressure and temperature have same volume and contain equal number of molecules.

$$V_{\text{rms}} = \sqrt{\frac{3RT}{m}}, \quad V_{\text{rms}} \propto \frac{1}{\sqrt{m}}$$

rms speed will not same, neon has smallest mass therefore rms speed will be largest for neon.

38.  $V_1 = 10^{-6} \text{ m}^3$

Pressure on bubble  $P_1$  = Water pressure + Atmospheric pressure

$$= pgh + P_{\text{atm}}$$

$$= 4.93 \times 10^5 \text{ Pa}$$

$$T_1 = 285 \text{ K}, T_2 = 308 \text{ K}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$V_2 = \frac{4.93 \times 10^5 \times 1 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5} = 5.3 \times 10^{-6} \text{ m}^3$$

39. According to kinetic theory of gases,

$$PV = \frac{1}{3} m v_{\text{rms}}^2$$

At constant temperature,  $v_{\text{rms}}^2$  is constant. As  $v$  is also constant, so  $P \propto m$ .

When the mass of the gas increase by 50% pressure also increases by 50%,

$$\therefore \text{Final pressure} = 76 + \frac{50}{100} \times 76 = 114 \text{ cm of Hg.}$$

40. For monoatomic gas,  $C_v = \frac{3}{2} R, n = 1 \text{ mole}$

For diatomic gas,  $C_v' = \frac{5}{2} R, n' = 3 \text{ mole}$

From conservation of energy, the molecular specific heat of the mixture is

$$C_v' = \frac{n(C_v) + n'(C_v')}{(n + n')}$$

$$= \frac{1 \times \frac{3}{2} R + 3 \times \frac{5}{2} R}{(1 + 3)} = \frac{9}{4} R$$

or

$$C_v' = \frac{9}{4} \times 8.31 = 18.7 \text{ J mol}^{-1} \text{ K}^{-1}.$$

41.  $V_1 = 30 \text{ litre} = 30 \times 10^3 \text{ cm}^3 = 3 \times 10^{-2} \text{ m}^3$

$$P_1 = 15 \times 1.013 \times 10^5 \text{ N/m}^2$$

$$T_1 = 300 \text{ K}$$

$$\mu_1 = \frac{P_1 V_1}{RT_1} = 18.3$$

$$P_2 = 11 \times 1.013 \times 10^5 \text{ N/m}^2$$

$$V_2 = 3 \times 10^{-2} \text{ m}^3$$

$$T_2 = 290 \text{ K}$$

$$\mu_2 = \frac{P_2 V_2}{RT_2} = 13.9$$

$$\mu_2 - \mu_1 = 18.3 - 13.9 = 4.4$$

Mass of gas taken out of cylinder

$$= 4.4 \times 32 \text{ g}$$

$$= 140.8 \text{ g}$$

$$= 0.140 \text{ kg.}$$

$$42. \quad v_{\text{rms}} = \left[ \frac{3PV}{M} \right]^{1/2} = \left[ \frac{3RT}{M} \right]^{1/2}$$

Let r.m.s. speed of oxygen is  $(v_{\text{rms}})_1$  and of helium is  $(v_{\text{rms}})_2$  is equal at temperature  $T_1$  and  $T_2$  respectively.

$$\frac{(v_{\text{rms}})_1}{(v_{\text{rms}})_2} = \sqrt{\frac{M_2 T_1}{M_1 T_2}}$$

$$\left[ \frac{4T_1}{32 \times 263} \right]^{1/2} = 1$$

$$T_1 = \frac{32 \times 263}{4} = 2104 \text{ K.}$$

43. As Boltzmann's constant,

$$k_B = \frac{R}{N}, \quad \therefore R = k_B N$$

$$\text{Now} \quad PV = nRT = nk_B NT$$

$\therefore$  The number of molecules in the room

$$= nN = \frac{PV}{Tk_B}$$



$$= \frac{1.013 \times 10^5 \times 25.0}{300 \times 1.38 \times 10^{-23}} = 6.117 \times 10^{26}.$$

44. Number of mole in 0.014 kg of Nitrogen

$$n = \frac{0.014 \times 10^3}{28} = \frac{1}{2} \text{ mole}$$

$$C_v = \frac{5}{2}R = \frac{5}{2} \times 2 = 5 \text{ cal/mole } k$$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}}, \quad T_2 = 4T_1$$

$$\Delta T = T_2 - T_1 = 4T_1 - T_1 = 3T_1 \\ = 3 \times 300 = 900 \text{ K}$$

$$\Delta Q = n c_v \Delta T = \frac{1}{2} \times 5 \times 900 = 2250 \text{ cal.}$$

### ANSWER (OBJECTIVE TYPE QUESTIONS)

45. (c)    46. (c)    47. (a)    48. (d)    49. (a)    50. (d)  
 51. (a)    52. (a)    53. (a)    54. (b)    55. (a)    56. (a)  
 57. (c)    58. (b)    59. (c)    60. (a)    61. (a)    62. (b)  
 63. (c)    64. (c)

### HINTS :-

45.  $v_1 = \sqrt{\frac{3RT_1}{M_1}}, \quad v_2 = \sqrt{\frac{3RT_2}{M_2}}$  i.e.  $T_1 = \frac{M_1}{M_2} T_2 = \frac{2}{32} \times 320 = 20 \text{ K}$   
 46.  $K_1 = \frac{3}{2} kT_1, \quad K_2 = \frac{3}{2} kT_2$  i.e.  $\frac{K_1}{K_2} = \frac{T_1}{T_2} \Rightarrow K_2 = \frac{K_1 T_2}{T_1} = \frac{621 \times 10^{-21}}{300} \times 500 = 10.35 \times 10^{-21} \text{ J}$   
 48.  $P^2 V = \text{constant} \Rightarrow \left(\frac{RT}{V}\right)^2 V = \text{constant} \Rightarrow \frac{T^2}{V} = \text{constant.}$   
 50.  $\gamma = 1 + \frac{1}{f}$   
 51.  $P_0 = \frac{1}{3} \rho C^2$  As  $\rho \rightarrow$  halved and  $C$  is doubled then  $P' = 2P_0$

54.  $C^2 \propto T$

55.  $\frac{PV}{T} = R$

56.  $PV = RT$  here  $V = \text{Constant}$  therefore  $P \propto T$ . i.e.  $= \frac{P}{T}$  constant

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{P}{T} = \frac{P + 0.4\% \text{ of } P}{T + 1} \Rightarrow T = 250 \text{ K}$$

58.  $P \propto T$

60.  $C_v = \frac{dU}{dT}$  and  $U = \frac{3}{2} RT$

64.  $V \propto T$  as  $P$  is constant.

### ASSERTION - REASON BASED ANSWERS

1. (a)      2. (b)      3. (a)      4. (e)      5. (d)      6. (b)  
7. (e)      8. (d)      9. (a)      10. (a)      11. (a)

### HINTS :-

- (a) The motion of the container is known as the ordered motion of the gas and zig-zag motion of gas molecules within the container is called disordered motion. When the container suddenly stops, ordered kinetic energy gets converted into disordered kinetic energy which increases the temperature of the gas.
- (b) Average translational kinetic energy of a gas molecule is  $E = \frac{3}{2} kT$   
For all gas molecules  $E = \frac{3}{2} kT \times N_A = \frac{3}{2} RT = \frac{3}{2} PV$
- (a) Gases do not settle to the bottom because of its kinetic energy. They are always in random motion. Because of the small mass, the effect of gravity on them is negligible.

### CASE STUDY BASED ANSWERS

- (a)  $P = \frac{2E}{3}$
- (a) At 0 K, all molecular motion stops, so kinetic energy becomes zero.
- (d)  $V_{\text{rms}} = \sqrt{\frac{3K_3T}{m}}$  i.e.  $V_{\text{rms}} \propto m^{-1/2}$
- (b) As the ant can move on a plane, it has 2 degree of freedom.
- (c) A diatomic molecule has 3 degree of freedom due to translatory motion and 2 degrees of freedom due to rotatory motion.

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