

## Unit - 5

# Rotational Motion

### 5.1 Introduction

**Rigid body :** A rigid body is a body that can rotate with all the parts locked together and without any change in its shape.

### 5.2 Centre of Mass

Centre of mass of a system is a point that moves as though all the mass were concentrated there and all external forces were applied there.

- (1) **Position vector of centre of mass for  $n$  particle system :** If a system consists of  $n$  particles of masses  $m_1, m_2, m_3, \dots, m_n$ , whose positions  $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$  respectively then position vector of centre of mass

$$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

If two masses are equal i.e.,  $m_1 = m_2$ , then position vector of centre of mass of two particle system is  $\vec{r} = \frac{\vec{r}_1 + \vec{r}_2}{2}$ .

(2) **Important points about centre of mass**

- (i) The position of centre of mass is independent of the co-ordinate system chosen.
- (ii) The position of centre of mass depends upon the shape of the body and distribution of mass.
- (iii) In symmetrical bodies in which the distribution of mass is homogeneous, the centre of mass coincides with the geometrical centre or centre of symmetry of the body. Centre of mass of cone

or pyramid lies on the axis of the cone at point distance  $\frac{3h}{4}$  from the vertex where  $h$  is the height of cone.

- (iv) The centre of mass changes its position only under the translatory motion. There is no effect of rotatory motion on centre of mass of the body.

- (v) If the origin is at the centre of mass, then the sum of the moments of the masses of the system about the centre of mass is zero *i.e.*,

$$\sum m_i \vec{r}_i = 0$$

- (vi) If a system of particles of masses  $m_1, m_2, m_3, \dots$  move with velocity  $v_1, v_2, v_3, \dots$  then the velocity of centre of mass

$$v_{\text{cm}} = \frac{\sum m_i v_i}{\sum m_i}$$

- (vii) If a system of particles of masses  $m_1, m_2, m_3, \dots$  move with accelerations  $a_1, a_2, a_3, \dots$  then the acceleration of centre of mass

$$A_{\text{cm}} = \frac{\sum m_i a_i}{\sum m_i}$$

- (viii) If  $\vec{r}$  is a position vector of centre of mass of a system then velocity

of centre of mass  $\vec{v}_{\text{cm}} = \frac{d\vec{r}}{dt}$ .

- (ix) Acceleration of centre of mass  $\vec{A}_{\text{cm}} = \frac{d\vec{v}_{\text{cm}}}{dt} = \frac{d^2\vec{r}}{dt^2}$ .

- (x) Force on a rigid body  $\vec{F} = M\vec{A}_{\text{cm}} = M \frac{d^2\vec{r}}{dt^2}$ .

- (xi) For an isolated system external force on the body is zero

$$\vec{F} = M \frac{d}{dt}(\vec{v}_{\text{cm}}) = 0$$

$$\Rightarrow \vec{v}_{\text{cm}} = \text{constant.}$$

*i.e.*, centre of mass of an isolated system moves with uniform velocity along a straight-line path.

## 5.3 Equations of Linear Motion and Rotational Motion

### Rotational Motion

If angular acceleration is 0,  $\omega = \text{constant}$  and  $\theta = \omega t$

If angular acceleration  $\alpha = \text{constant}$  then

$$(i) \quad \theta = \frac{(\omega_1 + \omega_2)}{2} t$$

$$(ii) \quad \alpha = \frac{\omega_2 - \omega_1}{t}$$

$$(iii) \quad \omega_2 = \omega_1 + \alpha t$$

$$(iv) \quad \theta = \omega_1 t + \frac{1}{2} \alpha t^2$$

$$(v) \quad \omega_2^2 = \omega_1^2 + 2\alpha\theta$$

$$(vi) \quad \theta_{nth} = \omega_1 + (2n-1) \frac{\alpha}{2}$$

If acceleration is not constant, the above equation will not be applicable. In this case

$$(i) \quad \omega = \frac{d\theta}{dt}$$

$$(ii) \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$(iii) \quad \omega d\omega = \alpha d\theta$$

## 5.4 Moment of Inertia

Moment of inertia plays the same role in rotational motion as mass plays in linear motion. It is the property of a body due to which it opposes any change in its state of rest or of uniform rotation.

- (1) Moment of inertia of a particle  $I = mr^2$ ; where  $r$  is the perpendicular distance of particle from rotational axis.
- (2) Moment of inertia of a body made up of number of particles (discrete distribution)

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

- (3) Moment of inertia of a continuous distribution of mass,  $dI = dm r^2$  i.e.,

$$I = \int r^2 dm$$

- (4) Dimension :  $[ML^2T^0]$

- (5) S.I. unit :  $\text{kgm}^2$ .
- (6) Moment of inertia depends on mass, distribution of mass and on the position of axis of rotation.
- (7) Moment of inertia is a tensor quantity.

## 5.5 Radius of Gyration

Radius of gyration of a body about a given axis is the perpendicular distance of a point from the axis, where if whole mass of the body were concentrated, the body shall have the same moment of inertia as it has with the actual distribution of mass.

When square of radius of gyration is multiplied with the mass of the body gives the moment of inertia of the body about the given axis.

$$I = Mk^2 \text{ or } k = \sqrt{\frac{I}{M}}$$

Here  $k$  is called radius of gyration.

$$k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}}$$

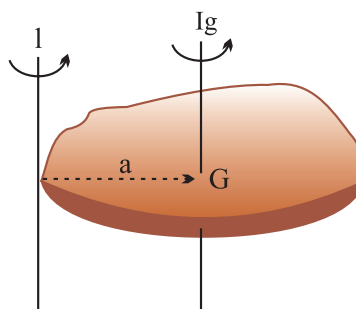
**Note :**

- For a given body inertia is constant whereas moment of inertia is variable.

## 5.6 Theorem of Parallel Axes

Moment of inertia of a body about a given axis  $I$  is equal to the sum of moment of inertia of the body about an axis parallel to given axis and passing through centre of mass of the body  $I_g$  and  $Ma^2$  where  $M$  is the mass of the body and  $a$  is the perpendicular distance between the two axes.

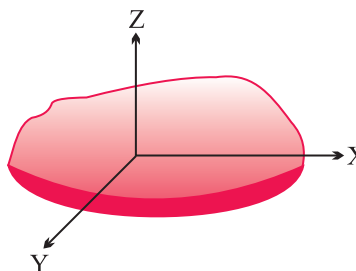
$$I = I_g + Ma^2$$



## 5.7 Theorem of Perpendicular Axes

According to this theorem the sum of moment of inertia of a plane lamina about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis perpendicular to the plane of lamina and passing through the point of intersection of first two axis.

$$I_z = I_x + I_y$$





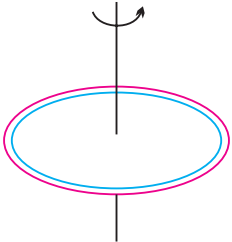
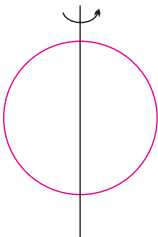
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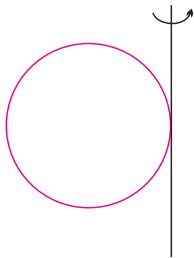
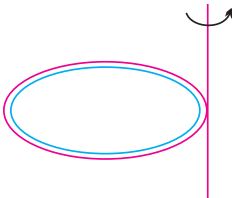
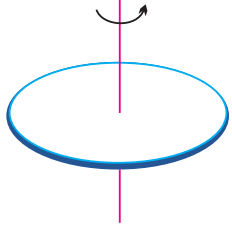
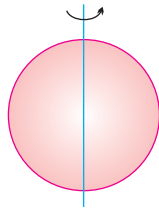
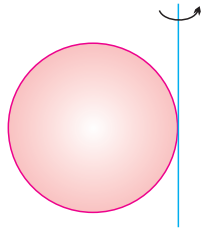
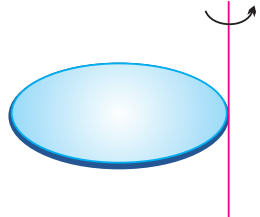
- In case of symmetrical two-dimensional bodies as moment of inertia for all axes passing through the centre of mass and in the plane of body will be same so the two axes in the plane of body need not be perpendicular to each other.

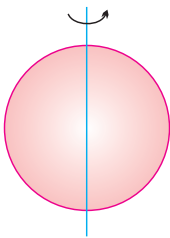
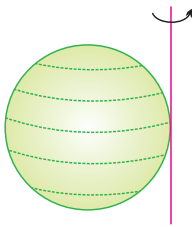
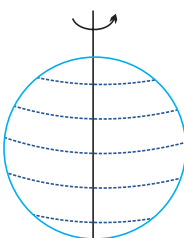
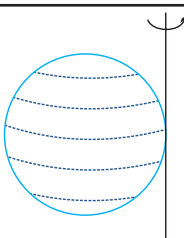
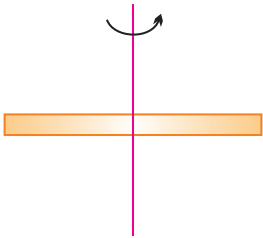

**5.8 Analogy between Translatory Motion and Rotational Motion**

Translatory motion		Rotatory motion	
Mass	(m)	Moment of Inertia	(I)
Linear	$P = mv$	Angular	$L = I\omega$
Momentum	$P = \sqrt{2mE}$	Momentum	$L = \sqrt{2IE}$
Force	$F = ma$	Torque	$\tau = I\alpha$
Kinetic energy	$E = \frac{1}{2}mv^2$		$E = \frac{1}{2}I\omega^2$
	$E = \frac{P^2}{2m}$		$E = \frac{L^2}{2I}$

**5.9 Moment of Inertia of Some Standard Bodies and Different Axes**

Body	Axis of Rotation	Figure	Moment of inertia	K	$K^2/R^2$
Ring (Cylindrical Shell)	About an axis Passing through C.G. and perpendicular to its plane		$MR^2$	R	1
Ring	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$

Body	Axis of Rotation	Figure	Moment of inertia	K	$K^2/R^2$
Ring	About a tangential axis in its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Ring	About a tangential axis perpendicular to its own plane		$2MR^2$	$\sqrt{2}R$	2
Disc (Solid cylinder)	About an axis Passing through C.G. and perpendicular to its plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Disc	About its diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Disc	About a tangential axis in its own plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$	$\frac{5}{4}$
Disc	About a tangential axis perpendicular to its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$

Body	Axis of Rotation	Figure	Moment of inertia	K	$K^2/R^2$
Solid Sphere	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$
Solid Sphere	About a tangential axis		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$	$\frac{7}{5}$
Spherical Shell	About its diametric axis		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$	$\frac{2}{3}$
Spherical Shell	About a tangential axis		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$	$\frac{5}{3}$
Long thin rod	About an axis passing through its centre of mass and perpendicular to the rod		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$	
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	

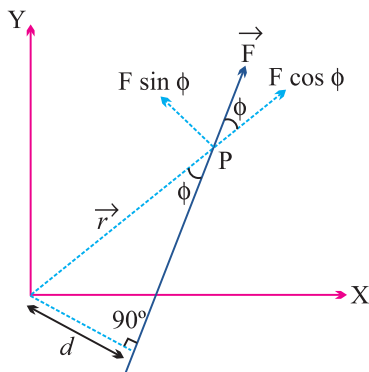
## 5.10 Torque

If the particle rotating in  $xy$  plane about the origin under the effect of force  $\vec{F}$  and at any instant the position vector of the particle is  $\vec{r}$  then,

$$\text{Torque } \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF \sin \phi$$

[where  $\phi$  is the angle between the direction of  $\vec{r}$  and  $\vec{F}$ ]



- (1) Torque is an axial vector *i.e.*, its direction is always perpendicular to the plane containing vector  $\vec{r}$  and  $\vec{F}$  in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

*i.e.*, Torque = Force  $\times$  Perpendicular distance of line of action of force from the axis of rotation.

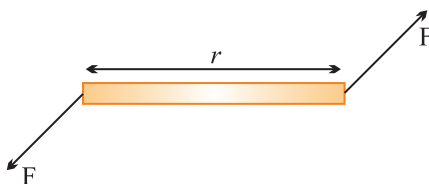
Torque is also called as moment of force and  $d$  is called moment or lever arm.

- (2) Unit : Newton-metre (M.K.S.) and Dyne-cm (C.G.S.)
- (3) Dimension :  $[ML^2T^{-2}]$ .
- (4) A body is said to be in rotational equilibrium if resultant torque acting on it is zero *i.e.*,  $\sum \vec{\tau} = 0$ .
- (5) Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translatory motion *i.e.*, torque is rotational analogue of force. This all is evident from the following correspondences between rotatory and translatory motion.

Rotatory Motion	Translatory Motion
$\vec{\tau} = I \vec{\alpha}$	$\vec{F} = m \vec{a}$
$W = \int \vec{\tau} \cdot d\vec{\theta}$	$W = \int \vec{F} \cdot d\vec{s}$
$P = \vec{\tau} \cdot \vec{\omega}$	$P = \vec{F} \cdot \vec{V}$
$\vec{\tau} = \frac{d\vec{L}}{dt}$	$\vec{F} = \frac{d\vec{P}}{dt}$

### 5.11 Couple

A couple is defined as combination of two equal but oppositely directed force not acting along the same line. The effect of couple is known by its moment of couple or torque by a couple  $\vec{\tau} = \vec{r} \times \vec{F}$ .



### 5.12 Angular Momentum

The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If  $\vec{P}$  is the linear momentum of particle and  $\vec{r}$  its position vector from the point of rotation then angular momentum.

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = rP \sin \phi \hat{n}$$

Angular momentum is an axial vector *i.e.*, always directed perpendicular to the plane of rotation and along the axis of rotation.

- (1) S.I. Unit :  $\text{kgm}^2 \text{s}^{-1}$  or J-sec.
- (2) Dimension :  $[\text{ML}^2\text{T}^{-2}]$  and it is similar to Planck's constant ( $h$ ).
- (3) Angular momentum = (Linear momentum)  $\times$  (Perpendicular distance of line of action of force from the axis of rotation)

(4) In vector form  $\vec{L} = I\vec{\omega}$ .

(5) From  $\vec{L} = I\vec{\omega}$ ,  $\therefore \frac{d\vec{L}}{dt} = I \frac{d\vec{\omega}}{dt} = I\vec{\alpha} = \vec{\tau}$

[Rotational analogue of Newton's second law]

(6) If a large torque acts on a particle for a small time then 'angular impulse'

of torque is given by  $\vec{J} = \int \vec{\tau} dt = \vec{\tau}_{av} \int_{t_1}^{t_2} dt$

$\therefore$  Angular impulse = Change in angular momentum

### 5.13 Law of Conservation of Angular Momentum

If the net external torque on a particle (or system) is zero then  $\frac{d\vec{L}}{dt} = 0$   
i.e.,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \dots = \text{constant.}$$

Angular momentum of a system (may be particle or body) remains constant if resultant torque acting on it zero.

As  $L = I\omega$  so if  $\vec{\tau} = 0$  then  $I\omega = \text{constant}$ .

### 5.14 Slipping, Spinning and Rolling

(1) **Slipping** : When the body slides on a surface without rotation then its motion is called slipping motion.

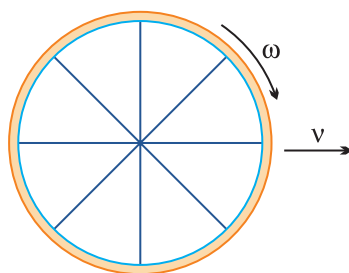
In this condition friction between the body and surface  $F = 0$ .

Body possess only translatory kinetic energy  $K_T = \frac{1}{2}mv^2$ .

(2) **Spinning** : When the body rotates in such a manner that its axis of rotation does not move then its motion is called spinning motion. In this condition axis of rotation of a body is fixed.

In spinning, body possess only rotatory kinetic energy  $K_R = \frac{1}{2}I\omega^2$ .

(3) **Rolling** : If in case of rotational motion of a body about a fixed axis, the axis of rotation also moves, the motion is called combined translatory and rotatory.



*Example :*

- (i) Motion of a wheel of cycle on a road.
- (ii) Motion of football rolling on a surface.

In this condition friction between the body and surface  $F \neq 0$ . Body possesses both translational and rotational kinetic energy.

Net kinetic energy = (Translatory + Rotatory) kinetic energy.

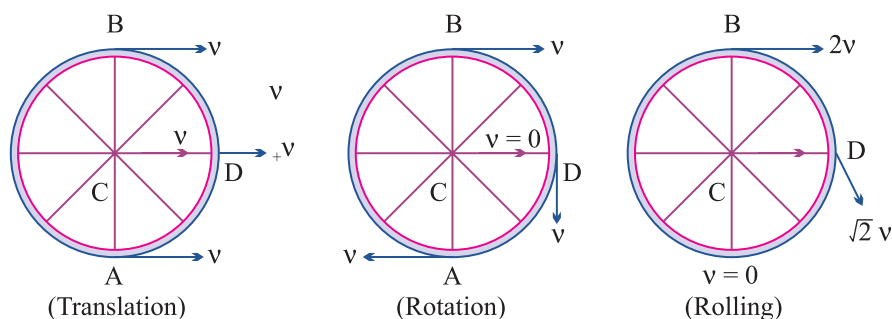
## 5.15 Rolling Without Slipping

In case of combined translatory and rotatory motion if the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping.

Friction is responsible for this type of motion but work done or dissipation of energy against friction is zero as there is no relative motion between body and surface at the point of contact.

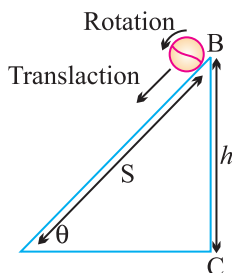
Rolling motion of a body may be treated as a pure rotation about an axis through point of contact with same angular velocity  $\omega$ . [ $v = R\omega$ ]

**Linear velocity of different points in rolling :** In case of rolling, all points of a rigid body have same angular speed but different linear speed. Let A, B, C and D are four points then their velocities are shown in the following figure.



## 5.16 Rolling on an Inclined Plane

When a body of mass  $m$  and radius  $R$  rolls down on inclined plane of height ' $h$ ' and angle of inclination  $\theta$ , it loses potential energy. However it acquires both linear and angular speeds and hence, gain kinetic energy of translation and that of rotation.



(1) Velocity at the lowest point :  $v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$

(2) Acceleration in motion : From equation  $v^2 = u^2 + 2as$ .

By substituting  $u = 0$ ,  $s = \frac{h}{\sin \theta}$  and  $v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$ , we get  $a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}}$ .

(3) Time of descent : From equation  $v = u + at$

By substituting  $u = 0$  and value of  $v$  and  $a$  from above expressions

$$t = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g} \left[ 1 + \frac{k^2}{R^2} \right]}$$

### Rotational Motion (1 mark)

1. About which axis a uniform cube will have minimum moment of inertia ?
2. State the principle of moments of rotational equilibrium.
3. Find the moment of inertia of a disc of radius  $R$  and mass  $m$  about an axis in its plane at a distance  $R/2$  from its centre.
4. Can the couple acting on a rigid body produce translator motion ?
5. Which component of linear momentum does not contribute to angular momentum ?
6. A system is in stable equilibrium. What can we say about its potential energy ?
7. Is radius of gyration a constant quantity ?



8. Two solid spheres of the same mass are made of metals of different densities. Which of them has a large moment of inertia about the diameter ?
9. The moment of inertia of two rotating bodies A and B are  $I_A$  and  $I_B$  ( $I_A > I_B$ ) and their angular momenta are equal. Which one has a greater kinetic energy ?
10. A particle moves on a circular path with decreasing speed. What happens to its angular momentum ?
11. What is the value of instantaneous speed of the point of contact during pure rolling ?
12. Which physical quantity is conserved when a planet revolves around the sun ?
13. What is the value of torque on the planet due to the gravitational force of sun ?
14. If no external torque acts on a body, will its angular velocity be constant ?
15. Why there are two propellers in a helicopter ?
16. A child sits stationary at one end of a long trolley moving uniformly with speed  $V$  on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, then what is the effect of the speed of the centre of mass of the (trolley + child) system ?

### Rotational Motion (2 marks)

17. Show that in the absence of any external force, the velocity of the centre of mass remains constant.
18. State the factors on which the position of centre of mass of a rigid body depends.
19. What is the turning effect of force called for ? On what factors does it depend ?
20. State the factors on which the moment of inertia of a body depends.
21. On what factors does radius of gyration of body depend ?
22. Why the speed of whirl wind in a Tornado is alarmingly high ?
23. Can a body be in equilibrium while in motion ? If yes, give an example.
24. There is a stick half of which is wooden and half is of steel. (i) it is pivoted at the wooden end and a force is applied at the steel end at right angle to its length (ii) it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular acceleration more and why ?
25. If earth contracts to half of its present radius what would be the length of the day at equator ?

26. An internal force can not change the state of motion of centre of mass of a body. How does the internal force of the brakes bring a vehicle to rest ?
27. When does a rigid body said to be in equilibrium ? State the necessary condition for a body to be in equilibrium.
28. How will you distinguish between a hard boiled egg and a raw egg by spinning it on a table top ?
29. Equal torques are applied on a cylinder and a sphere. Both have same mass and radius. Cylinder rotates about its axis and sphere rotates about one of its diameter. Which will acquire greater speed and why ?
30. In which condition a body lying in gravitational field is in stable equilibrium ?
31. Give the physical significance of moment of inertia. Explain the need of fly wheel in Engine.

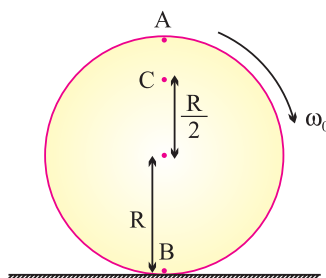
### Rotational Motion (3 marks)

32. Derive the three equation of rotational motion

$$(i) \omega = \omega_0 + \alpha t \quad (ii) \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad (iii) \omega^2 = \omega_0^2 + 2\alpha\theta$$

Under constant angular acceleration. Here symbols have usual meaning.

33. Obtain an expression for the work done by a torque. Hence write the expression for power.
34. Prove that the rate of change of angular momentum of a system of particles about a reference point is equal to the net torque acting on the system.
35. Three mass point  $m_1, m_2, m_3$  are located at the vertices of equilateral  $\Delta$  of side 'a'. What is the moment of inertia of system about an axis along the altitude of  $\Delta$  passing through  $m_1$  ?
36. Show that moment of a couple does not depend on the point about which moment is calculated.
37. A disc rotating about its axis with angular speed  $\omega_0$  is placed lightly (without any linear push) on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in figure. Will the disc roll ?



38. A uniform circular disc of radius  $R$  is rolling on a horizontal surface. Determine the tangential velocity (i) at the upper most point (ii) at the centre of mass and (iii) at the point of contact.
39. Explain if the ice on the polar caps of the earth melts, how will it affect the duration of the day ?
40. A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0.1 m. If the height of the incline plane is 4 m, what is rotational K.E. when it reaches the foot of the plane ?
41. Find the torque of a force  $7i - 3j - 5k$  about the origin which acts on a particle whose position vector is  $i + j - k$ .

### Numericals

42. Three masses 3 kg, 4 kg and 5 kg are located at the corners of an equilateral triangle of side 1 m. Locate the centre of mass of the system.
43. Two particles mass 100 g and 300 g at a given time have velocities  $10i - 7j - 3k$  and  $7i - 9j + 6k$   $\text{ms}^{-1}$  respectively. Determine velocity of COM.
44. From a uniform disc of radius  $R$ , a circular disc of radius  $R/2$  is cut out. The centre of the hole is at  $R/2$  from the centre of original disc. Locate the centre of gravity of the resultant flat body.
45. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds, (i) What is its angular acceleration (assume the acceleration to be uniform) (ii) How many revolutions does the wheel make during this time ?
46. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm, what is the mass of the meter stick ?
47. A solid sphere is rolling on a frictionless plane surface about its axis of symmetry. Find ratio of its rotational energy to its total energy.
48. Calculate the ratio of radii of gyration of a circular ring and a disc of the same radius with respect to the axis passing through their centres and perpendicular to their planes.
49. Two discs of moments of inertia  $I_1$  and  $I_2$  about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed  $\omega_1$  and  $\omega_2$  are brought into contact face to face with their axes of rotation coincident, (i) What is the angular speed of the two-disc system ? (ii) Show

that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy ? Take  $\omega_1 \neq \omega_2$ .

- 50.** In the HCl molecule, the separating between the nuclei of the two atoms is about  $1.27 \text{ \AA}$  ( $1 \text{ \AA} = 10^{-10} \text{ m}$ ). Find the approximate location of the CM of the molecule, given that the chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in all its nucleus.
- 51.** A child stands at the centre of turn table with his two arms out stretched. The turn table is set rotating with an angular speed of 40 rpm. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to  $2/3$  times the initial value ? Assume that the turn table rotates without friction. (ii) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation.

How do you account for this increase in kinetic energy ?

- 52.** To maintain a rotor at a uniform angular speed of  $200 \text{ rad s}^{-1}$ , an engine needs to transmit a torque of  $180 \text{ Nm}$ . What is the power required by the engine ? Assume that the engine is 100% efficient.
- 53.** A car weighs  $1800 \text{ kg}$ . The distance between its front and back axles is  $1.8 \text{ m}$ . Its centre of gravity is  $1.05 \text{ m}$  behind the front axle. Determine the force exerted by the level ground on each front and back wheel.

### Rotational Motion (5 marks)

- 54.** Prove that the angular momentum of a particle is twice the product of its mass and areal velocity. How does it lead to the Kepler's second law of planetary motion ?
- 55.** Prove the result that the velocity  $V$  of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height

$$h \text{ is given by } v^2 = \frac{2gh}{1 + \frac{k^2}{R^2}}.$$

where  $K$  = Radius of gyration of body about its symmetry axis, and  $R$  is radius of body. The body starts from rest at the top of the plane.

- 56.** (i) Establish the relation between torque and angular acceleration.

Hence define moment of inertia.

- 57.** State the theorem of :

- Find the moment of inertia of a rod of mass  $M$  and length  $L$  about an axis perpendicular to it through one end. Given the moment of inertia about an axis perpendicular to rod and through COM is  $\frac{1}{12}ML^2$ .

58. For which of the following does the center of mass lie outside the body?  
(a) Pencil (b) A Short put  
(c) A dice (d) A bangle
59. When a disc rotates with uniform angular velocity, which of the following is not true?  
(a) Sense of rotation remains same.  
(b) Orientation of the axis of rotation remains same.  
(c) The speed of rotation is non-zero and remains same.  
(d) The angular acceleration is non-zero and remains same.
60. Two identical particles move towards each other with velocities  $2V$  and  $V$  respectively. The velocity of centre of mass is  
(a)  $V$  (b)  $V/3$   
(c)  $V/2$  (d) Zero
61. A circular disc of radius  $R$  is removed from a bigger circular disc of radius  $2R$ , such that the circumference of the disc coincides. The centre of mass of the new disc is  $\alpha R$  from the centre of bigger disc. The value of  $\alpha$  is  
(a)  $\frac{1}{3}$  (b)  $\frac{1}{2}$   
(c)  $\frac{1}{6}$  (d)  $\frac{1}{4}$

62. Distance of the centre of mass of a solid uniform cone from its vertex is  $Z_o$ . If the radius of its base is  $R$  and its height is  $h$ , the  $Z_o$  is equal to
- (a)  $\frac{h^2}{4R}$  (b)  $\frac{3h}{4}$   
 (c)  $\frac{5h}{8}$  (d)  $\frac{3h^2}{8R}$
63. Angular momentum of the particle rotating with a central force is constant due to
- (a) Constant force (b) Constant linear momentum  
 (c) Constant torque (d) Zero torque
64. Four point masses each of the value  $m$ , are placed at the corner of a square ABCD of side  $l$ . The moment of inertia of this system about an axis passing through (diagonal BD) B D is
- (a)  $3 ml^2$  (b)  $ml^2$   
 (c)  $2 ml^2$  (d)  $\sqrt{3} ml^2$
65. A couple is acting on a two particle system. The resultant motion will be
- (a) Purely rotational motion (b) Purely linear motion  
 (c) Both (a) & (b) (d) Neither (a) nor (b)
66. The dimension of angular momentum are
- (a)  $[MLT^{-2}]$  (b)  $[ML^2T^{-1}]$   
 (c)  $[ML^2T^{-2}]$  (d)  $[ML^2T]$
67. Moment of Inertia of an object does not depend up on
- (a) Mass of object (b) Mass distribution  
 (c) Angular velocity (d) Axis of rotation
68. One circular ring and one circular disc both having same mass and radius. The ratio of their moment of inertia about the axis passing through their centres and perpendicular to their planes will be
- (a)  $1 : 1$  (b)  $2 : 1$   
 (c)  $1 : 2$  (d)  $4 : 1$
69. What is the ratio of the moments of inertia of two rings radii  $r$  and  $nr$  about an axis perpendicular to their plane and passing through their centres?
- (a)  $1 : n^2$  (b)  $1 : n$   
 (c)  $1 : 2n$  (d)  $n^2 : 1$

70. Two rings of radii  $R$  and  $nR$  made from the same wire have the ratio of moments of inertia about an axis passing through their centres equal to  $1:8$ . The value of  $n$  is
- (a) 2 (b)  $2\sqrt{2}$   
 (c) 4 (d)  $\frac{1}{2}$
71. The moment of inertia of a ring about one of its diameters is  $I$ . What will be the moment of inertia about a tangent parallel to the diameter?
- (a)  $4I$  (b)  $2I$   
 (c)  $\frac{3}{2}I$  (d)  $3I$
72. A person standing on a rotating disc stretches out his hands, the angular speed will
- (a) Increase (b) Decrease  
 (c) Remains same (d) None of these
73. A sphere of radius ' $r$ ' is rolling without sliding. What is the ratio of rotational kinetic energy and total kinetic energy associated with sphere
- (a)  $\frac{2}{7}$  (b)  $\frac{2}{5}$   
 (c) 1 (d)  $\frac{1}{2}$
74. A solid sphere of radius ' $r$ ' is rolling with velocity  $V$  on a smooth plane. The total kinetic energy of sphere is
- (a)  $\frac{7}{10}mv^2$  (b)  $\frac{3}{4}mv^2$   
 (c)  $\frac{1}{2}mv^2$  (d)  $\frac{1}{4}mv^2$
75. Two bodies have their moment of inertia  $I$  and  $2I$  respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momentum will be in the ratio
- (a)  $1:2$  (b)  $\sqrt{2}:1$   
 (c)  $2:1$  (d)  $1:\sqrt{2}$
76. An inclined plane makes an angle of  $30^\circ$  with horizontal. A solid sphere rolling down this inclined plane has a linear acceleration of
- (a)  $\frac{5g}{14}$  (b)  $\frac{2g}{3}$   
 (c)  $\frac{2}{3}g$  (d)  $\frac{5g}{7}$

77. Planetary motion in the solar system describes
- (a) Conservation of kinetic energy
  - (b) Conservation of linear momentum
  - (c) Conservation of angular momentum
  - (d) All of the above.

### ASSERTION - REASON BASED QUESTIONS

#### Direction for Q.No. 1 to Q.No. 12

The following questions from 1 to 12 consists of two statements each, labelled Assertion (A) and the other labelled Reason (R). While answering these questions, you are required to choose any of the following from options (a), (b), (c) & (d).

- (a) If both A and R are true & R is the correct explanation of A.
  - (b) If both A and R are true & R is NOT the correct explanation of A.
  - (c) If A is true but R is false.
  - (d) If A is false and R is also false.
- 
- 1. Assertion (A) : In rolling, all points of a rigid body have the same linear velocity.  
Reason (R) : The rotational motion does not affect the linear velocity.
  - 2. Assertion (A) : The speed of a whirl wind in tornado is alarmingly high.  
Reason (R) : If no external torque acts on a body, its angular velocity remains constant.
  - 3. Assertion (A) : The velocity of a body at the bottom of an inclined plane of given height is more when it slides down the plane compared when it is rolling down the same plane.  
Reason (R) : In rolling down, a body acquires both kinetic energy of translation and rotation.
  - 4. Assertion (A) : Torque is due to the transverse component of force only. The radial component has no role to play.  
Reason (R) : This is because transverse component is not perpendicular to radial component.
  - 5. Assertion (A) : Power associated with torque is product of torque and angular speed of the body about the axis of rotation.  
Reason (R) : It is rotational analogue of power in translatory motion.



6. Assertion (A) : A wheel moving down a perfectly frictionless inclined plane shall undergo slipping (not rolling)  
Reason (R) : For rolling torque is required, which is provided by tangential frictional force.
7. Assertion (A) : Torque is a vector whose direction is along the applied force.  
Reason (R) :  $\vec{\tau} = \vec{F} \times \vec{r}$
8. Assertion (A) : The centre of mass of a body may lie where there is no mass.  
Reason (R) : The centre of mass has nothing to do with the mass of the body.
9. Assertion (A) : To unscrew a rusted nut, we need a wrench with longer arm.  
Reason (R) :  $\vec{\tau} = \vec{r} \times \vec{F}$
10. Assertion (A) : Moment of inertia of a body is same, whatever be the axis of rotations.  
Reason (R) : Moment of inertia depends only a distribution of mass.
11. Assertion (A) : If polar ice melts, days will be longer.  
Reason (R) : Moment of inertia decreases and these angular velocity increases.
12. Assertion (A) : Value of radius of gyration of body depends upon axis on rotation.  
Reason (R) : Radius of gyration is root near square distance of particles of the body from the axis of rotation.

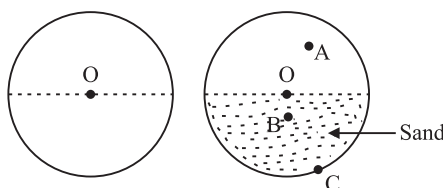
### CASE STUDY BASED QUESTIONS

#### 1. Motion of a rigid body and centre of mass :-

A Point object is only a hypothetical concept. In actual practice, we have bodies or objects which have definite size. An extended object or a real objects is made up of large number of particles. Whereas point object/mass can have only translation motion, extended object can have translational motion, rotational motion & combination of translational & rotational motion as well.

The motion of system of particles or an extended object is quite complicated. This is because every particle of the system moves in a different manner than the other particles of the system. Therefore to describe the overall motion of a body or a system of particles in a simple manner. We define a concept 'Centre of Mass'.

- (i) The centre of mass of a system of particles does not depend on
- position of particles
  - relative distance between the particles
  - forces acting on the particles
  - mass of the particles
- (ii) The centre of mass of a hallow sphere is at the its centre. Centre of mass of the hollow sphere when filled half with sand :
- shifts to A
  - shifts to B
  - shifts to C
  - remains at O
- (centre of sphere)



- (iii) For which of the following does the centre of mass outside the body
- a pencil
  - a dice
  - a bangle
  - a shot put
- (iv) Two identical particles move towards each other with velocities  $2v$  and  $v$  respectively. The velocity of centre of mass is
- $v$
  - $\frac{v}{3}$
  - $\frac{v}{2}$
  - zero
- (v) A couple is acting on a two particle system. The resultant motion will be
- purely rotational motion
  - purely translatory motion
  - both (a) & (b)
  - neither (a) nor (b)

## 2. Torque :-

We know that when an external force is applied on a body. It accelerates the body and the body has translation motion. Similarly, a body has rotational motion, upon a force is applied on a body at a certain distance from the axis of rotation of the body. The product of the applied force & distance of the point of application of the force from the axis of rotation its expressed

by a physical quality known as a 'Torque'. Torque is rotational analogue of force. Torque is an axial vector as it points along the axis of rotation.

- (i) The torque about the origin of the force  $\vec{F} = mg \hat{j}$  and  $\vec{r} = x\hat{i} + y\hat{j}$  is  
 (a)  $\vec{\tau} = mgx\hat{i}$       (b)  $\vec{\tau} = mgy\hat{j}$       (c)  $\vec{\tau} = mgx\hat{k}$       (d)  $\vec{\tau} = 0$
- (ii) Which of the following statement is false for torque  
 (a)  $\vec{\tau} = \frac{d\vec{l}}{dt}$       (b)  $\vec{\tau} = \vec{r} \times \vec{f}$       (c)  $\vec{\tau} = \vec{f} \times \vec{r}$       (d)  $\vec{\tau} = I\vec{\alpha}$
- (iii) Let  $\vec{F}$  be the force acting on a particle having position vector  $\vec{r}$  and  $\vec{\tau}$  be the torque of this force about the origin, then  
 (a)  $\vec{r} \cdot \vec{F} = 0$  and  $\vec{F} \cdot \vec{\tau} \neq 0$       (b)  $\vec{r} \cdot \vec{\tau} \neq 0$  and  $\vec{F} \cdot \vec{\tau} = 0$   
 (c)  $\vec{r} \cdot \vec{\tau} \neq 0$  and  $\vec{F} \cdot \vec{\tau} \neq 0$       (d)  $\vec{r} \cdot \vec{\tau} = 0$  and  $\vec{F} \cdot \vec{\tau} = 0$
- (iv) Angular momentum of the particle rotating with a central force is constant due to  
 (a) constant force      (b) constant linear momentum  
 (c) constant torque      (d) zero torque
- (v) When a disc rotates with uniform angular velocity, which of the following is correct?  
 (a) The sense of rotation remains the same  
 (b) The orientation of the axis of the rotation remains the same  
 (c) The speed of rotation is non-zero & remains the same  
 (d) The angular acceleration is non-zero & remains the same.
- (vi) The separation between the C and O atoms in CO is  $1.2 \text{ \AA}$ . The distance of carbon atom from the centre of mass is  
 (a)  $0.3 \text{ \AA}$       (b)  $0.7 \text{ \AA}$       (c)  $0.5 \text{ \AA}$       (d)  $0.9 \text{ \AA}$

### 3. Equilibrium of a rigid body :-

A rigid body is said to be in translational equilibrium, if the net external force acting on the body is zero and it is said to be in rotational equilibrium, if net external torque acting on the body is zero. A rigid body is said to be in partial equilibrium, if either it is in rotational equilibrium but not in translational equilibrium or it is in translational equilibrium but not in rotational equilibrium.

- (i) A couple is acting on rigid body then which statement is true  
 (a) The body is in translational equilibrium but not in rotational equilibrium

- (b) The body is in rotational equilibrium but not in translational equilibrium
  - (c) The body is in translational as well as in rotational equilibrium
  - (d) The body is neither in translational equilibrium nor in rotational equilibrium.
- (ii) For a body in translational equilibrium, which of the following statement is false
- (a)  $\sum \vec{F}_{\text{ext}} = 0$       (b)  $\vec{p} = 0$       (c)  $\vec{v} = 0$  or constant      (d)  $\vec{a} \neq 0$
- (iii) For a body in rotational equilibrium which of the following is true
- (a)  $\sum \vec{\tau}_{\text{ext}} = 0$       (b)  $\vec{L} \neq 0$       (c)  $\vec{r} \neq 0$       (d)  $\vec{\omega} \neq 0$
- (iv) A merry go round, made of a ring like platform of radius R and mass M, is revolving with angular speed  $\omega$ . A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen the ground). The speed of the merry go round afterwards is
- (a)  $2\omega$       (b)  $\omega$       (c)  $\frac{\omega}{2}$       (d) 0
- (iv) A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc.
- (a) Continuously decrease
  - (b) Continuously increase
  - (c) First increases and then decrease
  - (d) remaining unchanged

#### 4. Moment of inertia, parallel and perpendicular axis theorem :-

A body rotating about a given axis tends to maintain its state of uniform rotation, unless an external torque is applied on it to change this state. The property of a body by virtue of which it opposes to change its state of rest or uniform rotation about an axis is called rotational inertia or moment of inertia. Moment of inertia is the rotational analogue of mass.

There are two theorem namely theorem of perpendicular axes and theorem of parallel axes, which are used to find the MI of a regular shaped bodies about any axis of rotation. If the value of MI of the given body is known about a certain axis of rotation.

- (i) The moment of inertia (MI) of a body does not depend upon its
- Axis of rotation
  - different orientation of axis of rotation
  - Nature of distribution of mass
  - Angular velocity
- (ii) Which physical quantity is represented by the product of MI and angular acceleration
- Centre of mass
  - Torque
  - Angular momentum
  - Linear momentum
- (iii) The MI of a this uniform rod of length  $L$  and mass  $M$ , about an axis passing through its one end and perpendicular to its length is
- $\frac{3}{2}ML^2$
  - $\frac{1}{2}ML^2$
  - $\frac{1}{12}ML^2$
  - $\frac{1}{3}ML^2$
- (iv) The radius of gyration of a solid cylinder about its own axis is
- $\frac{R}{2}$
  - $\frac{R}{\sqrt{2}}$
  - $2R$
  - $\frac{R}{\sqrt{3}}$
- (v) The MI of a circular ring of radius  $R$  and mass  $M$  about its diameter is
- $MR^2$
  - $\frac{1}{2}MR^2$
  - $\frac{3}{2}MR^2$
  - $2MR^2$

## 5. Rolling Motion :-

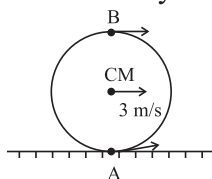
One of the most common motion observed in daily life is the rolling motion. All wheels used in transportation have rolling motion. Rolling motion can be regarded as a combination of pure rotation and pure translation (without slipping). This means that at any instant of time, the bottom of the body which is in contact with the surface is at rest always.

Total kinetic energy of a rolling body is the sum of translational kinetic energy and rotational kinetic energy of the rolling body.

- (i) For a pure rolling (without slipping) of a spherical rigid body when of the following friction is not correct.
- During pure rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body
  - The instantaneous speed of the point of contact during rolling is zero.
  - For perfect rolling, work done against friction is zero.
  - A sphere moving down a perfectly frictional inclined plane will undergo pure rolling motion.

- (ii) A spherical body is rolling over a horizontal surface w/o slipping such that the velocity of centre of mass (CM) of the body is 3 m/s. The velocity of point A & B on the body at any instant is

- (a) 0 m/s, 6 m/s  
 (b) 0 m/s, 3 m/s  
 (c) 3 m/s, 0 m/s  
 (d) 6 m/s, 0 m/s



- (iii) A sphere of radius 1m is rolling over a horizontal floor w/o slipping with an angular velocity of 1 rad/s. The linear velocity of the centre of mass (CM) of the sphere is

- (a)  $\frac{1}{2}$  m/s      (b) 1 m/s      (c) 2 m/s      (d) at rest

- (iv) A solid sphere and hollow sphere both of same mass and same radius and made of same material are allowed to roll down on inclined plane, then

- (a) solid sphere reaches the bottom first  
 (b) hollow sphere reaches the bottom first  
 (c) both reach the bottom at the same time  
 (d) cannot say

- (v) If a solid sphere of mass 1 kg and radius 0.1 m rolls w/o slipping at a uniform velocity of 1 m/s, along a straight line on a horizontal floor, then kinetic energy of the sphere is

- (a)  $\frac{2}{5}$  J      (b) 1 J      (c)  $\frac{7}{10}$  J      (d)  $\frac{7}{5}$  J

### ANSWERS (ROTATIONAL MOTION) 1 MARK

1. It will be about an axis passing through the centre of the cube and connecting the opposite corners.
2.  $\sum \vec{\tau} = 0$ .
3.  $\frac{1}{2}MR^2$ .
4. No. It can produce only rotatory motion.
5. Radial Component.
6. P.E. is minimum.
7. No, it changes with the position of axis of rotation.

8. Sphere of small density will have large moment of inertia.
9.  $K = \frac{L^2}{2I} \Rightarrow K_B > K_A$ .
10. as  $\vec{L} = \vec{r} \times m \vec{v}$  i.e., magnitude  $\vec{L}$  decreases but direction remains constant.
11. Zero.
12. Angular momentum of planet.
13. Zero.
14. No.  $\omega \propto \frac{1}{I}$ .
15. Due to conservation of angular momentum.
16. No change in speed of system as no external force is working.

### ANSWERS (2 MARKS)

18. (i) Shape of body  
(ii) mass distribution
19. Torque  
Factors  
(i) Magnitude of force  
(ii) Perpendicular distance of force vector from axis of rotation.
20. (i) Mass of body  
(ii) Size and shape of body  
(iii) Mass distribution w.r.t. axis of rotation  
(iv) Position and orientation of rotational axis
21. Mass distribution.
22. In this, air from nearly regions get concentrated in a small space, so  $I \downarrow$  considerably. Since  $I.W = \text{constant}$  so  $W \uparrow$ , so high.
23. Yes, if body has no linear and angular acceleration. Hence a body in uniform straight line motion will be in equilibrium.

24.  $I \text{ (first case)} > I \text{ (Second case)}$

$\therefore \tau = I\alpha$

$\Rightarrow \alpha \text{ (first case)} < \alpha \text{ (second case)}$

25.  $I_1 = \frac{2}{5}MR^2 \Rightarrow I_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2 \Rightarrow I_2 = \frac{I_1}{4}$

$L = I_1\omega_1 = I_2\omega_2$

or  $I\left(\frac{2\pi}{T_1}\right) = \frac{1}{4}\left(\frac{2\pi}{T_2}\right)$

or  $T_2 = \frac{T_1}{4} = \frac{24}{4} = 6 \text{ hours}$

26. In this case the force which bring the vehicle to rest is friction, and it is an external force.

27. For translation equilibrium

$$\Sigma \vec{F}_{\text{ext}} = 0$$

For rotational equilibrium

$$\Sigma \vec{\tau}_{\text{ext}} = 0$$

28. For same external torque, angular acceleration of raw egg will be small than that of Hard boiled egg.

29.  $\tau = I\alpha, \alpha = \frac{\tau}{I}$

$\alpha \text{ in cylinder, } \alpha_C = \frac{\tau}{I_C}$

$\alpha \text{ in sphere, } \alpha_S = \frac{\tau}{I_S}$

$$\frac{\alpha_C}{\alpha_S} = \frac{I_S}{I_C} = \frac{\frac{2}{5}MR^2}{MR^2} = \frac{2}{5}$$

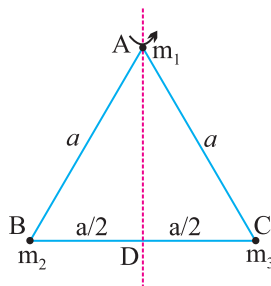
30. When vertical line through centre of gravity passes through the base of the body.

31. It plays the same role in rotatory motion as the mass does in translatory motion.



## ANSWERS (3 MARKS)

35.



$$\begin{aligned}
 I &= \sum_{i=1}^n m_i r_i^2 \\
 &= m_1 \times 0 + m_2 \times (BD)^2 + m_3 \times (DC)^2 \\
 &= 0 + m_2 \left(\frac{a}{2}\right)^2 + m_3 \left(\frac{a}{2}\right)^2 \\
 I &= \frac{1}{4}(m_2 + m_3)a^2
 \end{aligned}$$

37. For A,  $V_A = R\omega_0$  in forward direction

For B,  $V_B = R\omega_0$  in backward direction

For C,  $V_C = \frac{R}{2}\omega_0$  in forward direction disc will not roll.

$$41. \quad \vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 7 & -3 & -5 \end{vmatrix} = -8\hat{i} - 2\hat{j} - 10\hat{k}$$

## ANSWERS (NUMERICALS)

42.  $(x, y) = (0.54 \text{ m}, 0.36 \text{ m})$

43. Velocity of COM =  $\frac{31\hat{i} - 34\hat{j} + 15\hat{k}}{2} \text{ ms}^{-1}$ .

44. COM of resulting portion lies at  $R/6$  from the centre of the original disc in a direction opposite to the centre of the cut out portion.

45.  $\alpha = 4\pi \text{ rad s}^{-1}$

$$n = 576$$

46.  $m = 66.0 \text{ gm}$ .

47. 
$$\text{Rot. K.E.} = \frac{1}{2} I \omega^2 = \frac{1}{2} \times \frac{2}{5} M R^2 \times \frac{V^2}{R^2}$$

$$\left( \text{as } \omega = \frac{V}{R}, I = \frac{2}{5} M R^2 \right)$$

$$= \frac{1}{5} m v^2$$

$$\text{Total energy} = \text{Translational K.E.} + \text{Rot. K.E.}$$

$$= \frac{1}{2} m v^2 + \frac{1}{5} m v^2 = \frac{7}{10} m v^2$$

$$\therefore \frac{\text{Rot. K.E.}}{\text{Total Energy}} = \frac{\frac{1}{5} m v^2}{\frac{7}{10} m v^2} = \frac{2}{7}$$

48.  $\sqrt{2} : 1$

49. (i) Let  $\omega$  be the angular speed of the two-disc system. Then by conservation of angular momentum.

$$(I_1 + I_2) \omega = I_1 \omega_1 + I_2 \omega_2$$

or 
$$\omega = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2}$$

- (ii) Initial K.E. of the two discs.

$$K_1 = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2$$

Final K.E. of the two disc system.

$$K_2 = \frac{1}{2} (I_1 + I_2) \omega^2$$

$$= \frac{1}{2} (I_1 + I_2) \left( \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2} \right)^2$$

$$\text{Loss in K.E.} = K_1 - K_2 = \frac{1}{2} (I_1 \omega_1^2 + I_2 \omega_2^2) - \frac{1}{2(I_1 + I_2)} (I_1 \omega_1^2 + I_2 \omega_2^2)$$

$$= \frac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 - \omega_2)^2 = \text{a positive quantity } [\because \omega_1 \neq \omega_2]$$

Hence there is a loss of rotational K.E. which appears as heat.

When the two discs are brought together, work is done against friction between the two discs.

50. As shown in Fig. suppose the H nucleus is located at the origin. Then

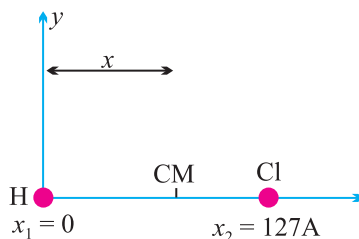
$$x_1 = 0, x_2 = 1.27 \text{ \AA}, m_1 = 1, m_2 = 35.5$$

The position of the CM of HCl molecule is

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\frac{1 \times 0 + 35.5 \times 1.27}{1 + 35.5} = 1.239 \text{ \AA}$$

Thus the CM of HCl is located on the line joining H and Cl nuclei at a distance of 1.235 Å from the H nucleus.



51. Here  $\omega_1 = 40 \text{ rpm}$ ,  $I_2 = \frac{2}{5} I_1$

By the principle of conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2 \text{ or } I_1 \times 40 = \frac{2}{5} I_1 \omega_2 \text{ or } \omega_2 = 100 \text{ rpm.}$$

- (ii) Initial kinetic energy of rotation

$$\frac{2}{5} I_1 \omega_1^2 = \frac{2}{5} I_1 (40)^2 = 800 I_1$$

New kinetic energy of rotation

$$\frac{2}{5} I_2 \omega_2^2 = \frac{1}{2} \times \frac{2}{3} I_1 (100)^2 = 2000 I_1$$

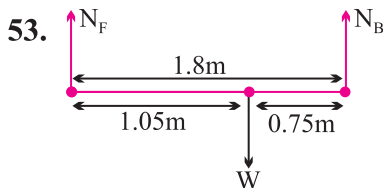
$$\frac{\text{New K.E.}}{\text{Initial K.E.}} = \frac{2000 I_1}{800 I_1} = 2.5$$

Thus the child's new kinetic energy of rotation is 2.5 times its initial kinetic energy of rotation. This increase in kinetic energy is due to the internal energy

of the child which he uses in folding his hands back from the out stretched position.

52. Here  $\omega = 200 \text{ rad s}^{-1}$ ,  $\tau = 180 \text{ Nm}$

$\therefore$  Power,  $P = \tau \omega = 180 \times 200 = 36,000 \text{ W} = 36 \text{ kW}$ .



For translation equilibrium of car

$$N_F + N_B = W = 1800 \times 9.8 = 17640 \text{ N}$$

For rotational equilibrium of car

$$1.05 N_F = 0.75 N_B$$

$$1.05 N_F = 0.75 (17640 - N_F)$$

$$1.8 N_F = 13230$$

$$N_F = 13230/1.8 = 7350 \text{ N}$$

$$N_B = 17640 - 7350 = 10290 \text{ N}$$

$$\text{Force on each front wheel} = \frac{7350}{2} = 3675 \text{ N}$$

$$\text{Force on each back wheel} = \frac{10290}{2} = 5145 \text{ N}$$

### ANSWERS (5 MARKS)

56. (ii) Yes, a body in translatory motion shall have angular momentum unless fixed point about which angular momentum is taken lies on the line of motion of body

$$\begin{aligned} \therefore |\vec{L}| &= rp \sin \theta \\ &= 0 \text{ only when } \theta = 0^\circ \text{ or } 180^\circ \end{aligned}$$

(iv) MI of stone  $I = ml^2$  ( $l \rightarrow$  length of string)

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{\tau}{ml^2}$$

if  $l$  is large  $\alpha$  is very small

$\therefore$  more difficult to revolve.

**Answer (MCQ) Key :**

58. (d)    59. (d)    60. (c)    61. (a)    62. (b)    63. (d)  
 64. (a)    65. (a)    66.    67.    68. (b)    69. (a)  
 70. (a)    71. (d)    72. (b)    73. (a)    74. (a)    75. (d)  
 76. (a)    77. (c)

**HINTS AND SOLUTION (MCQ)**

58. (d) In bangle centre of mass lies at it's centre.

59. (d)  $\alpha = \frac{dw}{dt}$ , given  $w = \text{constant}$   
 Hence  $\alpha = 0$

60. (c)  $V_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m(2v) + m(-v)}{2m} = \frac{V}{2}$

61. (a) Mass of original disc =  $m$

Mass of disc removed,  $m_1 = \frac{m}{4}$

Mass of remaining disc =  $\frac{3m}{4}$

Mass  $m_1$  and  $m_2$  are concentrated at  $O_1$  and  $O_2$  respectively and  $O$  is their centre of mass.

Moment of  $m_1$  about  $O$  = moment of  $m_2$  about  $O$ .

$$\frac{m}{4} \times R = \frac{3m}{4} \times \alpha R \Rightarrow \alpha = \frac{1}{3}$$

62. Mass of elementary disc,

$$dm = \pi r^2 dz$$

$$\frac{r}{R} = \frac{z}{h} \Rightarrow r = \frac{Rz}{h}$$

$$Z_o = \frac{\int z dm}{\int dm} = \frac{\int_0^h \pi r^2 dz}{\int_0^h \pi r^2 dz} = \frac{3}{4} h$$

63. (d) Torque due to central force is zero

$$\tau = \frac{dL}{dt} = 0 \Rightarrow L = \text{constant.}$$

64. (a)  $AC = BD = \sqrt{2} \ell$   
 $I_{BD} = m (AO)^2 + m (CO)^2 = ml^2$
65. (a) Net force for a couple is zero.  
 So, couple produces only rotational motion.
66. (b)  $\vec{L} = \vec{r} \times \vec{p} \quad [L] = [M L^2 T^{-1}]$
67. (c) M.I. does not depend up on angular velocity.
68. (b)  $\frac{I_{\text{ring}}}{I_{\text{disc}}} = \frac{mR^2}{\frac{1}{2}mR^2} = 2 : 1$
69. (a)  $\frac{I_1}{I_2} = \frac{Mr^2}{m(nr)^2} = 1 : n^2$
70. (a) As radius of round ring is n times, length and hence mass of wire is also n times  

$$\frac{I_1}{I_2} = \frac{mR^2}{nM(nR)^2} = \frac{1}{n^3} = \frac{1}{8} \Rightarrow n = 2$$
71. (d)  $I_T = I + mR^2 = \frac{1}{2}mR^2 + mR^2$   

$$= 3 \times \frac{1}{2}mR^2 = 3 I$$
72. (b) As person stretches his hands outward, hence moment of inertia,  $I$  = increases  
 $L = I\omega = \text{constant}$ , So  $\omega$  decreases.
73. (a)

$$E_T = E_{\text{trans}} + E_{\text{rot}} = \frac{1}{2} mV^2 + \frac{1}{2} I\omega^2$$

$$I = \frac{2}{5}mr^2, E_T = \frac{7}{10} mV^2$$

$$\frac{E_{\text{rot}}}{E_{\text{trans}}} = \frac{\frac{1}{5} mV^2}{\frac{7}{10} mV^2} = \frac{2}{7}$$

74. (a)  $E_T = E_{\text{trans}} + E_{\text{rot}} = \frac{7}{10} mV^2$
75. (d) Rotational K.E. =  $\frac{1}{2} I\omega^2$   
 $\frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} I_2 \omega_2^2 \Rightarrow \omega_2 = \frac{\omega_1}{\sqrt{2}}$   
 $\frac{L_1}{L_2} = \frac{1}{\sqrt{2}}$
76. (a)  $a = \frac{5}{7} g \sin 30^\circ = \frac{5g}{14}$
77. (c)  $T_{\text{ext}} = 0$  So  $\frac{d\vec{L}}{dt} = 0$   
 $\Rightarrow \vec{L} = \text{constant.}$

### ASSERTION - REASON BASED ANSWERS

- (d) both A & R are false. In case of rolling body, all fruits on it have same angular velocity but different linear speed. Linear speed is different at different places. It is the maximum at highest point and minimum at lowest point.
- (c) here assertion is true but reason is false.  
 Assertion is true as per conservation of angular momentum but when no external torque acts on the body, then its angular momentum is constant not angular velocity.
- (b) both A & R are true but R is not the correct explanation of A. When a body slides its PE is converted into KE of translation, but when it rolls down, some part of its PE is converted into its KE of rotation, thereby reducing the KE of translation and hence its linear velocity.
- (c) A is true but R false.
- (a) both A & R are true & R is the correct explanation of A.  
 In translatory motion  $P = fV$ .  
 In rotatory motion  $P = I\omega$ .
- (a) both A & R are true and R is the correct explanation of A.
- (d) both A & R are false.  
 $\vec{\tau} = \vec{r} \times \vec{F}$  so  $\vec{\tau}$  is  $\perp$  to the force.

8. (c) A is true but R is false.  
CM depends upon the distribution of mass.
9. (a) both A & R are true and R is the correct explanation of A.
10. (d) both A & R are false.
11. (c) A is true but R is false. When ice melts, MI increases and hence angular velocity decreases.  
$$L = I\omega = \text{constant}$$
12. (a) both A & R are true and R is the correct explanation of A.

### CASE STUDY BASED ANSWERS

1. i. (c) forces acting on the particle  
ii. (b) shifts to B.  
iii. (c) a bangle  
iv. (c)  $\frac{v}{2} = \left[ \text{use } V_{\text{CM}} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} \right]$   
v. (a) purely rotational motion
2. i. (c)  $\vec{\tau} = \vec{r} \times \vec{F}$   
$$\vec{\tau} = (x\hat{i} + y\hat{j}) \times mg\hat{j}$$
$$= mgx(\hat{i} \times \hat{j})$$
$$= mgx \hat{k}$$
  
ii. (c)  $\vec{\tau} = \vec{F} \times \vec{r}$   
iii. (d)  $\vec{r} \cdot \vec{\tau} = 0$  and  $\vec{F} \cdot \vec{\tau} = 0$  [as vector  $\vec{\tau}$  is  $\perp$  to both  $\vec{r}$  &  $\vec{F}$ ]  
[so  $\vec{r} \cdot \vec{\tau} = 0$  and also  $\vec{F} \cdot \vec{\tau} = 0$ ]  
iv. (d) Zero torque [as  $\frac{d\vec{\tau}}{dt} = \vec{\tau}_{\text{ent}}$  so then  $\vec{\tau}_{\text{ent}} = 0$   
then  $\frac{d\tau}{dt} = 0 \Rightarrow \tau = \text{constant}$   
v. (d) The angular acceleration is non-zero & remains the same.
3. i. (a) ii. (d) iii. (a)  
iv. (a) v. (c)