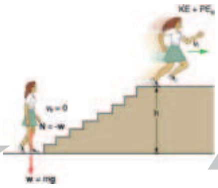


Unit - 4

Work, Energy And Power

4.1 Introduction

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

4.2 Work Done by a Constant Force

Let a constant force F be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance s.

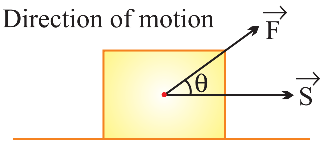
Then work done by the force in displacing the body through a distance s is given by

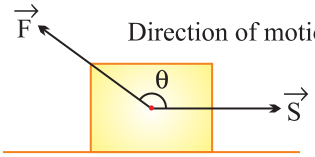
$$W = (F \cos \theta)\, s = Fs \cos \theta \Rightarrow W = (F \cos \theta)\, s = Fs \cos \theta$$

$$W = \vec{F} . \vec{s}$$

4.3 Nature of Work Done

Positive work	Negative work
Positive work means that force (or its component) is parallel to displacement $0^\circ \leq \theta < 90^\circ$	Negative work means that force (or its component) is opposite to displacement *i.e.*, $90^\circ < \theta \leq 180^\circ$





The positive work signifies that the external external force favours the motion of the body.	The negative work signifies that the force opposes the motion of the body.

4.4 Work Done by a Variable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinite simal displacement is given by $dW = \vec{F}.\vec{ds}$.

The total work done in going from A to B is $W = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\cos\theta)\, ds$.

Area under force displacement curve with proper algebraic sign represents work done by the force.

4.5 Work Depends on Frame of Reference

With change of frame of reference (inertial) force does not change while displacement may change. So the work done by a force will be different in different frames.

Examples : If a person is pushing a box inside a moving train, the work done in the frame of train will $\vec{F}.\vec{s}$ while in the frame of earth will be $\vec{F}.(\vec{s}+\vec{s}_0)$ where $\vec{s}_0$ is the displacement of the train relative to the ground.

4.6 Energy

The energy of a body is defined as its capacity for doing work.

(1) It is a scalar quantity.

(2) Dimension : $[ML^2T^{-2}]$ it is same as that of work or torque.

(3) Units : Joule [S.I.], erg [C.G.S.]

Practical units : electron volt (eV), Kilowatt hour (KWh), Calories (Cal)

Relation between different units :

$$1 \text{ Joule} = 10^7 \text{ erg}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joule}$$

$$1 \text{ KWh} = 3.6 \times 10^6 \text{ Joule}$$

$$1 \text{ Calorie} = 4.18 \text{ Joule}$$

(4) Mass energy equivalence : The relation between the mass of a particle m and its equivalent energy is given as $E = mc^2$ where c = velocity of light in vacuum.

4.7 Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.

Let m = mass of the body, v = velocity of the body then K.E. = $\frac{1}{2}mv^2$.

(1) Kinetic energy depends on frame of reference : The kinetic energy of a person of mass m, sitting in a train moving with speed v, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

(2) Work-energy theorem : It states that work done by a force acting on a body is equal to the change produced in the kinetic energy of the body.

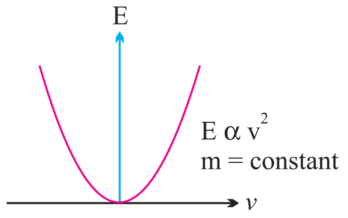
This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

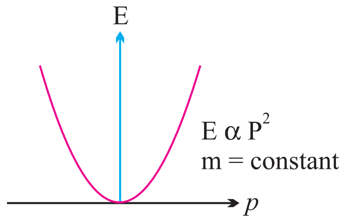
(3) Relation of kinetic energy with linear momentum : As we know

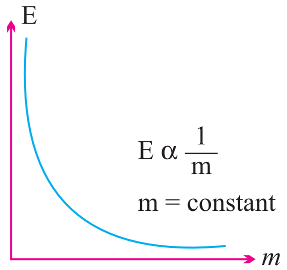
$$E = \frac{p2}{2m} \Rightarrow p = \sqrt{2mE}$$

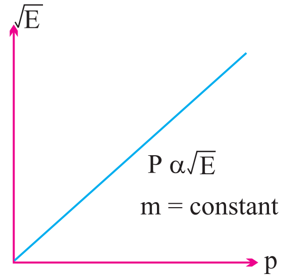
(4) Various graphs of kinetic energy

4.8 Kinetic Energy









4.9 Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types : Elastic potential energy and Gravitational potential energy etc.

(1) Change in potential energy : Change in potential energy between any two points is defined in terms of the work done by the force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_1^2 \vec{F}.d\vec{r} = -W \qquad ...(1)$$

(2) Potential energy curve : A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve. Negative gradient of the potential energy gives force.

$$-\frac{dU}{dx} = F$$

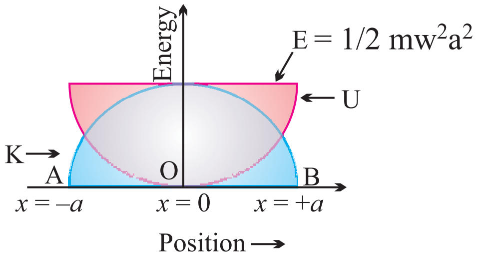
(5) Types of equilibrium : If net force acting on a particle is zero, it is said to be in equilibrium.

For equilibrium, $\frac{dU}{dx} = 0$, but the equilibrium of particle can be of three types :

Stable	Unstable	Neutral
When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.
Potential energy is minimum.	Potential energy is maximum.	Potential energy is constant.
$F = -\frac{dU}{dx} = 0$	$F = -\frac{dU}{dx} = 0$	$F = \frac{dU}{dx} = 0$
$\frac{d^2U}{dx^2} = \text{positive}$ *i.e.,* rate of change of $\frac{dU}{dx}$ is positive.	$\frac{d^2U}{dx^2} = \text{negative}$ *i.e.,* rate of change of $\frac{dU}{dx}$ is negative.	$\frac{d^2U}{dx^2} = 0$ *i.e.,* rate of change of $\frac{dU}{dx}$ is zero.
Example : A marble placed at the bottom of a hemi-spherical bowl.	*Example :* A marble balanced on top of a hemi-spherical bowl.	*Example :* A marble placed on horizontal table.

4.10 Elastic Potential Energy

(1) Restoring force and spring constant : When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance x, a restoring force is produced in the spring to bring it to the normal position. According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.



i.e., $\vec{F} \propto \vec{x}$

or $\vec{F} = k\vec{x}$ ...(i)

where k is called spring constant.

(2) Expression for elastic potential energy :

Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Note :

- If spring is stretched from initial position x_1 to final position x_2 then work done = Increment in elastic potential energy

$$= \frac{1}{2}k(x_2^2 - x_1^2)$$

(3) Energy graph for a spring : It mean kinetic energy changes parabolically w.r.t. position but total energy remain always constant irrespective to position of the mass.

4.11 Law of Conservation of Energy

(1) Law of conservation of energy : For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depends upon time. This is known as the law of conservation of mechanical energy.

(2) Law of conservation of total energy : If the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical etc., which is conserved.

4.12 Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av}) = \frac{\Delta W}{\Delta t} = \frac{W}{t}$.

Instantaneous power $(P_{inst.}) = \frac{dW}{dt} = \frac{\vec{F}.d\vec{s}}{dt}$

[As $dW = \vec{F}.d\vec{s}$]

$P_{inst} = \vec{F}.\vec{v}$.

i.e., power is equal to the scalar product of force with velocity.

(1) Dimension : $[P] = [ML^2T^{-3}]$

(2) Units : Watt or Joule/sec [S.I.]

Practical Units : Kilowatt (kW), Mega watt (MW) and Horse power (*hp*)

Relations between different units : 1 watt = 1 Joule/sec = 10^7 erg/sec

$$1hp = 746 \text{ Watt}$$

(3) The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan\theta$

(4) Area under power time curve gives the work done as $P = \frac{dW}{dt}$

$\therefore$ $W = \int P dt$

$\therefore$ W = Area under $P - t$ curve

4.13 Collision

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

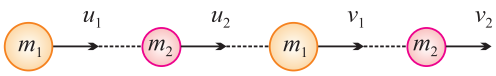
In collision particles may or may not come in real touch.

(3) Types of collision : (i) On the basis of conservation of kinetic energy.

Perfectly Elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
$(KE)_{final} = (KE)_{initial}$	Here kinetic energy appears in other forms. In some cases $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released.	The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation).
Examples : (1) Collision between atomic particles **(2)** Bouncing of ball with same velocity after the collision with earth.	**Examples :** (1) Collision between two billiard balls. (2) Collision between two automobile on a road. In fact all majority of collision belong to this category.	**Example :** Collision between a bullet and a block of wood into which it is fired. When the bullet remains embeded in the block.

4.14 Perfectly Elastic Head on Collision

Let two bodies of masses m_1 and m_2 moving initial velocities u_1 and u_2 in the same direction they collide such that after collision their final velocities are v_1 and v_2 respectively.



Before collision After collision

According to law of conservation of momentum and conservation of kinetic energy.

Note :

- The ratio of relative velocity of separation and relative velocity of approach is defined as coefficient of restitution.

$$e = \frac{v_2 - v_1}{u_1 - u_2} \text{ or } v_2 - v_1 = e\,(u_1 - u_2).$$

- For perfectly elastic collision $e = 1$

$$\therefore \quad v_2 - v_1 = u_1 - u_2 \quad \text{[As shown in eq. (vi)]}$$

- For perfectly inelastic collision $e = 0$

$$\therefore \quad v_2 - v_1 = 0 \text{ or } v_2 = v_1$$

  It means that two body stick together and move with same velocity.

- For inelastic collision $0 < e < 1$

$$\therefore \quad v_2 - v_1 = (u_1 - u_2)$$

  In short we can say that e is the degree of elasticity of collision and it is dimension less quantity.

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \quad \text{...(vii)}$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_1 + \frac{2m_1u_1}{m_1 + m_2} \quad \text{...(viii)}$$

- When two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

(2) Kinetic energy transfer during head on elastic collision : Fractional decrease in kinetic energy

$$\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 - m_2) + 4m_1m_2} \quad \text{...(iv)}$$

Note :

- Greater the difference in masses less will be transfer of kinetic energy and vice versa.

- Transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal.

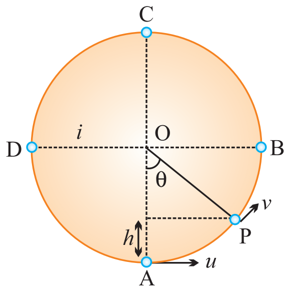
4.15 Motion in Vertical Circle

This is an example of non-uniform circular motion. In this motion body is under the influence of gravity of earth.

(1) Velocity at any point on vertical loop : If u is the initial velocity imparted to body at lowest point then, velocity of body at height h is given by

$$v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gl(1-\cos\theta)}$$

where l is the length of the string.



(2) Tension at any point on vertical loop : Tension at general point P,

$$T = mg\cos\theta + \frac{mv^2}{l}$$

(3) Various conditions for vertical motion :

Velocity at lowest point	Condition
$u_A > \sqrt{5gl}$	Tension in the string will not be zero at any of the point and body will continue the circular motion.
$u_A = \sqrt{5gl}$,	Tension at highest point C will be zero and body will just complete the circle.
$\sqrt{2gl} < u_A < \sqrt{5gl}$,	Particle will not follow circular motion. Tension in string become zero somewhere between points B and C whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory.
$u_A = \sqrt{2gl}$	Both velocity and tension in the string becomes zero between A and B and particle will oscillate along semi-circular path.
$u_A < \sqrt{2gl}$	Velocity of particle becomes zero between A and B but tension will not be zero and the particle will oscillate about the point A.

(6) Various quantities for a critical condition in a vertical loop at different positions :

Quantity	Point A	Point B	Point C	Point D	Point P
Linear velocity (v)	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$	$\sqrt{3gl}$	$\sqrt{gl(3+2\cos\theta)}$
Angular velocity (ω)	$\sqrt{\frac{5g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}(3+2\cos\theta)}$
Tension in String (T)	$6\,mg$	$3\,mg$	0	$3\,mg$	$3\,mg(1+\cos\theta)$
Kinetic Energy (KE)	$\frac{5}{2}mgl$	$\frac{3}{2}mgl$	$\frac{1}{2}mgl$	$\frac{3}{2}mgl$	$\frac{mu^2}{1} - 5mg = 0$
Potential Energy (PE)	0	mgl	$2mgl$	mgl	$mgl(1-\cos\theta)$
Total Energy (TE)	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$

VERY SHORT ANSWER QUESTIONS (1 MARK)

1. Define the conservative and non-conservative forces. Give examples of each.

2. A light and heavy body have same linear momentum. Which one has greater kinetic energy?

3. How much work is done by a coolie walking a horizon lat platform with a load on his head?

4. A body is moving along a circular path. How much work is done by the centripetal force ?

5. Which spring has greater value of spring constant – a hard spring or a delicate spring ?

6. Two bodies stick together after collision. What type of collision is in between these two bodies ?

7. State the two conditions under which a force does no work ?

8. How will the momentum of a body changes if its K.E. is doubled ?

9. K.E. of a body is increased by 300 %. Find the % increase in its momentum ?

10. A light and a heavy body have same K.E., which of the two have more momentum and why ?

11. Does the P.E. of a spring decreases or increases when it is compressed or stretched?

12. Name a process in which momentum changes but K.E. does not.

13. What happens to the P.E. of a bubble when it rises in water ?

14. A body is moving at constant speed over a frictionless surface. What is the work done by the weight of the body ?

15. Define spring constant of a spring.

SHORT ANSWER QUESTIONS (2 MARKS)

16. The momentum of body is doubled. What % does it K.E. charge?

17. Mountain roads orally go straight up the slope but wind up gradually. Why?

18. A truck and a car moving with the same K.E. on a straight road. Their engines are simultaneously switched off which one will stop at a lesser distance ?

19. Is it necessary that work done in the motion of a body over a closed loop is zero for every force in nature ? Why?

20. Derive an expression for K.E. of a body of mass '*m*' moving with velocity '*v*' by calculus method.

21. How high must a body be lifted to gain an amount of P.E. equal to the K.E. it has when moving at speed 20 ms^{-1}. (The value of acceleration due to gravity at a place is 9.8 ms^{-2}).

22. Give an example in which a force does work on a body but fails to change its K.E.

23. A bob is pulled sideway so that string becomes parallel to horizontal and released. Length of the pendulum is 2 m. If due to air resistance loss of energy is 10%, what is the speed with which the bob arrived at the lowest point.

24. Two springs A and B are identical except that A is harder than B ($K_A > K_B$) if these are stretched by the equal force. In which spring will more work be done ?

25. Two springs A and B are identical except that A is harder than B i.e., $K_A > K_B$. In which spring is more work expanded if they are stretched by same amount?

26. Find the work done if a particle moves from position $\vec{r_1} = (3\hat{i} + 2\hat{j} - 6\hat{k})$ to a position $\vec{r_2} = (14\hat{i} + 13\hat{j} - 9\hat{k})$ under the effect of force $\vec{F} = (4\hat{i} + \hat{j} + 3\hat{k})N$.

27. A ball at rest is dropped from a height of 12 m. It loses 25% of its kinetic energy in striking the ground, find the height to which it bounces. How do you account for the loss in kinetic energy ?

28. State and prove work energy theorem.

29. Which of the two kilowatt hour or electron volt is a bigger unit of energy and by what factor ?

30. A spring of force constant K is cut into two equal pieces. Calculate force constant of each part.

SHORT ANSWER QUESTIONS (3 MARKS)

31. A elastic spring is compressed by an amount x. Show that its P.E. is $1/2\ kx^2$ where k is the spring constant.

32. A car of mass 2000 kg is lifted up a distance of 30 m by a crane in 1 min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel ? What is the power supplied by each crane ? Neglect Power dissipation against friction.

33. Prove that bodies of identical masses exchange their velocities after head-on elastic collision.

34. A bullet of mass 0.012 kg and horizontal speed 70 m/s strikes a block of wood of mass 0.4 kg and instantly comes to rest w.r.t. the block. The block is suspended from the ceiling by wire. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

35. Define elastic and inelastic collision. A lighter body collides with a much more massive body at rest. Prove that the direction of lighter body is reversed and massive body remains at rest.

36. 20 J work is required to stretch a spring through 0.1 m. Find the force constant of the spring. If the spring is further stretched through 0.1 m. Calculate work done.

37. A body of mass M at rest is struck by a moving body of mass m. Prove that fraction of the initial K.E. of the mass m transferred to the struck body is $4\ \text{m M}/(m + \text{M})^2$ in an elastic collision.

38. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, how much electric power is consumed by the pump. The efficiency of the pump is 30%.

39. Show that in an elastic one dimensional collision the relative velocity of approach before collision is equal to the relative velocity of separation after collision.

40. A ball bounces to 80% of its original height. Calculate the mechanical energy lost in each bounce.

LONG ANSWER QUESTIONS (5 MARKS)

41. Show that at any instant of time during the motion total mechanical energy of a freely falling body remains constant. Show graphically the variation of K.E. and P.E. during the motion.

42. Define spring constant, write the characteristics of the force during the elongation of a spring. Derive the relation for the P.E. stored when it is elongated by X. Draw the graphs to show the variation of P.E. and force with elongation.

43. How does a perfectly inelastic collision differ from perfectly elastic collision ? Two particles of mass m_1 and m_2 having velocities U_1 and U_2 respectively make a head on collision. Derive the relation for their final velocities. Discuss the following special cases.

(i) $m_1 = m_2$

(ii) $m_1 >> m_2$ and $U_2 = 0$

(iii) $m_1 << m_2$ and $U_1 = 0$

NUMERICALS

44. A body is moving along z-axis of a coordinate system under the effect of a constant force $F = (2\hat{i} + 3\hat{j} + \hat{k})$N. Find the work done by the force in moving the body a distance of 2 m along z-axis.

45. Water is pumped out of a well 10 m deep by means of a pump rated 10 KW. Find the efficiency of the motor if 4200 kg of water is pumped out every minute. Take $g = 10$ m/s^2.

46. A railway carriage of mass 9000 kg moving with a speed of 36 km h^{-1} collides with a stationary carriage of same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision ? What type of collision is this ?

47. In lifting a 10 kg weight to a height of 2m, 230 J energy is spent. Calculate the acceleration with which it was raised ?

48. A bullet of mass 0.02 kg is moving with a speed of 10 ms^{-1}. It can penetrate 10 cm of a wooden block, and comes to rest. If the thickness of the target would be 6 cm only, find the K.E. of the bullet when it comes out.

49. A man pulls a lawn roller through a distance of 20 m with a force of 20 kg weight. If he applies the force at an angle of 60º with the ground, calculate the power developed if he takes 1 min in doing so.

50. A body of mass 0.3 kg is taken up an inclined plane to length 10 m and height 5 m and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.15. What is the

(i) work done by the gravitational force over the round trip.

(ii) work done by the applied force over the upward journey.

(iii) work done by frictional force over the round trip.

(iv) kinetic energy of the body at the end of the trip.

How is the answer to (iv) related to the first three answer ?

51. Two identical 5 kg blocks are moving with same speed of 2 ms^{-1} towards each other along a frictionless horizontal surface. The two blocks collide, stick together and come to rest. Consider the two blocks as a system. Calculate work done by (i) external forces and (i) Internal forces.

52. A truck of mass 1000 kg accelerates uniformly from rest to a velocity of 15 ms^{-1} in 5 seconds. Calculate (i) its acceleration, (ii) its gain in K.E., (iii) average power of the engine during this period, neglect friction.

53. An elevator which can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 ms^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power.

54. To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 kmh^{-1} on a smooth road and colliding with a horizontally mounted spring of spring constant 6.25×10^{-3} Nm^{-1}. What is the maximum compression of the spring.

MCQ (Work, Energy and Power)

55. A man is squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is

(a) constant and equal to 'mg' in magnitude.

(b) constant and greater than 'mg' in magnitude.

(c) variable but always greater than 'mg'

(d) at first greater than 'mg' and later becomes equal to 'mg'

56. A body of mass 0.5 kg travels in straight line with velocity $V = ax^{3/2}$ where $a = 5\ m^{-½}\ s^{-1}$. The work done by the net force during it's displacement from x = 0 to x = 2m is

(a) 15 J (b) 50 J

(c) 10 J (d) 100 j

57. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 rev/min. it's kinetic energy would be

(a) $250\ \pi^2$ (b) $100\ \pi^2$

(c) $5\ \pi^2$ (d) 0

58. An Athlete in the Olympic games covers a distance of 100 m in 10s, this kinetic energy can be estimated to be in the range (assume m = 60 kg)

(a) 200J – 500 J (b) 2×10^5J – 3×10^5J

(c) 20000J – 50000J (d) 2000J – 5000J

59. A block of mass 0.5 kg is moving with a speed of 2m/s on a smooth surface. It strikes another mass of 1 kg at rest and then they move together as a single body. The energy loss during the collision is

(a) 0.16J (b) 1.00 J

(c) 0.67 J (d) 0.34J

60. A bullet fired in to a fixed target losses half of it's velocity after penetrating distance of 3 cm. How much further it will penetrate before coming to rest assuming that if faces constant resistance to it's motion?

(a) 3 cm (b) 2.0 cm

(c) 1.5 cm (d) 1.0 cm

61. A uniform chain of length 2m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

(a) 7.2 J (b) 3.6 J

(c) 120 J (d) 1200 J

62. If the linear momentum is increased by 50%, then kinetic energy will increase by

(a) 50% (b) 100%

(c) 125% (d) 25%

63. A block having mass 'm' collides with another stationary block having mass 2m. The lighter block comes to rest after collision. If the velocity of first block is V, than the value of co-efficient of restitution will be

(a) 0.5 (b) 0.4

(c) 0.6 (d) 0.8

64. A body of mass 50 kg is at rest. The work done to accelerate it by 20 m/s in 10 s is

(a) 10^3 J (b) 10^4 J

(c) 2×10^3 J (d) 4×10^4 J

65. A spring of force constant 800 Nm^{-1} has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(a) 16 J (b) 8 J

(c) 35 J (d) 24 J

66. A particle is projected at an angle of 60° to the horizontal with a kinetic energy E. The kinetic energy at the highest point is

(a) E (b) E/4

(c) E/2 (d) Zero

67. A child is sitting on a swing. Its minimum and maximum heights from the ground 0.75 m and 2 m respectively, it's maximum speed will be

(a) 10 m s^{-1} (b) 5 m s^{-1}

(c) 8 m/s (d) 15 m s^{-1}

68. 300 J of work is done in sliding a 2 kg block up on inclined plane of height 10 m. Work done against friction is ($g = 10\ ms^{-2}$)

(a) 1000 J (b) 200 J

(c) 100 J (d) Zero

69. During inelastic collision between two bodies, which of the following quantities always remain conserved

(a) Total kinetic energy (b) Total mechanical energy

(c) Total linear momentum (d) Speed of each body

70. Two bodies with kinetic energies in the ratio 4 : 1 are moving with equal linear momentum. The ratio of their masses is

(a) 4 : 1 (b) 1 : 1

(c) 1 : 2 (d) 1 : 4

71. A position dependent force, $F = 7 - 2x + 3x^2$ N acts on a small body of mass 2 kg and displaces it from x = 0 to x = 5 m. The work done in joule is

(a) 135 (b) 270

(c) 35 (d) 70

72. A ball is dropped from height 'h' on the ground where co-efficient of restitution is 'e'. After one bounce the maximum height is

(a) $e^2 h$ (b) $e\sqrt{h}$

(c) eh (d) $\sqrt{eh}$

73. How much water a pump of 2 KW can raise in one minute to a height of 10 m? (g = 10 ms)?

(a) 1000 litres (b) 1200 litres

(c) 10 litres (d) 2000 litres

74. A bomb of mass 30 kg at rest explodes in to two pieces of masses 18 kg and 12 kg. The velocity of 18 kg mass is $6\ ms^{-1}$. The kinetic energy of the other mass is

(a) 324 J (b) 486 J

(c) 256 J (d) 5245 J

ASSERTION AND REASON TYPE QUESTIONS

(Work, Power and Energy)

Directions:- The following questions consist of two statements each, labelled as Assertion (A) and the other labelled Reason (R). While answering these questions. You are required to choose any of the following four options (a), (b), (c) & (d).

(a) If both A and R are true and R is the correct explanation of A.

(b) If both A and R are true but R is NOT the correct explanation of A.

(c) If A is true but R is false.

(d) If A is false and R is also false/True.

1. Assertion (A) : When two equal masses undergo a glancing elastic collision in 2D with one of them at rest, then after the collision, they will be at 90° to each other.

Reason (R) : It follows from the principle of conservation of linear momentum.

2. Assertion (A) : According to law of conservation of mechanical energy, change in potential energy is equal to and opposite for the change kinetic energy.

Reason (R) : Mechanical energy is not a conserved quantity.

3. Assertion (A) : Friction is a non-conservative force.

Reason (R) : This is because work done against friction in moving a body over a curved path is never zero.

4. Assertion (A) : Mass and energy are not conserved separately, but are conserved as a single entity called mass-energy.

Reason (R) : This is because one can be obtained at the cost of the other as per Einstein equation ($E = mc^2$).

5. Assertion (A) : Work done by the centripetal free in moving a body along a circle is always zero.

Reason (R) : Because displacement of the body is along the force.

6. Assertion (A) : A spring has potential energy, both when it is compressed or stretched.

Reason (R) : This is because in compressing or stretching work is done by the spring against the restoring force.

7. Assertion (A) : Graph between potential energy of a spring v/s the extension / compression (x) of the spring is a straight line.

Reason (R) : Potential energy is directly proportional to x.

8. Assertion (A) : In an elastic collision between the two bodies, the relative speed of the bodies after collisions is equal to the relative speed before the collision.

Reason (R) : In a elastic collision the linear momentum of the system is conserved.

9. Assertion (A) : Work done by the frictional force is negative.

Reason (R) : Frictional force acts along the direction of motion.

10. Assertion (A) : A body cannot have energy without possessing momentum but it can have momentum without having energy.

Reason (R) : Momentum and energy have same dimensions.

11. Assertion (A) : Time taken by a body to complete a given work has nothing to do with energy of the body.

Reason (R) : Power of a body is the rate of doing work.

12. Assertion (A) : Two particles moving in the same direction do not lose all their energy in a perfectly inelastic collision.

Reason (R) : Principle of conservation of linear momentum holds true for all kind of collisions.

CASE STUDY BASED QUESTIONS

Work :-

Q1. The term work is frequently used in everybody language. A farmer ploughing the field, a construction worker carrying bricks on his head, a student studying for a competitive examination, an artist painting a beautiful landscape. All are said to be working but in the language of physics they are not doing any work! In physics however the word 'Work' covers a definite & precise meaning. Energy is capacity of a body to do the work & power is the rate of doing work.

Though work is scalar quantity, yet its value may positive, negative and Zero.

(i) In physics work is defined as -

(a) Product of component of force in the direction of displacement and the magnitude of displacement.

(b) Product of component of force perpendicular to the direction of displacement and its magnitude of the displacement.

(c) Cross product of force vector and displacement vector.

(d) Product of the component of the force in the direction of displacement and magnitude of velocity.

(ii) Which of the following is not an example of zero work done -

(a) Work done by centripetal force.

(b) Work done by tension in a spring of simple pendulum.

(c) Word done by a frictional force.

(d) The work done in pushing a immovable stone.

(iii) A body is subjected to a constant force.

$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}\ (N)$ is constrained to move along the z axis of a coordinate system. The work done by the force in moving the body though a distance of 4m along z axis is

(a) 12 J (b) –12 J (c) 0 J (d) 16 J

(iv) When a body is thrown up, during the upward journey the work done by gravity an the body is

(a) + Positive (b) Zero (c) negative (d) cannot say

(v) A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(a) $t^{1/2}$ (b) t (c) $t^{3/2}$ (d) t^2

Conservative Forces & Non-conservative Forces, Transformation & Conservation of energy :-

Q2. The process of changing or converting one form of energy into another form is known as transformation of energy. We know, mechanical energy of a system is conserved, if forces acting on it are conservative forces. However, if a system is acted upon by conservative & non-conservative forces, then some of the mechanical energy of the system is converted into other forms of energy like sound, heat & hight energies but total mechanical energy of the system remains constant. Mechanical energy is the sum of kinetic energy & potential energy where $K.E = \frac{1}{2}mv^2$ and potential energy = mgh, where symbols have usual meanings.

(i) Which one the following is non-conservative force?

(a) gravitational force (b) electrostatic force

(c) magnetic force (d) frictional force

(ii) A body falling freely under the action of gravity alone in vacuum. Which of the following quantities remains constant during the fall :-

(a) kinetic energy

(b) potential energy

(c) total mechanical energy

(d) total linear momentum

(iii) A mass of 5 kg is moving along a circular path of radius 1 m. If mass moves with 300 rev./min., its KE would be

(a) 250 π^2J (b) 100 π^2 J (c) 5 π^2 J (d) 0 J

(iv) In which case the potential energy decreases ?

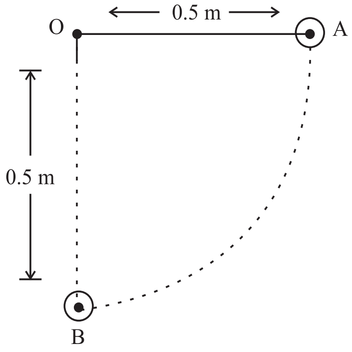
(a) on compressing the string

(b) on stretching the string

(c) on moving a body against gravitational pull

(d) on rising of an air bubble in water

(v) The bob of simple pendulum is held in the horizontal position A. Assuming no loss of energy, speed of bob at the lowest position B when released is



(a) $\sqrt{9.8}$ m/s

(b) 9.8 m/s

(c) 0 m/s

(d) $\sqrt{2\times9.8}$ m/s

Motion in a Vertical Circle :-

Q3. A uniform circular motion is the motion of a particle travelling at a constant (uniform) speed along circular path and hence its kinetic energy remain same everywhere. But when a particle moves in a vertical circle completing the loop, then its speed goes on changing at every point & hence, its kinetic energy goes on changing but total mechanical energy remains constant.

(i) Uniform circular motion is a example of

(a) accelerated motion

(b) uniform motion

(c) non-accelerated motion

(d) None of the above

(ii) The minimum velocity with which a body of mass m must enter a vertical loop of radius r, so that it can complete the loop is

(a) $\sqrt{2gr}$ (b) $\sqrt{3gr}$ (c) $\sqrt{gr}$ (d) $\sqrt{5gr}$

(iii) A bucket of water of mass m is rotated in a vertical circle of radius r such that the bucket is upside down at the highest point. The minimum angular velocity so that the water does not spill out is

(a) $w = \sqrt{\frac{r}{g}}$ (b) $w = \sqrt{\frac{g}{r}}$ (c) $w = \sqrt{rg}$ (d) $w = \sqrt{3rg}$

(iv) Particle of mass m executing a circular motion in a vertical plane (of radius r) has the tension in the string at the lowest point equal to

(a) $T_L = mg$ (b) $T_L = 0$

(c) $T_L = \frac{mv_L^2}{r} + mg$ (d) $T_L = \frac{mv_L^2}{r} - mg$

(v) The ratio of kinetic energy at the lowest point to the kinetic energy of the highest point of a vertical circle of radius r looping by a particle of mass m is

(a) 1 : 5 (b) 1 : 3 (c) 3 : 1 (d) 5 : 1

Collision :-

Q4. The laws of momentum & energy conservation are successfully applied to a commonly encountered phenomenon; namely – collisions. Several games such as billiards, marble or carom involve collisions. In all types of collisions; the linear momentum is conserved, on the other hand the total kinetic energy of the system is not necessarily conserved. The impact & deformation during collision may generate heat & sound.

The degree of elasticity of a collision is determined by a quantity called coefficient of restitution (e).

(i) In an elastic collision

(a) both momentum & kinetic energy are conserved

(b) both momentum & kinetic energy are non-conserved

(c) only energy is conserved

(d) Only momentum is conserved.

(ii) A body of mass M_1 collides elastically with another man M_2 at rest. There is 100% transfer of energy takes place when (assuming perfectly elastic collision)

(a) $M_1 > M_2$ (b) $M_1 < M_2$

(c) $M_1 = M_2$ (d) Same of all values of M_1 & M_2

(iii) A bullet hits and gets embedded in a solid block resting on a frictionless surface. In this process which one of the following is correct?

(a) only momentum is conserved

(b) only KE is conserved

(c) Neither momentum nor KE is conserved

(d) Both momentum & KE are conserved.

(iv) Two identical balls A & B collide head on elastically of velocities of A & B, before the collision are +0.5 m/s and –0.3 m/s respectively, then their velocities after the collision are respectively -

(a) –0.5 m/s, +0.3 m/s (b) +0.5 m/s, +0.3 m/s

(c) +0.3 m/s, –0.5 m/s (d) –0.3 m/s, +0.5 m/s

(v) Two bodies of mass 0.25 kg each moving towards each other with velocities 3 m/s & 1 m/s respectively. After collision, they stick together. The velocity of the combination will be -

(a) 0.1 cm/s (b) 1 cm/s

(c) 1 m/s (d) cannot be predicted

Work energy theorem :-

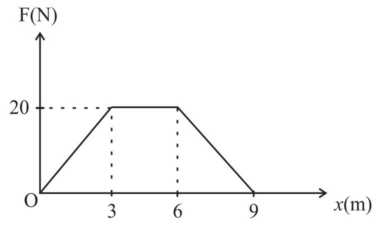
Q5. Work energy theorem states that the work done by the net force acting on the body is equal to the change produced in kinetic energy of the body. Work energy theorem is not independent of Newton's second law. It may be viewed as scalar form of second law. By using work energy theorem, the work done by a force can be calculated, even if the exact nature of the force is not known!

(i) When work is done on the system, then kinetic energy of the system -

(a) decreases (b) increases

(c) remains same (d) becomes zero

(ii) A body of mass 2.4 kg is subjected to a force which varies with distance as shown. The body starts from rest at x = 0. Its velocity at x = 9m is -



(a) $5\sqrt{3}$ m/s

(b) $20\sqrt{3}$ m/s

(c) 10 m/s

(d) 40 m/s

(iii) If the kinetic energy of the body becomes four times of its initial value, then new momentum will

(a) became twice its initial value

(b) became four times

(c) became thrice its initial value

(d) remains same

(iv) Two bodies with kinetic energies in the ratio 4 : 1 are moving with equal momentum. The ratio of their masses is

(a) 4 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 4

(v) A ball of mass 50 g is moving over the surface with velocity 10 m/s. Velocity of ball became 5 m/s after travelling some distance over the surface. The work done on the ball by the force of friction between the ball and the surface is

(a) –2 J (b) + 2 J (c) 3 J (d) –3 J

VERY SHORT ANSWERS (1 MARKS)

1. **Conservative force :** *e.g.,* Gravitational force, electrostatic force.

   Non-Conservative force : *e.g.,* forces of friction, viscosity.

2. Lighter body has more K.E. as K.E. = $\frac{p^2}{2m}$ and for constant p, K.E. $\propto \frac{1}{m}$.

3. W = 0 as angle between the force and despoilment is 90°.

   and W = FS cos θ = W = 0

4. W = FS cos 90º = 0.

5. Hard spring.

6. Inelastic collision.

7. (i) Displacement is zero or it is perpendicular to force.

   (ii) Conservative force moves a body over a closed path.

8. Momentum becomes $\sqrt{2}$ times.

9. K.E. = $\frac{p^2}{2m}$ so $p = \sqrt{2mk}$

$$\text{Increase in K.E.} = 300\% \text{ of } k = 3k$$

$$\text{Final K.E., } k' = k + 3k = 4k$$

$$\text{Final momentum, } p' = \sqrt{2mk'} = \sqrt{2m \times 4k} = 2\sqrt{2mk}$$

$$= 2p$$

$$\% \text{ Increase in momentum} = \frac{p' - p}{p} \times 100 = 100\%$$

10. Heavier body; for some kinetic energy $p \propto \sqrt{m}$

11. Increases because W.D. on it when it is compressed or stretched.

12. Uniform circular motion.

13. Decreases.

14. W = 0.

15. It is the restoring force set up in a string per unit extension.

SHORT ANSWERS (2 MARKS)

16. K.E = $p^2/2m$ when p is doubled, K.E becomes 4 times.

17. If roads go straight up then angle of slope θ would be large so frictional force $f = \mu\, mg \cos\theta$ would be less and the vehicles may slip. Also greater power would be required.

18. By Work - Energy Theorem,

Loss in K.E. = W.D. against the force × distance of friction

or K.E. $= \mu\, mg\, S$

For constant K.E., $S \propto \dfrac{1}{m}$

∴ Truck will stop in a lesser distance.

19. No. W.D. is zero only in case of a conservative force.

21. $$mgh = \frac{1}{2}mv^2$$

so $h = 20.4$ m

22. When a body is pulled on a rough, horizontal surface with constant velocity. Work is done on the body but K.E. remains unchanged.

23. $$\frac{1}{2}mv^2 = 90\% \text{ of } mgh$$

∴ $v = 6$ m/s

24. $F = Kx$ so $x = \dfrac{F}{K}$

For same F, $$W_A = \frac{1}{2}K_A x^2 = \frac{1}{2}\frac{F^2}{K_A}$$

and $$W_B = \frac{F^2}{2K_B}$$

∴ $$\frac{W_A}{W_B} = \frac{K_B}{K_A}$$

As $K_A > K_B$ so $W_A < W_B$.

25. $W = \dfrac{1}{2}Kx^2$; $\dfrac{W_A}{W_B} = \dfrac{K_A}{K_B}$, for some x

As $K_A > K_B$ So $W_A > W_B$

26. $\vec{r} = \vec{r_2} - \vec{r_1} = 11\hat{i} + 11\hat{j} - 3\hat{k}$

$\vec{F} = (4\hat{i} + \hat{j} + 3\hat{k})\ N$

$W = \vec{F}.\vec{r} = 46J$

27. If ball bounces to height h', then

$$mgh' = 75\% \text{ of } mgh$$

$$\therefore \quad h' = 0{\cdot}75\ h = 9 \text{ m.}$$

29. *kwh* is a bigger unit of energy.

$$\frac{1kwh}{1eV} = \frac{3{\cdot}6\times10^6 \text{J}}{1{\cdot}6\times10^{-19}\text{J}} = 2{\cdot}25\times10^{25}$$

30. Force constant of each half becomes twice the force constant of the original spring.

SHORT ANSWERS (3 MARKS)

32. t_1 = 1 min = 60 s, t_2 = 2 min = 120 s

$W = F_s = mgs = 5{\cdot}88 \times 10^5$ J

As both cranes do same amount of work so both consume same amount of fuel.

$$P_1 = \frac{W}{t_1}$$

and $$P_1 = \frac{W}{t_2}$$

$$\therefore \quad P_1 = 9800 \text{ W} \ \& \ P_2 = 4900 \text{ W.}$$

36. P.E. of spring when stretched through a distance 0·1 m,

$$U = \text{W.D.}$$

$$= \frac{1}{2}Kx^2 = 20 \text{ J}$$

$$K = 4000 \text{ N/m}$$

when spring is further stretched through 0·1 m, then P.E. will be :

$$U' = \frac{1}{2}k(0{\cdot}2)^2 = 80 \text{ J}$$

$$\therefore \quad \text{W.D.} = U' - U = 80 - 20 = 60 \text{ J.}$$

38. $$30\% \text{ of Power} = \frac{W}{t} = \frac{mgh}{t} = \frac{V\rho gh}{t}$$

$$\frac{30}{100} \times P = \frac{V\rho gh}{t}$$

$$\therefore \quad P = 43{\cdot}6 \text{ KW.}$$

40. Let Initial P.E. = mgh

$$\text{P.E. after first bounce} = mg \times 80\% \text{ of } h$$

$$= 0{\cdot}80\, mgh$$

$$\text{P.E. lost in each bounce} = 0{\cdot}20\, mgh$$

∴ Fraction of P.E. lost in each bounce

$$= \frac{0{\cdot}20 mgh}{mgh} = 0{\cdot}20$$

NUMERICAL ANSWERS

44. $$\vec{F} = (2\hat{i} + 3\hat{j} + \hat{k})\text{N}, \ \vec{S} = 2\hat{k}$$

$$W = \vec{F}.\vec{S} = 2 \text{ J.}$$

45. $$\text{Input power} = 10 \text{ KW}$$

$$\text{Output power} = \frac{W}{t} = \frac{mgh}{t} = 7 \text{ KW}$$

$$\therefore \quad \text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100 = 70\%$$

46. $m_1 = 9000$ kg, $u_1 = 36$ km/h = 10 m/s

$m_2 = 9000$ kg, $u_2 = 0$, $v = v_1 = v_2 = ?$

By conservation of momentum :

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$$

$$\therefore \quad v = 5 \text{ m/s}$$

$$\text{Total K.E. before collision} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$= 45000 \text{ J}$$

$$\text{Total K.E. after collision} = \frac{1}{2}(m_1 + m_2)v^2 = 225000 \text{ J}$$

As total K.E. after collision < Total K.E. before collision

$\therefore$ Collision is inelastic.

47. $W = mgh + mah = m(g + a)h$

$\therefore \quad a = 1{\cdot}5 \text{ m/s}^2.$

48. For $x = 10$ cm $= 0{\cdot}1$ m, $Fx = \frac{1}{2}mv_1^2 = 1$ J

$\therefore \quad F = 10$ N

For $x = 6$ cm $= 0{\cdot}06$ m, $Fx = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2$

or $\quad Fx = \frac{1}{2}mv_1^2 -$ Final K.E.

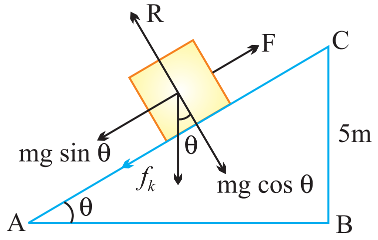
or $\quad$ Final K.E. $= \frac{1}{2}mv_1^2 - Fx = 1 - 10 \times 0{\cdot}06$

$= 1 - 0{\cdot}6$

$= 0{\cdot}4$ J

49. $P = \frac{W}{t} = \frac{Fs\cos\theta}{t} = 32{\cdot}66$ W

50.



$\sin\theta = \frac{CB}{CA} = 0{\cdot}5$

$\therefore \quad \theta = 30^\circ.$

(i) W = FS = – mg sin q × h = – 14·7 J is the W.D. by gravitational force in moving the body up the inclined plane.

W′ = FS = + mg sin θ × h = 14·7 J is the W.D. by gravitational force in moving the body down the inclined plane.

$\therefore$ Total W.D. round the trip, $W_1 = W + W' = 0$.

(ii) Force needed to move the body up the inclined plane,

$$F = mg \sin\theta + f_k$$
$$= mg \sin\theta + \mu_k R$$
$$= mg \sin\theta + \mu_k mg \cos\theta$$

∴ W.D. by force over the upward journey is

$$W_2 = F \times l = mg(\sin\theta + \mu_k \cos\theta)\, l$$
$$= 18{\cdot}5 \text{ J}$$

(iii) W.D. by frictional force over the round trip,

$$W_3 = -f_k(l + l) = -2f_k l$$
$$= -2\mu_k mg \cos\theta\, l = -7{\cdot}6 \text{ J}$$

(iv) K.E. of the body at the end of round trip

= W.D. by net force in moving the body down the inclined plane

$$= (mg \sin\theta - \mu_k mg \cos\theta)\, l$$
$$= 10{\cdot}9 \text{ J}$$

$$\Rightarrow \quad \text{K.E. of body} = \text{net W.D. on the body.}$$

51. Here no external forces are acting on the system so :

$$\vec{F}_{ext.} = 0 \Rightarrow W_{ext.} = 0$$

According to work-energy theorem :

$$\text{Total W.D.} = \text{Change in K.E.}$$

or $$W_{ext.} + W_{int.} = \text{Final K.E.} - \text{Initial K.E.}$$

$$0 + W_{int.} = 0 - \left(\frac{1}{2}mu^2 + \frac{1}{2}mu^2\right)$$

or $$W_{int.} = -mu^2 = -20 \text{ J}$$

52. (i) $a = \frac{v-u}{t} = 3 \text{ m/s}^2$

(ii) Gain in K.E. $= \frac{1}{2}m(v^2 - u^2) = 1{\cdot}125 \times 10^5$ J

(iii) $P = \frac{W}{t} = 22500$ W

53. Downward force on the elevator is :

$$F = mg + f = 22000 \text{ N}$$

∴ Power supplied by motor to balance this force is :

$$P = Fv = 44000 \text{ W}$$

$$= \frac{44000}{746} = 59\ hp.$$

54. At maximum compression x_m, the K.E. of the car is converted entirely into the P.E. of the spring.

∴ $$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2$$

or $$x_m = 2 \text{ m.}$$

Answer Key :

55. (d)	56. (b)	57. (a)	58. (d)	59. (c)	60. (d)
61. (b)	62. (c)	63. (a)	64. (b)	65. (b)	66. (b)
67. (b)	68. (c)	69. (c)	70. (d)	71. (a)	72. (a)
73. (b)	74. (b)				

HINTS AND SOLUTION TO MCQ

55. (d) In squatting, he is tilted some what, hence also has to balance frictional force besides his weight in this case R = friction + mg

$\Rightarrow$ R > mg

When he set straight up in that case,

R = mg.

56. (b)

$$\vec{A} = \frac{d\vec{v}}{dt} = \frac{2}{2} a^2 x^2, \vec{F} = m\vec{A}$$

$$w = \int_0^2 F\, ds = 50J$$

57. (a) $V = \omega R, \omega = \frac{2\pi}{T} = 10\pi \text{ rod/s}$

$$V = 10\pi \text{ ms}^{-1} \Rightarrow K.E = \frac{1}{2} mv^2 = 250\pi^2 J$$

58. (d) $V_{av} = \frac{S}{t} = 10\text{ms}^{-1}, m = 60\text{ kg} \Rightarrow \text{Av. K.E. } 3000J$

59. (c) $m_1 v_1 + m_2 v_2 = (m_1 + m_2)V \Rightarrow v = \frac{2}{3} \text{ms}^{-1}$

$$\text{Energy loss} = \frac{1}{2} \times 0.5 \times 2^2 - \frac{1}{2} \times 1.5 \times \left(\frac{2}{3}\right)^2 = 0.67 \text{ J}$$

60. (d) $W = \Delta K$

$$\text{Case I} : -F \times 3 = \frac{1}{2} m \left(\frac{Vo}{2}\right)^2 - \frac{1}{2} mV^2,$$

Where $F \rightarrow$ resistive force

$V_0 \rightarrow$ initial velocity

Case II : Let further distance be 's'

$$-F(3 + s) = K_f - K_i = -\frac{1}{2} mV_0^2$$

$$s = 1 \text{ cm}$$

61. (b) Mass per unit length $= \frac{4\text{ kg}}{2\text{ m}} = 2\text{ kg m}^{-1}$

Mass of 60 cm length = 1.2 kg.

weight of hanging part = $1.2\times10=12N$

$W = F\times S = 12\times0.3 = 3.6$ J.

62. (c) $P^1 = 1.5P$, Initial K.E. $= \frac{P^2}{2\text{ m}}$, Find K.E., $K^1 = \frac{P^{12}}{2\text{ m}}$

$K^1 = 2.25\text{ K}$

$\%\text{ increase} = \frac{\Delta K}{K}\times 100 = 125\%$

63. (a) By conservation of momentum

$mV = 2mV^1 \Rightarrow V^1 = \frac{V}{2}$

$e = \frac{\text{Vel. of separation}}{\text{Velocity of approch}} = 0.5$

64. (b) $a = \frac{v-u}{t} = 2\text{ ms}^{-2},\ 5 = ut + \frac{1}{2}at^2 = 100\text{ m}$

$W = F\times S = 10^4\text{ J}$

65. (b) $W = \frac{1}{2}\times 800\,(0.15^2 - 0.05^2) = 8\text{ J.}$

66. (b) K.E. at highest point $= \frac{1}{2}m\,(4\cos 60°)^2 = \frac{E}{4}$.

67. (b) Maximum K.E. = Drop in P.E.

$\frac{1}{2}m\,V_{max}^2 = mg\,(h_2 - h_1) \Rightarrow V_{max} = 5\text{ ms}^{-1}$

68. (c) Total work done = Gain in P.E.+Work done against friction
$300 = 2\times10\times10+W \Rightarrow W = 100$ J.

70. (d) $E \alpha \frac{1}{m}$ or $m \alpha \frac{1}{E}$

$$\frac{m_1}{m_2} = \frac{E_2}{E_1} = \frac{1}{4}$$

71. (a) $W = \int F\,dx = 135\text{ J}$

72. (a) Velocity with which the ball strikes the ground, $u = \sqrt{2gh}$

If the ball re-bounces with velocity, V, then $V = eu = e\sqrt{2gh}$

If $h' \rightarrow$ max. height after one bounce, then

$$0^2 - V^2 = 2(-g)h' \Rightarrow h' = e^2 h$$

73. (b) $P = \frac{w}{t} \Rightarrow w = Pt = mgh \Rightarrow m = 1200\text{ kg}$

74. (b) By conservation of momentum

$30 \times 0 = 18 \times 6 + 12 + V \rightarrow V = -9\text{ ms}^{-1}$

$$K.E. = \frac{1}{2}mV^2 = 486\text{ J}$$

ASSERTION - REASON BASED ANSWERS

1. (a) both A & R are true and R is the correct explanation of A.

2. (c) A is true but the R is false.
According to Law of conservation of mechanical on energy (If No external forces do work on a system),

$K + U = \text{constant}$

$\Delta K + \Delta U = 0$ or $\Delta K = -\Delta U$.

3. (a) both A & R are true and R is the correct explanation of assertion.

4. (a) both A & R are true and R is the correct explanation of A.

5. (c) A is true but R is false.
Displacement (tangential) of the body is perpendicular to the centripetal force (acts along radius towards centre).

6. (c) A is true but R is false.
In compressing or stretching the spring, work is done on the spring against the restoring force.

7. (d) both A & R are false.
P.E. of a spring; $U = \frac{1}{2}Kx^2$ [symbols have then usual meaning]
so $U \propto x^2$
so graph is a parabola.

8. (d) Hence Assertion is false, because is elastic collision the relative velocity of separation (not relative speed of separation) is equal to the relative velocity of approach (Not the relative speed of approach) of two bodies during elastic collision.
Reason is though true but cannot explain the assertion.

9. (c) A is true but R is false.
Force is friction acts is the apposite direction of motion.

10. (d) both A & R are false.
A raised body at rest has energy (PE) but no momentum (p) but to possess momentum (p), a body should have KE on $KE = \frac{P^2}{2m}$.
Also dimensions of momentum is $[MLT^{-1}]$ and of energy of $[ML^2T^{-2}]$.

11. (b) both A & R are true but R is not the correct explanation of A.

12. (b) both A & R are true but R is not the correct explanation of A.

CASE STUDY BASED ANSWERS

1. i. (a) $w = fd\cos\theta$

ii. (c) Work done by a frictional force is an example of negative work done.

iii. (a) $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{d} = 4k$

$w = \vec{F}.\vec{d}$

$= 3\hat{k}.4\hat{k} = 12J$

iv. (c) negative

v. (b) $v = u + at$

$v = at$ as $u = 0$

As power $P = fV = (ma)(at)$

$$P = Ma^2t \implies p \propto t \text{ as a is constant.}$$

2. i. (d)

ii. (c)

iii. (a) $KE = \frac{1}{2}mV^2 = \frac{1}{2}m(rw)^2 = \frac{1}{2}mr^2 4\pi^2 f^2 = 250\,\pi^2$ J

iv. (d) PE decreases when an air bubble rises in water because work is done by upthrust.

v. (a) Applying Law of Conservation of Energy at A and B.

$$\frac{1}{2}mV^2 = mgh$$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.5} = \sqrt{9.8} \text{ m/s}$$

3. i. (a) accelerated motion

ii. (d) velocity at the lowest point = $\sqrt{5gr}$

iii. (b) at the highest point $mrw^2 = mg \quad \Rightarrow w = \sqrt{\frac{g}{r}}$

iv. (c) $T_L = \frac{mv_L^2}{r} + mg$

v. (d) $\frac{(KE)_L}{(KE)_H} = \frac{v_L^2}{v_H^2} = \frac{5gr}{gr} = \frac{5}{1}$

4. i. (a) both momentum & KE are conserved.

ii. (c) $M_1 = M_2$ [when both the colliding bodies have equal masses, maximum transfer of energy occurs].

iii. (a) only momentum is conserved.

(as this is an example of perfectly inelastic collisions)

iv. (d) –0.3 m/s, +0.5 m/s

(In elastic collision, two identical bodies exchange then velocities after collision)

v. (c) 1 m/s

[Common velocity $V = \dfrac{m_1u_1 - m_2u_2}{m_1 + m_2}$, $u_1 = 3$ m/s, $u_2 = -1$ m/s]

5. i. (a) decreases

ii. (c) 10 m/s

Work done = change is KE

Area under f – x = $\frac{1}{2}mv^2 - 0$

$\frac{1}{2}(9 + 3) \times 20 = \frac{1}{2} \times 2.4 \times v^2$

$\Longrightarrow v = 10$ m/s

iii. (a) become twice its initial value [Using $p = \sqrt{2mk}$].

iv. (d) 1 : 4 [again using $p = \sqrt{2mk} \Rightarrow k = \dfrac{p^2}{2m}$]

v. (a) –2J [Work done = change is KE = $\frac{1}{2}m(v^2 - u^2) = -2J$]

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