METHOD OF DIFFERENTIATION

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ and $h : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then- **[JEE(Advanced) 2016]**

(A) $g'(2) = \frac{1}{15}$ (B) $h'(1) = 666$ (C) $h(0) = 16$ (D) $h(g(3)) = 36$

2. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1,3) is **[JEE(Advanced) 2014]**

SOLUTIONS

1. Ans. (B, C)

Sol. (A) $f'(x) = 3x^2 + 3$

so, $g'(2) = \dfrac{1}{f'(0)}$ (Given $g(x) = f^{-1}(x)$)

$\Rightarrow g'(2) = \dfrac{1}{3}$

(B) $h(g(g(x)) = x$

$$h'(g(g(x)) = \frac{1}{g'(g(x)).g'(x)}$$

Now, $g(g(x)) = 1$

$g(x) = f(1) = 6$

$\therefore x = f(6) = 236$

$$\text{so } h'(1) = \frac{1}{g'(6).g'(236)} = \frac{1}{\frac{1}{6}.\frac{1}{111}}$$

$\Rightarrow h'(1) = 666$

(C) $g(g(x)) = 0$

$\therefore g(x) = g^{-1}(0) \Rightarrow g(x) = f(0) \Rightarrow g(x) = 2$

$\Rightarrow x = g^{-1}(2) \Rightarrow x = f(2) \Rightarrow x = 16$

so $h(0) = 16$

(D) $g(x) = 3 \Rightarrow x = g^{-1}(3) \Rightarrow x = f(3)$

$\Rightarrow x = 38$ so $h(g(3)) = 38$

2. Ans. (8)

Sol. $(y - x^5)^2 = x(1 + x^2)^2$

$\Rightarrow 2(y - x^5)$

$$\left(\frac{dy}{dx} - 5x^4\right) = (1 + x^2)^2 + 2x(1 + x^2).2x$$

Put $x = 1, y = 3$

$$\frac{dy}{dx} = 8$$