

TRIGONOMETRIC EQUATION

1. Consider the following lists:

List-I		List-II	
(I)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P)	has two elements
(II)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3} \tan 3x = 1\right\}$	(Q)	has three elements
(III)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2 \cos(2x) = \sqrt{3}\right\}$	(R)	has four elements
(IV)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S)	has five elements
		(T)	has six elements

The correct option is:

[JEE(Advanced) 2022]

- (A) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)
 (B) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)
 (C) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)
 (D) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

2. Let $f : [0, 2] \rightarrow \mathbb{R}$ be the function defined by

[JEE(Advanced) 2020]

$$f(x) = (3 - \sin(2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$$

If $\alpha, \beta \in [0, 2]$ are such that $\{x \in [0, 2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is _____.

3. Answer the following by appropriately matching the lists based on the information given in the paragraph. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List-I contains the sets X, Y, Z and W. List -II contains some information regarding these sets.

[JEE(Advanced) 2019]

List-I		List-II	
(I)	X	(P)	$\cong \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithmetic progression
(III)	Z	(R)	NOT an arithmetic progression
(IV)	W	(S)	$\cong \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\cong \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\cong \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?

Options :

- (A) (II), (R), (S) (B) (I), (P), (R) (C) (II), (Q), (T) (D) (I), (Q), (U)

4. Answer the following by appropriately matching the lists based on the information given in the paragraph.
 Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order : [JEE(Advanced) 2019]

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\}.$$

$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List-I contains the sets X, Y, Z and W. List -II contains some information regarding these sets.

List-I	List-II
(I) X	(P) $\cong \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\cong \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$
	(T) $\cong \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$
	(U) $\cong \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$

Which of the following is the only CORRECT combination ?

Options :

- (A) (IV), (Q), (T) (B) (IV), (P), (R), (S) (C) (III), (R), (U) (D) (III), (P), (Q), (U)
5. Let a, b, c be three non-zero real numbers such that the equation [JEE(Advanced) 2018]

$$\sqrt{3}a \cos x + 2b \sin x = c, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then the value of $\frac{b}{a}$ is _____.

6. Let $S = \left\{ x \in (-\pi, \pi) : x \neq 0, \pm \frac{\pi}{2} \right\}$. The sum of all distinct solution of the equation [JEE(Advanced) 2016]
- $$\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$$
- in the set S is equal to -
- (A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

7. The number of distinct solutions of equation [JEE(Advanced) 2015]
- $$\frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$
- in the interval $[0, 2\pi]$ is

8. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has [JEE(Advanced) 2014]
- (A) infinitely many solutions (B) three solutions
 (C) one solution (D) no solution

SOLUTIONS

1. Ans. (B)

Sol.

(I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$

$\cos x + \sin x = 1$

$\Rightarrow \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos \frac{\pi}{4}$

$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}; n \in Z$

$\Rightarrow x = 2n\pi; x = 2n\pi + \frac{\pi}{2}; n \in Z$

$\Rightarrow x \in \left\{0, \frac{\pi}{2}\right\}$ in given range has two solutions

(II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3} \tan 3x = 1\right\}$

$\sqrt{3} \tan 3x = 1 \Rightarrow \tan 3x = \frac{1}{\sqrt{3}}$

$\Rightarrow 3x = n\pi + \frac{\pi}{6}$

$\Rightarrow x = (6n+1)\frac{\pi}{18}; n \in Z$

$\Rightarrow x \in \left\{\frac{\pi}{18}, \frac{5\pi}{18}\right\}$ in given range has two solutions

(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$

$2\cos 2x = \sqrt{3}$

$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6}$

$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{6}; n \in Z$

$\Rightarrow x = n\pi \pm \frac{\pi}{12}; n \in Z$

$x \in \left\{\pm \frac{\pi}{12}, \pi \pm \frac{\pi}{12}, -\pi \pm \frac{\pi}{12}\right\}$

Six solutions in given range

(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$

$\cos x - \sin x = -1$

$\Rightarrow \cos\left(x + \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos \frac{3\pi}{4}$

$\Rightarrow x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}; n \in Z$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi - \pi; n \in Z$

$\Rightarrow x \in \left\{\frac{\pi}{2}, -\frac{3\pi}{2}, \pi, -\pi\right\}$

four solutions in given range

2. Ans. (1.00)

Sol. Let $\pi x - \frac{\pi}{4} = \theta \in \left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$

So, $\left(3 - \sin\left(\frac{\pi}{2} + 2\theta\right)\right) \sin \theta \geq \sin(\pi + 3\theta)$

$\Rightarrow (3 - \cos 2\theta) \sin \theta \geq -\sin 3\theta$

$\Rightarrow \sin \theta [3 - 4\sin^2 \theta + 3 - \cos 2\theta] \geq 0$

$\Rightarrow \sin \theta (6 - 2(1 - \cos 2\theta) - \cos 2\theta) \geq 0$

$\Rightarrow \sin \theta (4 + \cos 2\theta) \geq 0$

$\Rightarrow \sin \theta \geq 0$

$\Rightarrow \theta \in [0, \pi] \Rightarrow 0 \leq \pi x - \frac{\pi}{4} \leq \pi$

$\Rightarrow x \in \left[\frac{1}{4}, \frac{5}{4}\right]$

$\Rightarrow \beta - \alpha = 1$

3. Ans. (C)

4. Ans. (B)

Sol. $f(x) = \sin(\pi \cos x)$

$X : \{x : f(x) = 0\}$

$f(x) = 0 \Rightarrow \sin(\pi \cos x) = 0 \Rightarrow \cos x = n \Rightarrow \cos$

$x = 1, -1, 0 \Rightarrow x = \frac{n\pi}{2}$

$X = \left\{\frac{n\pi}{2} : n \in N\right\} = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \dots\right\}$

$g(x) = \cos(2\pi \sin x)$

$Z = \{x : g(x) = 0\}$

$\cos(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = (2n+1)\frac{\pi}{2}$

$$\Rightarrow \sin x = \frac{(2n+1)}{4}$$

$$\sin x = -\frac{1}{4}, \frac{1}{4}, -\frac{3}{4}, \frac{3}{4}$$

$$Z = \left\{ n\pi \pm \sin^{-1}\left(\frac{1}{4}\right), n\pi \pm \sin^{-1}\left(\frac{3}{4}\right), n \in I \right\}$$

$$Y = \{x : f(x) = 0\}$$

$$f(x) = \sin(\pi \cos x) \Rightarrow f'(x)$$

$$= \cos(\pi \cos x) \cdot (-\pi \sin x) = 0$$

$$\sin x = 0 \Rightarrow x = n\pi.$$

$$\cos(\pi \cos x) = 0 \Rightarrow \pi \cos x = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \cos x = \frac{(2n+1)}{2} \Rightarrow \cos x = -\frac{1}{2}, \frac{1}{2}$$

$$Y = \left\{ n\pi, n\pi \pm \frac{\pi}{3} \right\} = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi, \dots \right\}$$

$$W = \{x : g'(x) = 0\}$$

$$g(x) = \cos(2\pi \sin x) \Rightarrow g'(x)$$

$$= -\sin(2\pi \sin x) \cdot (2\pi \cos x) = 0$$

$$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$$

$$\sin(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = n\pi \Rightarrow \sin x = \frac{n}{2}$$

$$= -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$$

$$W = \left\{ \frac{n\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in I \right\}$$

$$= \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \dots \right\}$$

Now check the options

5. **Ans. (0.5)**

Sol. $\sqrt{3} \cos x + \frac{2b}{a} \sin x = \frac{c}{a}$

Now, $\sqrt{3} \cos \alpha + \frac{2b}{a} \sin \alpha = \frac{c}{a}$ (1)

$\sqrt{3} \cos \beta + \frac{2b}{a} \sin \beta = \frac{c}{a}$ (2)

$\sqrt{3} [\cos \alpha - \cos \beta] + \frac{2b}{a} (\sin \alpha - \sin \beta) = 0$

$$\sqrt{3} \left[-2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right) \right] +$$

$$\frac{2b}{a} \left[2 \cos \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right) \right] = 0$$

$$-\sqrt{3} + 2\sqrt{3} \cdot \frac{b}{a} = 0$$

$$\frac{b}{a} = \frac{1}{2} = 0.5$$

6. **Ans. (C)**

Sol. $\sqrt{3} \sin x + \cos x = 2 \cos 2x$

$$\Rightarrow \cos 2x = \cos \left(x - \frac{\pi}{3} \right)$$

$$\Rightarrow 2x = 2n\pi \pm \left(x - \frac{\pi}{3} \right)$$

$$x = (6n-1)\frac{\pi}{3} \text{ or } (6n+1)\frac{\pi}{9}$$

$$\Rightarrow x = -\frac{\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9} \text{ and } -\frac{5\pi}{9} \text{ in } (-\pi, \pi)$$

$$\Rightarrow \text{sum} = 0$$

7. **Ans. (8)**

Sol. Given equation

$$\frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

$$\frac{5}{4} \cos^2 2x + 1 - 2 \sin^2 x \cos^2 x + 1 - 3 \sin^2 x \cos^2 x = 2$$

$$\frac{5}{4} \cos^2 2x - \frac{1}{2} (1 - \cos^2 2x) - \frac{3}{4} (1 - \cos^2 2x) = 0$$

$$\frac{5}{2} \cos^2 2x = \frac{5}{4} \Rightarrow \cos^2 2x = \frac{1}{2} \Rightarrow \cos 4x = 0$$

Clearly having 8 solutions in $[0, 2\pi]$

8. **Ans. (D)**

Sol. $\sin x - \sin 3x + 2 \sin 2x = 3$

$$-2 \sin x \cos 2x + 4 \sin x \cos x = 3$$

$$2 \sin x \{-\cos 2x + 2 \cos x\} = 3$$

$$2 \sin x \{-(2 \cos^2 x - 1) + 2 \cos x\} = 3$$

$$2 \sin x \{-2 \cos^2 x + 2 \cos x + 1\} = 3$$

$$2 \sin x \left\{ \frac{3}{2} - 2 \left(\cos x - \frac{1}{2} \right)^2 \right\} = 3$$

for equality to hold

$$\sin x = 1 \text{ \& } \cos x = \frac{1}{2}$$

which is not possible

hence no solution