STRAIGHT LINE

Question Stem for Question No. 1 and 2

Question Stem

Consider the lines L_1 and L_2 defined by

$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line y = 2x + 1 meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the **square** of the distance between R' and S'.

1. The value of λ^2 is __________. **[JEE(Advanced) 2021]**

2. The value of D is __________. **[JEE(Advanced) 2021]**

3. Let a, λ, m $\in \mathbb{R}$. Consider the system of linear equations

$ax + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is(are) correct ? **[JEE(Advanced) 2016]**

(A) If a = –3, then the system has infinitely many solutions for all values of λ and μ

(B) If a $\neq$ –3, then the system has a unique solution for all values of λ and μ

(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for a = –3

(D) If $\lambda + \mu \neq 0$, then the system has no solution for a = –3

4. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines x – y = 0 and x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is **[JEE(Advanced) 2014]**

5. For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$. Then **[JEE(Advanced) 2014]**

(A) a + b – c > 0 (B) a – b + c < 0 (C) a – b + c > 0 (D) a + b – c < 0

SOLUTIONS

1. **Ans. (9.00)**

Sol. $$\left|\frac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\left|\frac{\sqrt{2}x-y+1}{\sqrt{3}}\right|=\lambda^2$$

$$\left|\frac{2x^2-(y-1)^2}{3}\right|=\lambda^2, C:\left|2x^2-(y-1)^2\right|=3\lambda^2$$

line $y = 2x + 1, RS=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$,

$R(x_1,y_1)$ and $S(x_2,y_2)$

$y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1$

$\Rightarrow (y_1 - y_2) = 2(x_1 - x_2)$

$$RS=\sqrt{5(x_1-x_2)^2}=\sqrt{5}|x_1-x_2|$$

solve curve C and line y = 2x + 1 we get

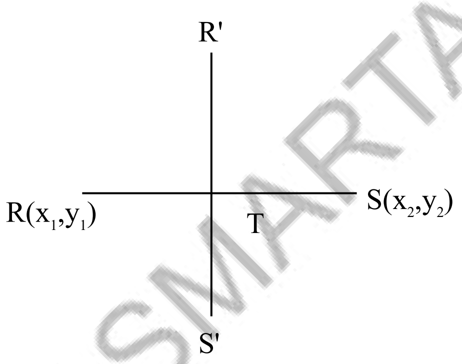
$$\left|2x^2-(2x)^2\right|=3\lambda^2 \Rightarrow x^2=\frac{3\lambda^2}{2}$$

$$RS=\sqrt{5}\left|\frac{2\sqrt{3}\lambda}{\sqrt{2}}\right|=\sqrt{30}\lambda=\sqrt{270}$$

$$\Rightarrow 30\lambda^2=270 \Rightarrow \lambda^2=9$$

2. **Ans. (77.14)**

Sol.



$\perp$ bisector of RS

$$T\equiv\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Here $x_1 + x_2 = 0$

T = (0,1)

Equation of

$$R'S' : (y-1)=-\frac{1}{2}(x-0)\Rightarrow x+2y=2$$

$R'(a_1,b_1)$ $S'(a_2,b_2)$

$$D = (a_1 - a_2)^2 + (b_1 - b_2)^2 = 5(b_1 - b_2)^2$$

solve $x+2y=2$ and $\left|2x^2-(y-1)^2\right|=3\lambda^2$

$$\left|8(y-1)^2-(y-1)^2\right|=3\lambda^2 \Rightarrow (y-1)^2=\left(\frac{\sqrt{3}\lambda}{\sqrt{7}}\right)^2$$

$$y-1=\pm\frac{\sqrt{3}\lambda}{\sqrt{7}} \Rightarrow b_1=1+\frac{\sqrt{3}\lambda}{\sqrt{7}}, b_2=1-\frac{\sqrt{3}\lambda}{\sqrt{17}}$$

$$D=5\left(\frac{2\sqrt{3}\lambda}{\sqrt{7}}\right)^2=\frac{5\times4\times3\lambda^2}{7}$$

$$=\frac{5\times4\times27}{7}=77.14$$

3. **Ans. (B, C, D)**

Sol. $ax + 2y = \lambda$

$3x - 2y = \mu$

for $a = -3$ above lies will be parallel or coincident

parallel for $\lambda + \mu \neq 0$ and coincident if $\lambda + \mu = 0$

and if $a \neq -3$ lies are intersecting

$\Rightarrow$ unique solution.

4. **Ans. (6)**

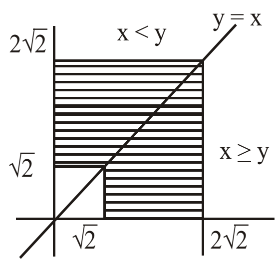
Sol. Let P(x,y) is the point in I quad.

Now $2\le\left|\frac{x-y}{\sqrt{2}}\right|+\left|\frac{x+y}{\sqrt{2}}\right|\le4$

$$2\sqrt{2}\le|x-y|+|x+y|\le4\sqrt{2}$$

Case-I : $x \ge y$

$$2\sqrt{2}\le(x-y)+(x+y)\le4\sqrt{2}$$



$\Rightarrow \quad x\in[\sqrt{2}, 2\sqrt{2}]$

Case-II : $x < y$

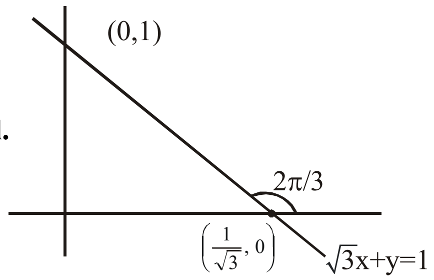
$$2\sqrt{2}\le y-x+(x+y)\le4\sqrt{2}$$

$$y\in[\sqrt{2}, 2\sqrt{2}]$$

$$A = \left(2\sqrt{2}\right)^2-\left(\sqrt{2}\right)^2=6$$

5. Ans. (A or C or A, C)

Sol.



Point of intersection of both lines is

$$\left(-\frac{c}{(a+b)}, -\frac{c}{(a+b)}\right)$$

Distance between $\left(-\frac{c}{(a+b)}, -\frac{c}{(a+b)}\right)$

& (1,1) is

Distance $= \sqrt{\frac{(a+b+c)^2}{(a+b)^2} \times 2} < 2\sqrt{2}$

$a + b + c < 2(a + b)$

$a + b - c > 0$

According to given condition option (C) also correct.