PROBABILITY

1. Let $X=\left\{(x,y)\in\mathbb{Z}\times\mathbb{Z}:\frac{x^2}{8}+\frac{y^2}{20}<1 \text{ and } y^2<5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is **[JEE(Advanced) 2023]**

(A) $\frac{71}{220}$ (B) $\frac{73}{220}$ (C) $\frac{79}{220}$ (D) $\frac{83}{220}$

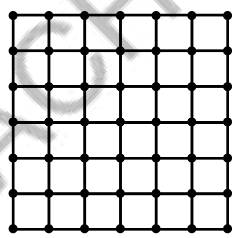
2. Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment stops with head is. **[JEE(Advanced) 2023]**

(A) $\frac{1}{3}$ (B) $\frac{5}{21}$ (C) $\frac{4}{21}$ (D) $\frac{2}{7}$

3. Let X be the set of all five digit numbers formed using 1,2,2,2,4,4,0. For example, 22240 is in X while 02244 and 44422 are not in X . Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of 38p is equal to- **[JEE(Advanced) 2023]**

Paragraph for Question No. 4 and 5

Consider the 6 × 6 square in the figure. Let $A_1, A_2, \ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.



4. Let p_i be the probability that a randomly chosen point has i many friends, $i = 0,1,2,3,4$. Let X be a random variable such that for $i = 0,1,2,3,4$, the probability $P(X = i) = p_i$. Then the value of $7E(X)$ is **[JEE(Advanced) 2023]**

5. Two distinct points are chosen randomly out of the points $A_1, A_2, \ldots, A_{49}$. Let p be the probability that they are friends. Then the value of $7p$ is **[JEE(Advanced) 2023]**

6. In a study about a pandemic, data of 900 persons was collected. It was found that

190 persons had symptom of fever,
220 persons had symptom of cough,
220 persons had symptom of breathing problem,
330 persons had symptom of fever or cough or both,
350 persons had symptom of cough or breathing problem or both,
340 persons had symptom of fever or breathing problem or both,
30 persons had all three symptoms (fever, cough and breathing problem).

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is _________. **[JEE(Advanced) 2022]**

7. Two players, P_1 and P_2, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by P_1 and P_2, respectively. If $x > y$, then P_1 scores 5 points and P_2 scores 0 point. If $x = y$, then each player scores 2 points. If $x < y$, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of P_1 and P_2, respectively, after playing the i^{th} round. **[JEE(Advanced) 2022]**

List-I		List-II	
(I)	Probability of $(X_2 \geq Y_2)$ is	(P)	$\frac{3}{8}$
(II)	Probability of $(X_2 > Y_2)$ is	(Q)	$\frac{11}{16}$
(III)	Probability of $(X_3 = Y_3)$ is	(R)	$\frac{5}{16}$
(IV)	Probability of $(X_3 > Y_3)$ is	(S)	$\frac{355}{864}$
		(T)	$\frac{77}{432}$

The correct option is:

(A) (I) → (Q); (II) → (R); (III) → (T); (IV) → (S)

(B) (I) → (Q); (II) → (R); (III) → (T); (IV) → (T)

(C) (I) → (P); (II) → (R); (III) → (Q); (IV) → (S)

(D) (I) → (P); (II) → (R); (III) → (Q); (IV) → (T)

8. Suppose that

Box-I contains 8 red, 3 blue and 5 green balls,
Box-II contains 24 red, 9 blue and 15 green balls,
Box-III contains 1 blue, 12 green and 3 yellow balls,
Box-IV contains 10 green, 16 orange and 6 white balls.

A ball is chosen randomly from Box-I ; call this ball b. If b is red then a ball is chosen randomly from Box-II, if b is blue then a ball is chosen randomly from Box-III, and if b is green then a ball is chosen randomly from Box-IV. The conditional probability of the event 'one of the chosen balls is white' given that the event 'at least one of the chosen balls is green' has happened, is equal to

[JEE(Advanced) 2022]

(A) $\frac{15}{256}$ (B) $\frac{3}{16}$ (C) $\frac{5}{52}$ (D) $\frac{1}{8}$

9. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements.

Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is **[JEE(Advanced) 2021]**

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Question Stem for Question Nos. 10 and 11

Question Stem

Three numbers are chosen at random, one after another with replacement, from the set S = {1, 2, 3, ..., 100}. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

10. The value of $\frac{625}{4}p_1$ is ________. **[JEE(Advanced) 2021]**

11. The value of $\frac{125}{4}p_2$ is ________. **[JEE(Advanced) 2021]**

12. Let E, F and G be three events having probabilities

$P(E)=\frac{1}{8}, P(F)=\frac{1}{6}$ and $P(G)=\frac{1}{4}$, and let $P(E\cap F\cap G)=\frac{1}{10}$.

For any event H, if H^C denotes its complement, then which of the following statements is(are) **TRUE ?**

[JEE(Advanced) 2021]

(A) $P(E\cap F\cap G^C)\le\frac{1}{40}$

(B) $P(E^C\cap F\cap G)\le\frac{1}{15}$

(C) $P(E\cup F\cup G)\le\frac{13}{24}$

(D) $P(E^C\cap F^C\cap G^C)\le\frac{5}{12}$

13. A number is chosen at random from the set {1, 2, 3, ..., 2000}. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of 500p is ________.

[JEE(Advanced) 2021]

14. Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose α is the number of heads that appear when C_1 is tossed twice, independently, and suppose β is the number of heads that appear when C_2 is tossed twice, independently, Then the probability that the roots of the quadratic polynomial $x^2-\alpha x+\beta$ are real and equal, is **[JEE(Advanced) 2020]**

(A) $\frac{40}{81}$ (B) $\frac{20}{81}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

15. The probability that a missile hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is NOT less than 0.95, is ________.

[JEE(Advanced) 2020]

16. Two fair dice, each with faces numbered 1,2,3,4,5 and 6, are rolled together and the sum of the numbers on the faces is observed. This process is repeated till the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If p is the probability that this perfect square is an odd number, then the value of 14p is ________.

[JEE(Advanced) 2020]

17. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls, Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ? **[JEE(Advanced) 2019]**

(A) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(B) Probability that the chosen ball is green equals $\frac{39}{80}$

(C) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(D) Probability that the selected bag is B_3, given that the chosen balls is green, equals $\frac{5}{13}$

18. Let S be the sample space of all 3×3 matrices with entries from the set $\{0, 1\}$. Let the events E_1 and E_2 be given by

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{sum of entries of A is } 7\}$.

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals ____

[JEE(Advanced) 2019]

19. Let |X| denote the number of elements in set X. Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are indepenent events associated with S, then the number of ordered pairs (A, B) such that $1 \le |B| < |A|$, equals **[JEE(Advanced) 2019]**

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on Paragraph "A". the question given below is one of them)

20. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 and **NONE** of the remaining students gets the seat previously allotted to him/her is - **[JEE(Advanced) 2018]**

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on Paragraph "A", the question given below is one of them*)

21. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is-

[JEE(Advanced) 2018]

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

22. Let X and Y be two events such that $P(X)=\frac{1}{3}$, $P(X\mid Y)=\frac{1}{2}$ and $P(Y\mid X)=\frac{2}{5}$. Then

[JEE(Advanced) 2017]

(A) $P(X'\mid Y)=\frac{1}{2}$ (B) $P(X\cap Y)=\frac{1}{5}$

(C) $P(X\cup Y)=\frac{2}{5}$ (D) $P(Y)=\frac{4}{15}$

23. Three randomly chosen non negative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is **[JEE(Advanced) 2017]**

(A) $\frac{36}{55}$ (B) $\frac{6}{11}$ (C) $\frac{5}{11}$ (D) $\frac{1}{2}$

24. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that **[JEE(Advanced) 2016]**

P(computer turns out to be defective given that is produced in plant T_1)

= 10P(computer turns out to be defective given that it is produced in plant T_2)

where P(E) denotes the probability of an event E. A computer produces in the factory is randomly selected and it does not turn out to be defective. Then the probabality that it is produced in plant T_2 is

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Paragraph For Questions No. 25 and 26

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games

25. $P(X > Y)$ is- **[JEE(Advanced) 2016]**

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{7}{12}$

26. $P(X = Y)$ is- **[JEE(Advanced) 2016]**

(A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{1}{2}$

27. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is **[JEE(Advanced) 2015]**

Paragraph For Questions Nos. 28 and 29

Let n_1 and n_2 be the number of red and black balls respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

28. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

[JEE(Advanced) 2015]

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ (B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ (D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

29. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)

[JEE(Advanced) 2015]

(A) $n_1 = 4$ and $n_2 = 6$ (B) $n_1 = 2$ and $n_2 = 3$

(C) $n_1 = 10$ and $n_2 = 20$ (D) $n_1 = 3$ and $n_2 = 6$

30. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is - **[JEE(Advanced) 2014]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Paragraph For Questions No. 31 and 32

Box 1 contains three cards bearing numbers, 1,2,3 ; box 2 contains five cards bearing numbers 1,2,3,4,5; and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, i = 1,2,3.

31. The probability that $x_1 + x_2 + x_3$ is odd, is **[JEE(Advanced) 2014]**

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

32. The probability that x_1, x_2, x_3 are in an arithmetic progression, is- **[JEE(Advanced) 2014]**

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

SOLUTIONS

1. Ans. (B)

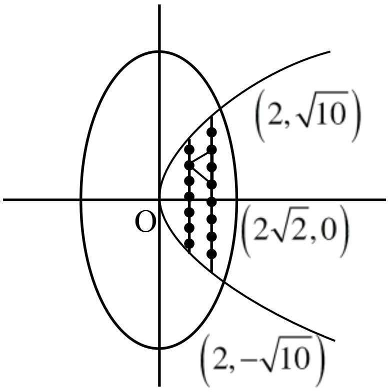
Sol. $\frac{x^2}{8}+\frac{y^2}{20}<1$ & $y^2<5x$

Solving corresponding equations

$\frac{x^2}{8}+\frac{y^2}{20}=1$ & $y^2=5x$

$\Rightarrow \begin{Bmatrix} x=2 \\ y=\pm\sqrt{10} \end{Bmatrix}$

X = {(1,1), (1,0), (1,–1), (1,2), (1,–2), (2, 3), (2,2), (2,1), (2,0), (2,–1), (2,–2), (2,–3)}



Let S be the sample space & E be the event n(S) = $^{12}C_3$

<u>For E</u>

Selecting 3 points in which 2 points are either or x = 1 & x = 2 but distance b/w then is even

Triangles with base 2 :

= 3 × 7 + 5 × 5 = 46

Triangles with base 4 :

= 1 × 7 + 3 × 5 = 22

Triangles with base 6 :

= 1 × 5 = 5

$$P(E)=\frac{46+22+5}{^{12}C_3}=\frac{73}{220}$$

2. Ans. (B)

Sol. $P(H)=\frac{1}{3};P(T)=\frac{2}{3}$

Req. prob = P(HH or HTHH or HTHTHH or ….)

+ P(THH or THTHH or THTHTHH or ….)

$$=\frac{\frac{1}{3}.\frac{1}{3}}{1-\frac{2}{3}.\frac{1}{3}}+\frac{\frac{2}{3}.\frac{1}{3}.\frac{1}{3}}{1-\frac{2}{3}.\frac{1}{3}}=\frac{5}{21}$$

3. Ans. (31)

Sol. No. of elements in X which are multiple of 5

$$\left.\begin{array}{l}
\underbrace{_\,_\,_\,_}_{1,2,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 3}=4 \\
\underbrace{_\,_\,_\,_}_{1,4,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 2}=12 \\
\underbrace{_\,_\,_\,_}_{4,2,2,2}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 3}=4 \\
\underbrace{_\,_\,_\,_}_{2,2,4,4}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 2\lfloor 2}=6 \\
\underbrace{_\,_\,_\,_}_{1,2,4,4}\ \underbrace{0}_{\text{fixed}} \rightarrow \frac{\lfloor 4}{\lfloor 2}=12
\end{array}\right\}\text{Total}=38$$

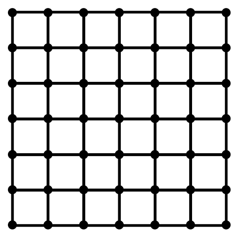
Among these 38 elements, let us calculate when element is not divisible by 20

$$\left.\begin{array}{l}
\underbrace{_\,_\,_}_{2,2,2}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \frac{\lfloor 3}{\lfloor 3}=1 \\
\underbrace{_\,_\,_}_{2,2,4}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \frac{\lfloor 3}{\lfloor 2}=3 \\
\underbrace{_\,_\,_}_{2,4,4}\ \underbrace{1\ \ 0}_{\text{fixed}} \rightarrow \frac{\lfloor 3}{\lfloor 2}=3
\end{array}\right\}\text{Total}=7$$

$\therefore\ p=\frac{38-7}{38}\quad \therefore\ 38p=31$

4. Ans. (24.00)

Sol.



P_i = Probability that randomly selected points has friends

P_0 = 0 (0 friends)

P_1 = 0 (exactly 1 friends)

$P_2=\frac{^4C_1}{^{49}C_1}=\frac{4}{9}$ (exactly 2 friends)

$P_3=\frac{^{20}C_1}{^{49}C_1}=\frac{20}{49}$ (exactly 3 friends)

$P_4=\frac{^{25}C_1}{^{49}C_1}=\frac{25}{49}$ (exactly 4 friends)

x	0	1	2	3	4
$P(x)$	0	0	$\frac{4}{49}$	$\frac{20}{49}$	$\frac{25}{49}$

Mean = E(x) =

$$\sum x_i P_i = 0+0+\frac{8}{49}+\frac{60}{49}+\frac{100}{49}=\frac{168}{49}$$

$$7\left(E\left(x\right)\right)=\frac{168}{49}\times 7=24$$

5. Ans. (0.50)

Sol. Total number of ways of selecting 2 persons $= {}^{49}C_2$

Number of ways in which 2 friends are selected $= 6 \times 7 \times 2 = 84$

$$7P=\frac{84\times 2}{49\times 48}\times 7=\frac{1}{2}$$

6. Ans. (0.80)

Sol. $n(U) = 900$

Let $A \equiv$ Fever, $B \equiv$ Cough

$C \equiv$ Breathing problem

$\therefore n(A) = 190, n(B) = 220, n(C) = 220$

$n(A \cup B) = 330, n(B \cup C) = 350,$

$n(A \cup C) = 340, n(A \cap B \cap C) = 30$

Now $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\Rightarrow 330 = 190 + 220 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 80$

Similarly,

$350 = 220 + 220 - n(B\cap C)$

$\Rightarrow n(B\cap C) = 90$

and $340 = 190 + 220 - n(A\cap C)$

$\Rightarrow n(A\cap C) = 70$

$\therefore n(A\cup B\cup C) = (190 + 220 + 220) - (80 + 90 + 70) + 30$

$= 660 - 240 = 420$

$\Rightarrow$ Number of person without any symptom

$= n(\cup) - n(A \cup B \cup C)$

$= 900 - 420 = 480$

Now, number of person suffering from exactly one symptom

$= (n(A) + n(B) + n(C)) - 2(n(A \cap B) + n(B \cap C) + n(C \cap A)) + 3n(A \cap B \cap C)$

$= (190 + 220 + 220) - 2(80 + 90 + 70) + 3(30)$

$= 630 - 480 + 90 = 240$

$\therefore$ Number of person suffering from atmost one symptom

$= 480 + 240 = 720$

$$\Rightarrow \text{Probability} = \frac{720}{900}=\frac{8}{10}=\frac{4}{5}=0.80$$

7. Ans. (A)

Sol. $P(\text{draw in 1 round}) = \frac{6}{36}=\frac{1}{6}$

$$P(\text{win in 1 round}) = \frac{1}{2}\left(1-\frac{1}{6}\right)=\frac{5}{12}$$

$$P(\text{loss in 1 round}) = \frac{5}{12}$$

$P(X_2 > Y_2) = P(10,0) + P(7,2)$

$$=\frac{5}{12}\times\frac{5}{12}+\frac{5}{12}\times\frac{1}{6}\times 2=\frac{45}{144}=\frac{5}{16}$$

$P(X_2 = Y_2) = P(5,5) + P(4,4)$

$$=\frac{5}{12}\times\frac{5}{12}\times 2+\frac{1}{6}\times\frac{1}{6}=\frac{25+2}{72}=\frac{3}{8}$$

$P(X_3 = Y_3) = P(6,6) + P(7,7)$

$$=\frac{1}{6\times 6\times 6}+\frac{5}{12}\times\frac{1}{6}\times\frac{5}{12}\times 6=\frac{2}{432}+\frac{75}{432}$$

$$=\frac{77}{432}$$

$$P(X_3 > Y_3) = \frac{1}{2}\left(1-\frac{77}{432}\right)=\frac{355}{864}$$

8. Ans. (C)

Sol. Box I 8(R) 3(B) 5(G)

Box II 24(R) 9(B) 15(G)

Box III 1(B) 12(G) 3(y)

Box IV 10(G) 16(o) 6(w)

A (one of the chosen balls is white)

B (at least one of the chosen ball is green)

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$$

$A \cap B \rightarrow (wG)$

$$=\frac{\frac{5}{16}\times\frac{6}{32}}{\frac{5}{16}\times 1+\frac{8}{16}\times\frac{15}{48}+\frac{3}{16}\times\frac{12}{16}}$$

$$=\frac{15}{156}=\frac{5}{52}$$

9. Ans. (A)

Sol. $P = \dfrac{P\left(S_1 \cap (E_1 = E_3)\right)}{P(E_1 = E_3)} = \dfrac{P(B_{1,2})}{P(B)}$

$P(B) = P(B_{1,2}) + P(B_{1,3}) + P(B_{2,3})$

↑ If 1,2 chosen at start; ↑ If 1,3 chosen at start; ↑ If 2,3 chosen at start

$P(B_{1,2}) = \dfrac{1}{3} \times \underbrace{\dfrac{1 \times {}^3C_1}{{}^4C_2}}_{\text{1 is definitely chosen from } F_2} \times \underbrace{\dfrac{1}{{}^5C_2}}_{\text{1,2 chosen from } G_2}$

$P(B_{1,3}) = \dfrac{1}{3} \times \underbrace{\dfrac{1 \times {}^2C_1}{{}^3C_2}}_{\text{1 is definitely chosen from } F_2} \times \underbrace{\dfrac{1}{{}^5C_2}}_{\text{1,2 chosen from } G_2}$

$P(B_{2,3}) = \dfrac{1}{3} \times \left[\underbrace{\dfrac{{}^3C_2 \times 1}{{}^4C_2}}_{\text{If 1 is not chosen from } F_2} \times \dfrac{1}{{}^4C_2} + \underbrace{\dfrac{1 \times {}^3C_1}{{}^4C_2}}_{\text{If 1 is chosen from } F_2} \times \dfrac{1}{{}^5C_2}\right]$

$\dfrac{P(B_{1,2})}{P(B)} = \dfrac{1}{5}$

10. Ans. (76.25)

Sol. p_1 = probability that maximum of chosen numbers is at least 81

p_1 = 1 – probability that maximum of chosen number is at most 80

$p_1 = 1 - \dfrac{80 \times 80 \times 80}{100 \times 100 \times 100} = 1 - \dfrac{64}{125}$

$p_1 = \dfrac{61}{125}$

$\dfrac{625p_1}{4} = \dfrac{625}{4} \times \dfrac{61}{125} = \dfrac{305}{4} = 76.25$

the value of $\dfrac{625p_1}{4}$ is 76.25

11. Ans. (24.50)

Sol. p_2 = probability that minimum of chosen numbers is at most 40

= 1 – probability that minimum of chosen numbers is at least 41

$= 1 - \left(\dfrac{60}{100}\right)^3$

$= 1 - \dfrac{27}{125} = \dfrac{98}{125}$

$\therefore \dfrac{125}{4} p_2 = \dfrac{125}{4} \times \dfrac{98}{125} = 24.50$

12. Ans. (A, B, C)

Sol. $P(E) = \dfrac{1}{8}$; $P(F) = \dfrac{1}{6}$; $P(G)$

$= \dfrac{1}{4}$; $P(E \cap F \cap G) = \dfrac{1}{10}$

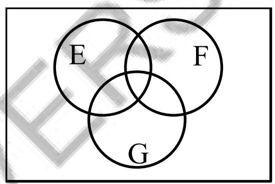
(C) $P(E \cup F \cup G) = P(E) + P(F) + P(G) -$

$P(E \cap F) - P(F \cap G) - P(G \cap E) + P(E \cap F \cap G)$

$= \dfrac{1}{8} + \dfrac{1}{6} + \dfrac{1}{4} - \sum P(E \cap F) + \dfrac{1}{10}$

$= \dfrac{3+4+6}{24} + \dfrac{1}{10} - \sum P(E \cap F)$

$= \dfrac{13}{24} + \dfrac{1}{10} - \sum P(E \cap F)$



$\Rightarrow P(E \cup F \cup G) \le \dfrac{13}{24}$ [(C) is Correct]

(D) $P(E^C \cap F^C \cap G^C)$

$= 1 - P(E \cup F \cup G) \ge 1 - \dfrac{13}{24}$

$\Rightarrow P(E^C \cap F^C \cap G^C) \ge \dfrac{11}{24}$ [(D) is Incorrect]

(A) $P(E) = \dfrac{1}{8} \ge P(E \cap F \cap G^C) + P(E \cap F \cap G)$

$\Rightarrow \dfrac{1}{8} \ge P(E \cap F \cap G^c) + \dfrac{1}{10}$

$\Rightarrow \dfrac{1}{8} - \dfrac{1}{10} \ge P(E \cap F \cap G^c)$

$\Rightarrow \dfrac{1}{40} \ge P(E \cap F \cap G^c)$ [(A) is Correct]

(B) $P(F) = \dfrac{1}{6} \ge P(E^C \cap F \cap G) + P(E \cap F \cap G)$

$\Rightarrow \dfrac{1}{6} - \dfrac{1}{10} \ge P(E^C \cap F \cap G)$

$\Rightarrow \dfrac{4}{60} \ge P(E^C \cap F \cap G)$

$\Rightarrow \dfrac{1}{15} \ge P(E^C \cap F \cap G)$ [(B) is Correct]

13. Ans. (214)

Sol. A = set of numbers divisible by 3

A = {3, 6, 9, 12, ... 1998}

$\therefore$ n (A) = 666

B = set of numbers divisible by 7

B = {7, 14, 21, ...1995}

$\therefore$ n(B) = 285

$A \cap B = \{21, 42, ...1995\}$

$\therefore\ n(A \cup B) = 606 + 285 - 95 = 856$

required probability $= \frac{856}{2000} = P$

so, 500 P $= \frac{856}{2000} \times 500 = 214$

14. Ans. (B)

Sol. $P(H) = \frac{2}{3}$ for C_1

$P(H) = \frac{1}{3}$ for C_2

for C_1

No. of Heads (α)	0	1	2
Probability	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$

for C_2

No. of Heads (β)	0	1	2
Probability	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

for real and equal roots

$\alpha^2 = 4\beta$

$(\alpha, \beta) = (0, 0), (2, 1)$

So, probability $= \frac{1}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{4}{9} = \frac{20}{81}$

15. Ans. (6)

Sol. Let P(r) = probability of r successes

$= {}^nC_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{n-r}$

$1 - (P(0) + P(1) + P(2)) \geq 0.95$

$\Rightarrow \quad 1 - {}^nC_0 \left(\frac{1}{4}\right)^n - {}^nC_1 \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{n-1} - {}^nC_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^{n-2} \geq 0.95$

$\Rightarrow \quad 1 - \left(\frac{1 + 3n + \frac{9n(n-1)}{2}}{4^n}\right) \geq 0.95$

$\Rightarrow \quad 9n^2 - 3n + 2 \leq 0.05 \times 4^n \times 2 \leq \frac{4^n}{10}$

for n = 5 $\quad 212 \leq 102.4$ (Not true)

for n = 6 $\quad 308 \leq 409.6$ true

$\therefore$ least value of n = 6

16. Ans. (8.00)

Sol. Prime : 2, 3, 5, 7, 11

1 2 4 6 2

$P(\text{Prime}) = \frac{15}{36}$

Perfect square = 4, 9

$P(\text{perfect square}) = \frac{7}{36}$

required probability

$$= \frac{\frac{4}{36} + \frac{14}{36} \times \frac{4}{36} + \left(\frac{14}{36}\right)^2 \frac{4}{36} + \ldots}{\frac{7}{36} + \frac{14}{36} \times \frac{7}{36} + \left(\frac{14}{36}\right)^2 \frac{7}{36} + \ldots}$$

$P = \frac{4}{7}$

$14P = 14.\frac{4}{7} = 8$

17. Ans. (B, C)

Sol.

Ball	Balls composition	$P(B_i)$
B_1	5R + 5G	$\frac{3}{10}$
B_2	3R + 5G	$\frac{3}{10}$
B_3	5R + 3G	$\frac{4}{10}$

(A) $P(B_3 \cap G) = P\left(\frac{G_1}{B_3}\right)P(B_3)$

$= \frac{3}{8} \times \frac{4}{10} = \frac{3}{20}$

(B) $P(G) = P\left(\frac{G_1}{B_1}\right)P(B_1) + P\left(\frac{G}{B_2}\right)P(B_2) + P\left(\frac{G}{B_3}\right)P(B_3)$

$= \frac{3}{20} + \frac{3}{16} + \frac{3}{20} = \frac{39}{80}$

(C) $P\left(\frac{G}{B_3}\right) = \frac{3}{8}$

(D) $P\left(\frac{B_3}{G}\right) = \frac{P(G \cap B_3)}{P(G)} = \frac{3/20}{39/80} = \frac{4}{13}$

18. Ans. (0.50)

Sol. $n(E_2) = {}^9C_2$ (as exactly two cyphers are there)

Now, det A = 0, when two cyphers are in the same column or same row

$\Rightarrow$ $n(E_1 \cap E_2) = 6 \times {}^3C_2$.

Hence, $P\left(\frac{E_1}{E_2}\right) = \frac{n(E_1 \cap E_2)}{n(E_2)} = \frac{18}{36} = \frac{1}{2}$

= 0.50

19. Ans. (422.00)

Sol. $P\left(\frac{B}{A}\right) = P(B)$

$\Rightarrow$ $n\frac{(A \cap B)}{n(A)} = \frac{n(B)}{n(s)}$ ....(1)

$\Rightarrow$ n(A) should have 2 or 3 as prime factors

$\Rightarrow$ n(A) can be 2, 3, 4 or 6 as n(A) > 1

n(A)= 2 does not satisfy the constraint (1).

for n(A) = 3. n(B) = 2 and $n(A \cap B) = 1$

$\Rightarrow$ No. of ordered pair = ${}^6C_4 \times \frac{4!}{2!} = 180$

for n(A) = 4. n(B) = 3 and $n(A \cap B) = 2$

$\Rightarrow$ No. of ordered pairs = ${}^6C_5 \times \frac{5!}{2!2!} = 180$

for n(A) = 6. n(B) can be 1, 2, 3, 4, 5.

$\Rightarrow$ No. of ordered pairs = $2^6 - 2 = 62$

Total ordered pair = 180 + 180 + 62 = 422.

20. Ans. (A)

Sol. Required probability = $\frac{4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)}{5!}$

$= \frac{9}{120} = \frac{3}{40}$

21. Ans. (C)

Sol. $n(T_1 \cap T_2 \cap T_3 \cap T_4)$

$= \text{Total} - n\left(\overline{T}_1 \cup \overline{T}_2 \cup \overline{T}_3 \cup \overline{T}_4\right)$

$= 5! - \left({}^4C_1 4!2! - \left({}^3C_1.3!2! + {}^3C_1 3!2!2!\right) + \left({}^2C_1 2!2! + {}^4C_1.2.2!\right) - 2\right)$

= 14

Probability = $\frac{14}{5!} = \frac{7}{60}$

22. Ans. (A, D)

Sol. $P(x) = \frac{1}{3}$; $\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$; $\frac{P(Y \cap X)}{P(X)} = \frac{2}{5}$

from this information, we get

$P(X \cap Y) = \frac{2}{15}$; $P(Y) = \frac{4}{15}$

$\therefore$ $P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$

$P(\overline{X}/Y) = \frac{P(\overline{X} \cap Y)}{P(Y)}$

$= \frac{P(Y) - P(X \cap Y)}{P(Y)}$

$\Rightarrow$ $P(\overline{X}/Y) = 1 - \frac{2/15}{4/15} = \frac{1}{2}$

23. Ans. (B)

Sol. Let z = 2k, where k = 0, 1, 2, 3, 4, 5

$\therefore$ $x + y = 10 - 2k$

Number of non negative integral solutions

$$\sum_{k=0}^{5} {}^{11-2k}C_1 = \sum 11 - 2k = 36$$

Total cases = ${}^{10+3-1}C_{3-1} = 66$

Reqd. prob. = $\frac{36}{66} = \frac{6}{11}$

24. Ans. (C)

Sol. $P(T_1)=\frac{20}{100}$ $\quad P(T_2)=\frac{80}{100}$

Let $P\left(\frac{D}{T_2}\right)=x$

$P\left(\frac{D}{T_1}\right)=10x$

$P(D)=\frac{7}{100}$ (given)

$P(T_1)P\left(\frac{D}{T_1}\right)+P(T_2)P\left(\frac{D}{T_2}\right)=\frac{7}{100}$

$\frac{20}{100}\times 10x+\frac{80}{100}\times x=\frac{7}{100}$

$x=\frac{1}{40}$

$P\left(\frac{D}{T_2}\right)=\frac{1}{40}\Rightarrow P\left(\frac{\bar{D}}{T_2}\right)=\frac{39}{40}$

$P\left(\frac{D}{T_1}\right)=\frac{10}{40}\Rightarrow P\left(\frac{\bar{D}}{T_1}\right)=\frac{30}{40}$

$P\left(\frac{T_2}{\bar{D}}\right)=\frac{\frac{80}{100}\times\frac{39}{40}}{\frac{20}{100}\times\frac{30}{40}+\frac{80}{100}\times\frac{39}{40}}=\frac{78}{93}$

25. Ans. (B)

Sol. $P(X>Y) = P(T_1 \text{ win})\, P(T_1 \text{ win}) + P(T_1 \text{ win})\, P(\text{match draw}) + P(\text{match draw}).P(T_1 \text{ win})$

$=\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{2}=\frac{5}{12}$

26. Ans. (C)

Sol. $P(X = Y) = P(\text{match draw})\, P(\text{match Draw}) + P(T_1 \text{ win})\, P(T_2 \text{ win}) + P(T_2 \text{ win})\, P(T_1 \text{ win})$

$=\frac{1}{6}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{2}=\frac{13}{36}$

27. Ans. (8)

Sol. Let the number of tosses be n

$\therefore$ Probability of getting at least two heads

$=1-\left(\frac{1}{2}\right)^n-{}^nC_1.\left(\frac{1}{2}\right)^{n-1}.\left(\frac{1}{2}\right)$

$\therefore \quad 1-\frac{(n+1)}{2^n}\geq\frac{24}{25} \quad\Rightarrow\quad \frac{n+1}{2^n}\leq\frac{1}{25}$

$\therefore \quad n=8$

28. Ans. (A, B)

Sol. Required probability $=\frac{\left(\frac{n_3}{n_3+n_4}\right)}{\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4}}=\frac{1}{3}$

now check options.

29. Ans. (C, D)

Sol. Required probability =

$\frac{n_1}{(n_1+n_2)}\frac{(n_1-1)}{(n_1+n_2-1)}+\frac{n_2}{(n_1+n_2)}\frac{n_1}{(n_1+n_2-1)}=\frac{1}{3}$

$\Rightarrow \frac{n_1^2+n_1n_2-n_1}{(n_1+n_2)(n_1+n_2-1)}=\frac{1}{3}$

now check options.

30. Ans. (A)

Sol. Total ways of arranging all boys & girls = 5! = 120

unfavourable case will be

I _ _ _ _ g } 2.4! = 48
_ _ _ g g }

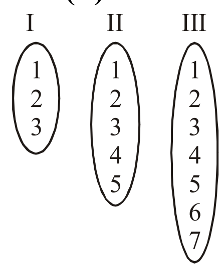
II _ _ g g B } 2!.3! = 12

Favourable ways are 120 – 48 – 12 = 60

$P=\frac{60}{120}=\frac{1}{2}$

31. Ans. (B)

Sol.



x_1 = number on the card drawn from I

x_2 = number on the card drawn from II

x_3 = number on the card drawn from III

$\because \quad x_1+x_2+x_3=\text{odd}$

odd + odd + odd $\Rightarrow \frac{2}{3}.\frac{3}{5}.\frac{4}{7}=\frac{24}{105}$

odd + even + even $\Rightarrow \frac{2}{3}.\frac{2}{5}.\frac{3}{7}=\frac{12}{105}$

even + odd + even $\Rightarrow \frac{1}{3}.\frac{3}{5}.\frac{3}{7}=\frac{9}{105}$

even + even + odd $\Rightarrow \frac{1}{3}.\frac{2}{5}.\frac{4}{7}=\frac{8}{105}$

$\Rightarrow$ Probability that $x_1+x_2+x_3$ is odd is

$\frac{24+12+9+8}{105}=\frac{53}{105}$

32. Ans. (C)

Sol. $2x_2 = x_1 + x_3$

$\Rightarrow$ $x_1 + x_3$ = even for every x_2

even + even $\Rightarrow \left(\frac{1}{3}.\frac{3}{7}\right)\frac{1}{5} = \frac{3}{105}$

odd + odd $\Rightarrow \left(\frac{2}{3}.\frac{4}{7}\right)\frac{1}{5} = \frac{8}{105}$

$\Rightarrow$ probability that x_1, x_2, x_3 are in AP is $\frac{3}{105} + \frac{8}{105} = \frac{11}{105}$