CONTINUITY

1. Let [x] be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ? **[JEE(Advanced) 2017]**

(A) $x = -1$ (B) $x = 0$

(C) $x = 2$ (D) $x = 1$

2. For every pair of continuous function $f, g : [0, 1] \rightarrow \mathbb{R}$ such that

$\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0,1]\}$,

the correct statement(s) is(are) : **[JEE(Advanced) 2014]**

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$

(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$

(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

SOLUTIONS

1. Ans. (A, C, D)

Sol. $f(x) = x\cos(\pi x + [x]\pi)$

$\Rightarrow f(x) = (-1)^{[x]}\, x\cos\pi x.$

Discontinuous at all integers except zero.

2. Ans. (A, D)

Sol. $f, g\ [0,1] \rightarrow R$

we take two cases.

Let f & g attain their common maximum value at p.

$\Rightarrow f(p) = g(p)$ where $p \in [0,1]$

let f & g attain their common maximum value at different points.

$\Rightarrow f(a) = M$ & $g(b) = M$

$\Rightarrow f(a) - g(a) > 0$ & $f(b) - g(b) < 0$

$\Rightarrow f(c) - g(c) = 0$ for some $c \in [0,1]$ as 'f' & 'g' are continuous functions.

$\Rightarrow f(c) - g(c) = 0$ for some $c \in [0,1]$ for all cases. ...(1)

Option (A) $\Rightarrow f^2(c) - g^2(c) + 3\,(f(c) - g(c)) = 0$ which is true from (1)

Option (D) $\Rightarrow f^2(c) - g^2(c) = 0$ which is true from (1)

Now, if we take $f(x) = 1$ & $g(x) = 1\ \forall\ x \in [0,1]$ options (B) & (C) does not hold. Hence option (A) & (D) are correct.