CONTINUITY 1. Let [x] be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(\mathbf{x}) = \mathbf{x}\cos(\pi(\mathbf{x} + [\mathbf{x}]))$ is discontinuous ? [JEE(Advanced) 2017] (A) x = -1(B) x = 0(C) x = 2(D) x = 12. For every pair of continuous function $f,g:[0,1] \rightarrow \mathbb{R}$ such that $\max\{f(\mathbf{x}): \mathbf{x} \in [0, 1]\} = \max\{g(\mathbf{x}): \mathbf{x} \in [0, 1]\},\$ the correct statement(s) is(are) : [JEE(Advanced) 2014] (A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$ (B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$ (C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$ (D) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

JEE Advanced Mathematics 10 Years Topicwise Questions with Solutions

Sol. $f(x) = x \cos(\pi x + [x]\pi)$ $\Rightarrow f(x) = (-1)^{[x]} x \cos\pi x.$ Discontinuous at all integers except zero. 2. Ans. (A, D) Sol. $f, g [0,1] \rightarrow \mathbb{R}$ we take two cases. Let $f \& g$ attain their common maximum val at p. $\Rightarrow f(p) = g(p)$ where $p \in [0,1]$ let $f \& g$ attain their common maximum val at different points. $\Rightarrow f(a) = M \& g(b) = M$ $\Rightarrow f(a) - g(a) > 0 \& f(b) - g(b) < 0$ $\Rightarrow f(a) - g(c) = 0$ for some $c \in [0,1]$ as 'f' & are continuous functions. $\Rightarrow f(c) - g(c) = 0$ for some $c \in [0,1]$ for a cases(1) $Option (A) \Rightarrow f^2(c) - g^2(c) + 3 (f(c) - g(c)) =$ which is true from (1) $Option (D) \Rightarrow f^2(c) - g^2(c) = 0$ which is true from (1) Now, if we take $f(x) = 1 \& g(x) = 1 \forall x \in [0, 1]$		SOLUTIONS
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