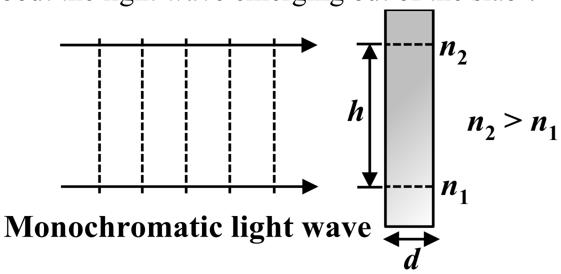
WAVE OPTICS

1. A monochromatic light wave is incident normally on a glass slab of thickness d, as shown in the figure. The refractive index of the slab increases linearly from n_1 to n_2 over the height h. Which of the following statement(s) is (are) true about the light wave emerging out of the slab ? **[JEE(Advanced) 2023]**



(A) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2^2 - n_1^2)d}{2h}\right]$

(B) It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2 - n_1)d}{h}\right]$

(C) It will not deflect.

(D) The deflection angle depends only on $(n_2 - n_1)$ and not on the individual values of n_1 and n_2.

2. The electric field associated with an electromagnetic wave propagating in a dielectric medium is given by $\vec{E} = 30(2\hat{x} + \hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ V m^{-1}. Which of the following option(s) is(are) correct ?

[Given: The speed of light in vacuum, $c = 3 \times 10^8$ ms^{-1}] **[JEE(Advanced) 2023]**

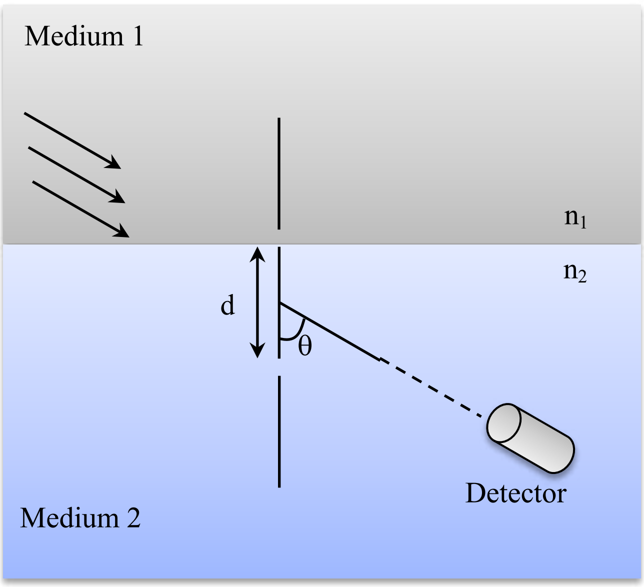
(A) $B_x = -2 \times 10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ Wbm^{-2}.

(B) $B_y = 2 \times 10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]$ Wbm^{-2}

(C) The wave is polarized in the xy-plane with polarization angle 30° with respect to the x-axis.

(D) The refractive index of the medium is 2.

3. A double slit setup is shown in the figure. One of the slits is in medium 2 of refractive index n_2. The other slit is at the interface of this medium with another medium 1 of refractive index $n_1(\neq n_2)$. The line joining the slits is perpendicular to the interface and the distance between the slits is d. The slit widths are much smaller than d. A monochromatic parallel beam of light is incident on the slits from medium 1. A detector is placed in medium 2 at a large distance from the slits, and at an angle θ from the line joining them, so that θ equals the angle of refraction of the beam. Consider two approximately parallel rays from the slits received by the detector.



[JEE(Advanced) 2022]

Which of the following statement(s) is (are) correct?

(A) The phase difference between the two rays is independent of d.

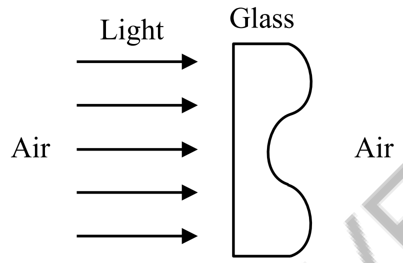
(B) The two rays interfere constructively at the detector.

(C) The phase difference between the two rays depends on n_1 but is independent of n_2.

(D) The phase difference between the two rays vanishes only for certain values of d and the angle of incidence of the beam, with θ being the corresponding angle of refraction.

4. A parallel beam of light strikes a piece of transparent glass having cross section as shown in the figure below. Correct shape of the emergent wavefront will be (figures are schematic and not drawn to scale)-

[JEE(Advanced) 2020]



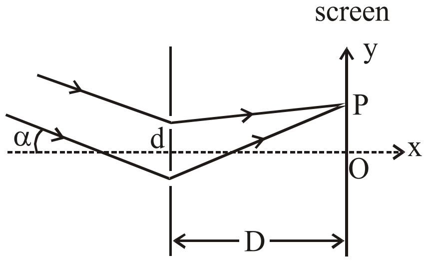
(A) 

(B) 

(C) 

(D) 

5. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1m. A parallel beam of light of wavelength 600nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct? **[JEE(Advanced) 2019]**



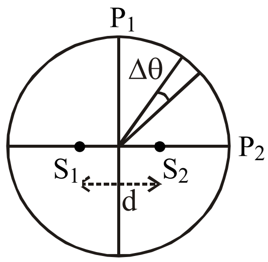
(A) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

(B) Fringe spacing depends on α

(C) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P

(D) For $\alpha = 0$, there will be constructive interference at point P.

6. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct ? **[JEE(Advanced) 2017]**



(A) A dark spot will be formed at the point P_2

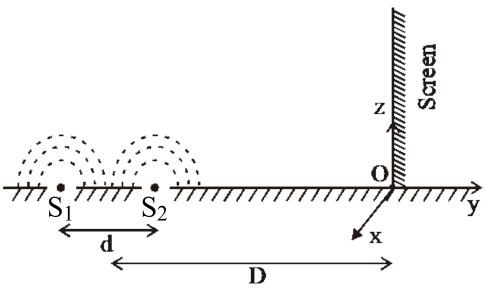
(B) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

(C) At P_2 the order of the fringe will be maximum

(D) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

7. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for z > 0) at a distance D = 3m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources d = 0.6003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is (are) true of the intensity pattern on the screen ?

[JEE(Advanced) 2016]



(A) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

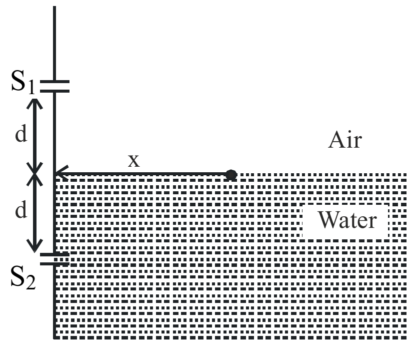
(B) Semi circular bright and dark bands centered at point O

(C) The region very close to the point O will be dark

(D) Straight bright and dark bands parallel to the x-axis

8. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), 2d is the separation between the slits and m is an integer. The value of p is.

[JEE(Advanced) 2015]



9. A light source, which emits two wavelengths λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then :-

[JEE(Advanced) 2014]

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

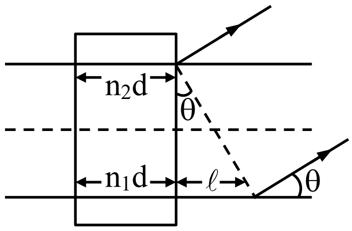
(C) From the central maximum, 3^{rd} maximum of λ_2 overlaps with 5^{th} minimum of λ_1

(D) The angular separation of fringes of λ_1 is greater than λ_2

SOLUTIONS

1. Ans. (B, D)

Sol.

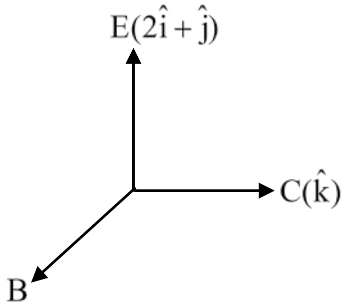


$n_1 d + \ell = n_2 d$

$$\tan\theta = \frac{\ell}{h} = \frac{(n_2 - n_1)d}{h}$$

2. Ans. (A, D)

Sol.



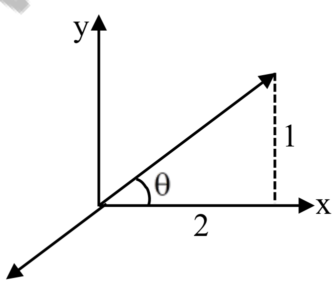
$$C_{medium} = \frac{5\times10^{14}}{10^7/3} = 1.5 \times 10^8 \text{ m/s} \quad (\therefore \mu = 2)$$

$$C_{medium} = \frac{E}{B}$$

$$\Rightarrow B = \frac{E}{C_m} = \frac{30\sqrt{5}}{1.5\times10^8} = 2\sqrt{5}\times10^{-7}$$

$$\vec{B}_{direction} \equiv \hat{k}\times(2\hat{i}+\hat{j}) \equiv \frac{2\hat{j}-\hat{i}}{\sqrt{5}}$$

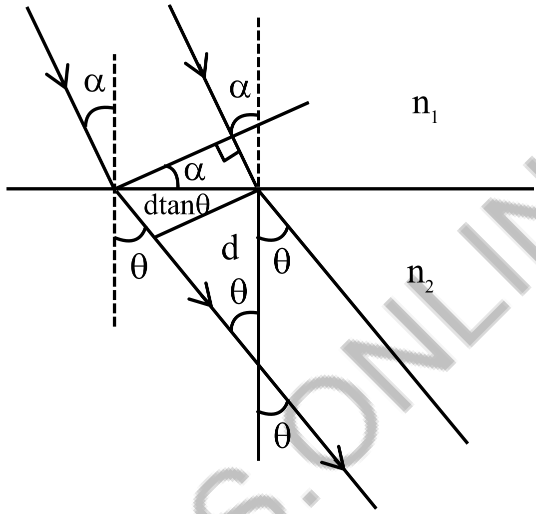
$$\therefore \vec{B} = 2\times10^{-7}(-\hat{i}+2\hat{j})\sin\left[27\left(5\times10^{17}t - \frac{10^7}{3}z\right)\right]$$



$$\tan\theta = \frac{1}{2}$$

3. Ans. (A, B)

Sol.



Optical path difference →

$\Delta x = n_1(d\tan\theta)\sin\alpha - n_2(d\tan\theta)\sin\theta$

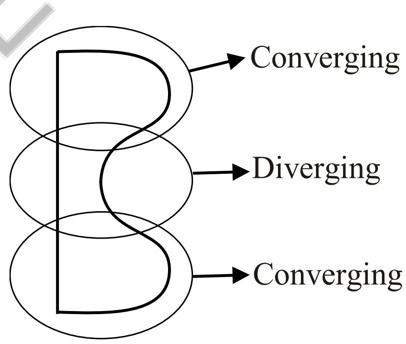
$= (n_1\sin\alpha - n_2\sin\theta)\, d\tan\theta$

$= 0$

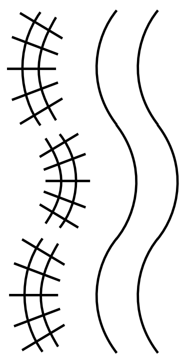
$\Rightarrow \Delta\phi = 0$

4. Ans. (A)

Sol.



(A)



5. Ans. (C)

Sol. (A) $\Delta x = d\sin\alpha = d\alpha$ (as α is very small)

$$\alpha = \frac{.36}{180} = (2\times10^{-3})\text{ rad}$$

$$\frac{\Delta x}{\lambda} = \frac{(3\times10^{-4})(2\times10^{-3})}{6\times10^{-7}} = 1$$

so constructive interference

(B) $\beta = \dfrac{D\lambda}{d}$

(C) $\Delta x_p = d\alpha + \dfrac{dy}{D}$

$= 3 \times 10^{-4}\,(2 \times 10^{-3} + 11 \times 10^{-3})$

$= 39 \times 10^{-7}$

$\dfrac{\Delta x_p}{\lambda} = \dfrac{39\times10^{-7}}{6\times10^{-7}} = 6.5$ so destructive

(D) $\Delta x_p = \dfrac{dy}{D} = (3\times10^{-4})\times11\times10^{-3}$

$= 33 \times 10^{-7}$

$\dfrac{\Delta x_p}{\lambda} = \dfrac{33\times10^{-7}}{6\times10^{-7}} = 5.5 \Rightarrow$ destructive

6. Ans. (C, D)

Sol. At point P_2; $\Delta x = d = 1.8$ mm $= 3000\,\lambda$

hence a (bright fringe) will be formed at P_2

Now, $\Delta x = d\cos\theta = n\lambda$

$\cos\theta = \dfrac{n\lambda}{d}$

$-\sin\theta\,\Delta\theta = (\Delta n)\,\dfrac{\lambda}{d}$

$\Delta\theta = -(\Delta n)\dfrac{\lambda}{d\sin\theta}$

$\Delta\theta$ increases as θ decreases

At P_2, the order of fringe will be maximum.

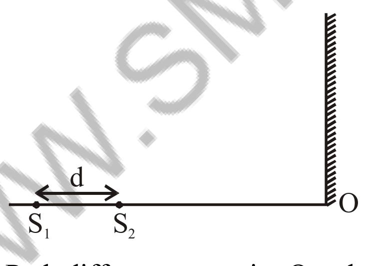
For total no. of bright fringes

$d = n\lambda \;\Rightarrow\; n = 3000$

$\therefore$ total no. of fringes = 3000

7. Ans. (B, C)

Sol.



Path difference at point O = d = .6003 mm

= 600300 nm

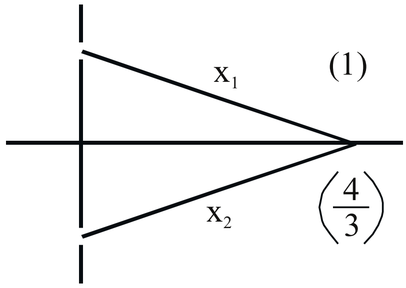
$= \dfrac{2001}{2}(600\text{nm}) = 1000\lambda + \dfrac{\lambda}{2}$

$\Rightarrow$ minima form at point O

Line S_1S_2 and screen are $\perp$ to each other so fringe pattern is circular (semi-circular because only half of screen is available)

8. Ans. (3)

Sol.



$x_1 = \sqrt{x^2 + d^2}$

$x_2 = \dfrac{4}{3}\sqrt{x^2 + d^2}$

$\Delta x = \left(\dfrac{4}{3} - 1\right)\sqrt{x^2 + d^2} = n\lambda$

$\dfrac{1}{3}\sqrt{x^2 + d^2} = n\lambda$

$(x^2 + d^2) = 9n^2\lambda^2$

$\therefore$ (P = 3)

9. Ans. (A, B, C)

Sol. $\beta = \dfrac{\lambda D}{d}$

$\lambda_1 < \lambda_2$

$\beta_2 > \beta_1$

$m_1\beta_1 = m_2\beta_2 = y$

$\Rightarrow m_1 > m_2$

$\left.\begin{aligned} y_1 &= 3\dfrac{\lambda_2 D}{d} \\ y_2 &= 4.5\dfrac{\lambda_1 D}{d}\end{aligned}\right]$ Here $y_1 = y_2$

$\theta = \dfrac{\beta}{D} = \dfrac{\lambda}{d}$

Hence A, B & C are correct choices