HEAT & THERMODYNAMICS

1. One mole of an ideal gas expands adiabatically from an initial state (T_A, V_0) to final state (T_f, $5V_0$). Another mole of the same gas expands isothermally from a different initial state (T_B, V_0) to the same final state (T_f, $5V_0$). The ratio of the specific heats at constant pressure and constant volume of this ideal gas is γ. What is the ratio T_A/T_B? **[JEE(Advanced) 2023]**

(A) $5^{\gamma-1}$ (B) $5^{1-\gamma}$ (C) 5^{γ} (D) $5^{1+\gamma}$

2. A closed container contains a homogeneous mixture of two moles of an ideal monatomic gas ($\gamma = 5/3$) and one mole of an ideal diatomic gas ($\gamma = 7/5$). Here, γ is the ratio of the specific heats at constant pressure and constant volume of an ideal gas. The gas mixture does a work of 66 Joule when heated at constant pressure. The change in its internal energy is ________ Joule.

[JEE(Advanced) 2023]

3. Match the temperature of a black body given in List-I with an appropriate statement in List-II, and choose the correct option. **[JEE(Advanced) 2023]**

[Given: Wien's constant as 2.9×10^{-3} m-K and $\frac{hc}{e} = 1.24 \times 10^{-6}$ V-m]

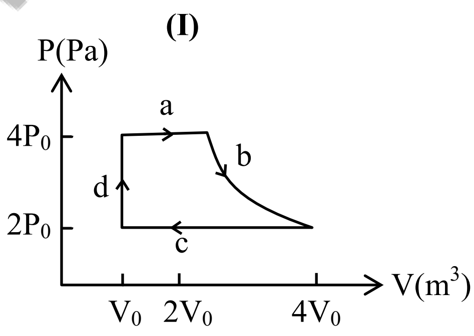
List-I		List-II	
(P)	2000 K	(1)	The radiation at peak wavelength can lead to emission of photoelectrons from a metal of work function 4 eV
(Q)	3000 K	(2)	The radiation at peak wavelength is visible to human eye.
(R)	5000 K	(3)	The radiation at peak emission wavelength will result in the widest central maximum of a single slit diffraction.
(S)	10000 K	(4)	The power emitted per unit area is 1/16 of that emitted by a blackbody at temperature 6000 K.
		(5)	The radiation at peak emission wavelength can be used to image human bones.

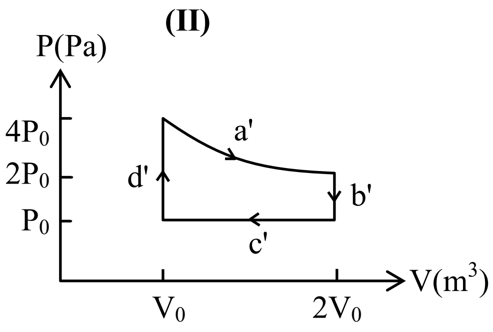
(A) $P \to 3, Q \to 5, R \to 2, S \to 3$ (B) $P \to 3, Q \to 2, R \to 4, S \to 1$

(C) $P \to 3, Q \to 4, R \to 2, S \to 1$ (D) $P \to 1, Q \to 2, R \to 5, S \to 3$

4. One mole of an ideal gas undergoes two different cyclic processes I and II, as shown in the P-V diagrams below. In cycle I, processes a, b, c and d are isobaric, isothermal, isobaric and isochoric, respectively. In cycle II, processes a′, b′, c′ and d′ are isothermal, isochoric, isobaric and isochoric, respectively. The total work done during cycle I is W_I and that during cycle II is W_{II}. The ratio W_I/W_{II} is _______.

[JEE(Advanced) 2023]





5. List-I describes thermodynamic processes in four different systems. List-II gives the magnitudes (either exactly or as a close approximation) of possible changes in the internal energy of the system due to the process. **[JEE(Advanced) 2022]**

List-I		List-II	
(I)	10^{-3} kg of water at 100°C is converted to steam at the same temperature, at a pressure of 10^5 Pa. The volume of the system changes from 10^{-6} m^3 to 10^{-3} m^3 in the process. Latent heat of water = 2250 kJ/kg.	(P)	2 kJ
(II)	0.2 moles of a rigid diatomic ideal gas with volume V at temperature 500 K undergoes an isobaric expansion to volume 3 V. Assume R = 8.0 J $mol^{-1}K^{-1}$.	(Q)	7 kJ
(III)	On mole of a monatomic ideal gas is compressed adiabatically from volume $V = \frac{1}{3}m^3$ and pressure 2 kPa to volume $\frac{v}{8}$.	(R)	4 kJ
(IV)	Three moles of a diatomic ideal gas whose molecules can vibrate, is given 9 kJ of heat and undergoes isobaric expansion.	(S)	5 kJ
		(T)	3 kJ

Which one of the following options is correct ?

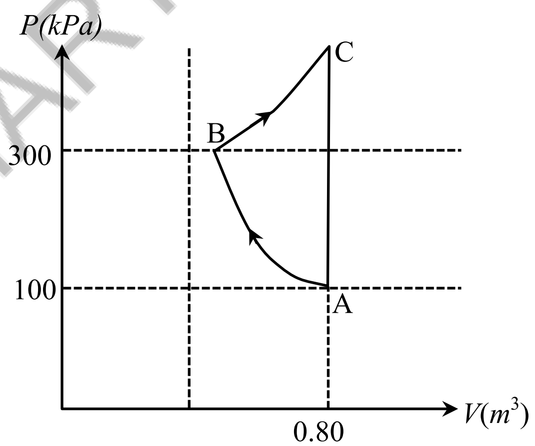
(A) I $\rightarrow$ T, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ Q

(B) I $\rightarrow$ S, II $\rightarrow$ P, III $\rightarrow$ T, IV $\rightarrow$ P

(C) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ Q

(D) I $\rightarrow$ Q, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ T

6. In the given P-V diagram, a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is first compressed adiabatically from state A to state B. Then it expands isothermally from state B to state C. [Given: $\left(\frac{1}{3}\right)^{0.6} \simeq 0.5$, $\ln 2 \simeq 0.7$].



Which of the following statement(s) is(are) correct? **[JEE(Advanced) 2022]**

(A) The magnitude of the total work done in the process A $\rightarrow$ B $\rightarrow$ C is 144 kJ.

(B) The magnitude of the work done in the process B $\rightarrow$ C is 84 kJ.

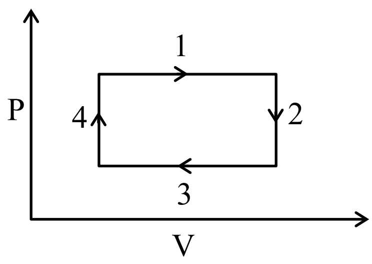
(C) The magnitude of the work done in the process A $\rightarrow$ B is 60 kJ.

(D) The magnitude of the work done in the process C $\rightarrow$ A is zero.

7. A small object is placed at the center of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time t = 0, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2/t_1) is_______.

[JEE(Advanced) 2021]

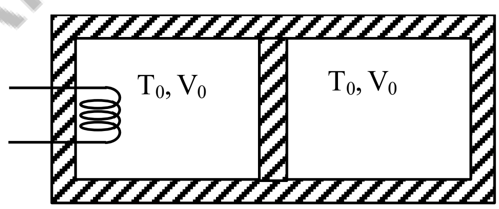
8. An ideal gas undergoes a four step cycle as shown in the P – V diagram below. During this cycle, heat is absorbed by the gas in **[JEE(Advanced) 2021]**



(A) steps 1 and 2 (B) steps 1 and 3 (C) steps 1 and 4 (D) steps 2 and 4

Paragraph for Question No. 9 and 10

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, $C_v = 2R$. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0. The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to $V_0/2$. Consequently, the gas temperatures on the left and the right sides become T_L and T_R, respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition.



9. The value of $\frac{T_R}{T_0}$ is **[JEE(Advanced) 2021]**

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 3

10. The value of $\frac{Q}{RT_0}$ is **[JEE(Advanced) 2021]**

(A) $4\left(2\sqrt{2}+1\right)$ (B) $4\left(2\sqrt{2}-1\right)$ (C) $\left(5\sqrt{2}+1\right)$ (D) $\left(5\sqrt{2}-1\right)$

11. The filament of a light bulb has surface area 64 mm^2. The filament can be considered as a black body at temperature 2500 K emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of 100 m. Assume the pupil of the eyes of the observer to be circular with radius 3 mm. Then **[JEE(Advanced) 2020]**

(Take Stefan-Boltzmann constant = 5.67×10^{-8} $Wm^{-2} K^{-4}$, Wien's displacement constant = 2.90×10^{-3} m-K, Planck's constant = 6.63×10^{-34} Js, speed of light in vacuum = 3.00×10^8 ms^{-1})

(A) power radiated by the filament is in the range 642 W to 645 W

(B) radiated power entering into one eye of the observer is in the range 3.15×10^{-8} W to 3.25×10^{-8} W

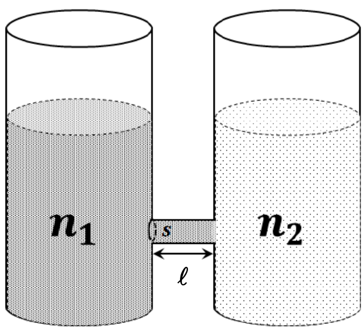
(C) the wavelength corresponding to the maximum intensity of light is 1160 nm

(D) taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range 2.75×10^{11} to 2.85×10^{11}

12. A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700Wm^{-2} and it is absorbed by the water over an effective area of 0.05 m^2. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in ºC) in the temperature of water and the surroundings after a long time will be ________. (Ignore effect of the container, and take constant for Newton's law of cooling = 0.001 s^{-1}, Heat capacity of water = 4200 J kg^{-1} K^{-1})

[JEE(Advanced) 2020]

13. As shown schematically in the figure, two vessels contain water solutions (at temperature T) of potassium permanganate ($KMnO_4$) of different concentrations n_1 and n_2 ($n_1 > n_2$) molecules per unit volume with $\Delta n = (n_1 - n_2) \ll n_1$. When they are connected by a tube of small length ℓ and cross-sectional area S, $KMnO_4$ starts to diffuse from the left to the right vessel through the tube. Consider the collection of molecules to behave as dilute ideal gases and the difference in their partial pressure in the two vessels causing the diffusion. The speed v of the molecules is limited by the viscous force $-\beta v$ on each molecule, where β is a constant. Neglecting all terms of the order $(\Delta n)^2$, which of the following is/are correct? (k_B is the Boltzmann constant)- **[JEE(Advanced) 2020]**



(A) the force causing the molecules to move across the tube is $\Delta n k_B T S$

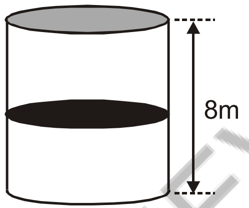
(B) force balance implies $n_1 \beta v \ell = \Delta n k_B T$

(C) total number of molecules going across the tube per sec is $\left(\frac{\Delta n}{\ell}\right)\left(\frac{k_B T}{\beta}\right) S$

(D) rate of molecules getting transferred through the tube does not change with time

14. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia}, respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f \ln 2$, then f is ________. **[JEE(Advanced) 2020]**

15. A thermally isolated cylindrical closed vessel of height 8 m is kept vertically. It is divided into two equal parts by a diathermic (perfect thermal conductor) frictionless partition of mass 8.3 kg. Thus the partition is held initially at a distance of 4 m from the top, as shown in the schematic figure below. Each of the two parts of the vessel contains 0.1 mole of an ideal gas at temperature 300 K. The partition is now released and moves without any gas leaking from one part of the vessel to the other. When equilibrium is reached, the distance of the partition from the top (in m) will be ______. (take the acceleration due to gravity = 10 ms^{-2} and the universal gas constant = 8.3 J $mol^{-1}K^{-1}$). **[JEE(Advanced) 2020]**



16. A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes $(R-a)$. For $a \ll R$ the magnitude of the work done in the process is given by $(4\pi p_0 R a^2)X$, where X is a constant and $\gamma = C_p/C_V = 41/30$. The value of X is________. **[JEE(Advanced) 2020]**

17. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as :

$$T(t) = T_0 (1 + \beta t^{1/4})$$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is : **[JEE(Advanced) 2019]**

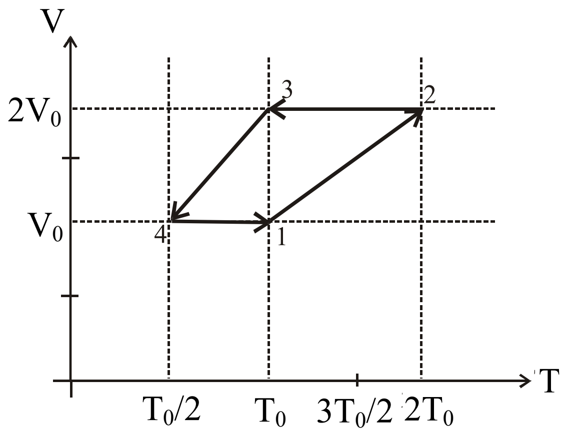
(A) $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$ (B) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

(C) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (D) $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

18. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be ______°C.

[Neglect the heat exchange with surrounding] **[JEE(Advanced) 2019]**

19. One mole of a monoatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are:
[R is the gas constant] **[JEE(Advanced) 2019]**



(A) Work done in this thermodynamic cycle ($1\to2\to3\to4\to1$) is $|W| = \frac{1}{2}RT_0$

(B) The ratio of heat transfer during processes $1\to2$ and $2\to3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(C) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(D) The ratio of heat transfer during processes $1\to2$ and $3\to4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

20. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0 and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are, **[JEE(Advanced) 2019]**
(Give $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)

(A) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

(B) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(C) The work |W| done during the process is $13RT_0$

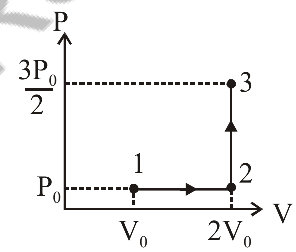
(D) Adiabatic constant of the gas mixture is 1.6

21. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**
In a thermodynamics process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is **temperature** of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities. **[JEE(Advanced) 2019]**

List-I	List-II
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3 + \ln 2)$
	(U) $\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(A) I $\to$ Q, II $\to$ R, III $\to$ P, IV $\to$ U

(B) I $\to$ S, II $\to$ R, III $\to$ Q, IV $\to$ T

(C) I $\to$ Q, II $\to$ R, III $\to$ S, IV $\to$ U

(D) I $\to$ Q, II $\to$ S, III $\to$ R, IV $\to$ U

22. Answer the following by appropriately matching the lists based on the information given in the paragraph.

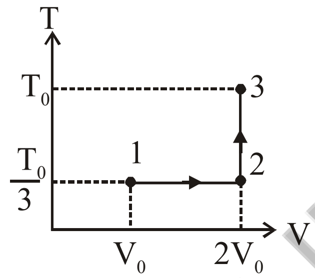
In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas

$X = \frac{3}{2} R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities. **[JEE(Advanced) 2019]**

List-I	List-II
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3 + \ln 2)$
	(U) $\frac{5}{6}RT_0$

If the process on one mole of monatomic ideal gas is an shown is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is



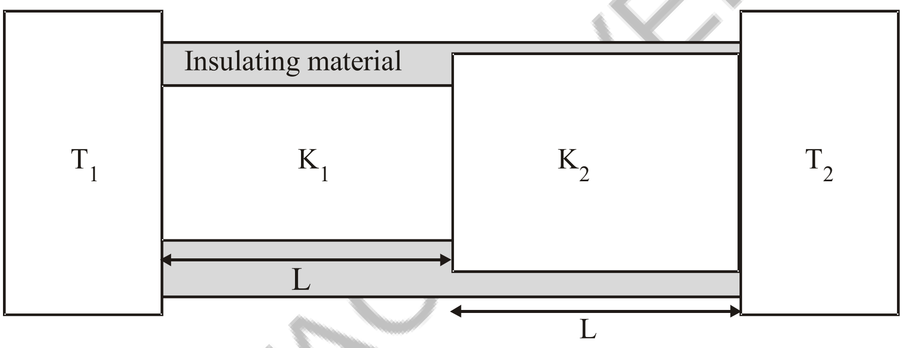
(A) I → S, II → T, III → Q, IV → U

(B) I → P, II → R, III → T, IV → S

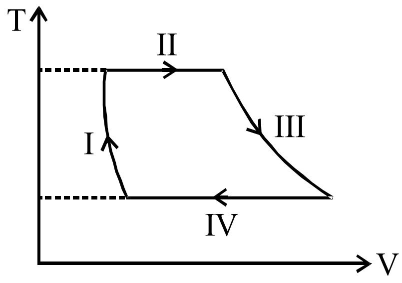
(C) I → P, II → T, III → Q, IV → T

(D) I → P, II → R, III → T, IV → P

23. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2 = _________. **[JEE(Advanced) 2018]**



24. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statements below is (are) true ? **[JEE(Advanced) 2018]**



(A) Process I is an isochoric process

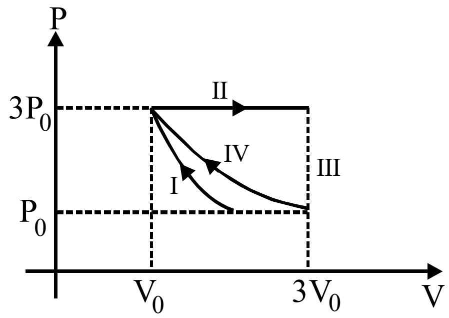
(B) In process II, gas absorbs heat

(C) In process IV, gas releases heat

(D) Processes I and II are not isobaric

25. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J mol^{-1} K^{-1}, the decrease in its internal energy, in Joule, is_______. **[JEE(Advanced) 2018]**

26. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II. **[JEE(Advanced) 2018]**



List–I		List–II
P.	In process I	1. Work done by the gas is zero
Q.	In process II	2. Temperature of the gas remains unchanged
R.	In process III	3. No heat is exchanged between the gas and its surroundings
S.	In process IV	4. Work done by the gas is 6 P_0V_0

(A) P → 4 ; Q → 3 ; R → 1 ; S → 2

(B) P → 1 ; Q → 3 ; R → 2 ; S → 4

(C) P → 3 ; Q → 4 ; R → 1 ; S → 2

(D) P → 3 ; Q → 4 ; R → 2 ; S → 1

27. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of σT_0^4 = 460 Wm^{-2} (where σ is the Stefan-Boltzmann constant). Which of the following options is/are **correct** ? **[JEE(Advanced) 2017]**

(A) The amount of energy radiated by the body in 1 second is close to 60 Joules

(B) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(C) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(D) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

28. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true ? **[JEE(Advanced) 2017]**

(A) The resistive force experienced by the plate is proportional to v

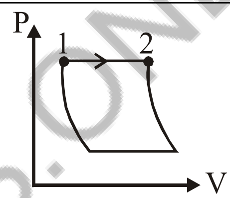
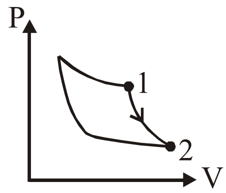
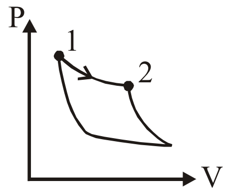
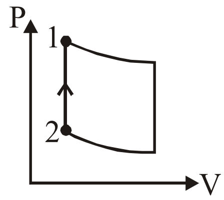
(B) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

(C) The plate will continue to move with constant non-zero acceleration, at all times

(D) At a later time the external force F balances the resistive force.

Answer Q.29, Q.30 and Q.31 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamics process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamics processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

Column-1	Column-2	Column-3
(I) $W_{1\to2} = \frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to2} = -nRT \ln \frac{V_2}{V_1}$	(iv) Adiabatic	(S) 

29. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$? **[JEE(Advanced) 2017]**

(A) (II) (iv) (R) (B) (II) (iii) (P) (C) (II) (iii) (S) (D) (III) (iii) (P)

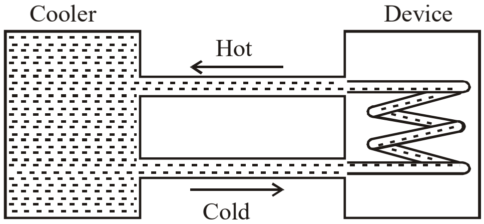
30. Which one of the following options is the correct combination ? **[JEE(Advanced) 2017]**

(A) (III) (ii) (S) (B) (II) (iv) (R) (C) (II) (iv) (P) (D) (IV) (ii) (S)

31. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas ? **[JEE(Advanced) 2017]**

(A) (III) (iv) (R) (B) (I) (ii) (Q) (C) (IV) (ii) (R) (D) (I) (iv) (Q)

32. A water cooler of storage capacity 120 litres can cool water at constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is : **[JEE(Advanced) 2016]**



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

33. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C ? **[JEE(Advanced) 2016]**

34. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K^{-1}, the change in length of the wire PQ is **[JEE(Advanced) 2016]**

(A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

35. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = \left(\frac{1}{32}\right)\times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumes V_f. The amount of heat supplied to the system in the two step process is approximately **[JEE(Advanced) 2016]**

(A) 112 J (B) 294 J (C) 588 J (D) 813 J

36. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is. **[JEE(Advanced) 2015]**

37. A container of fixed volume has a mixutre of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are) :- **[JEE(Advanced) 2015]**

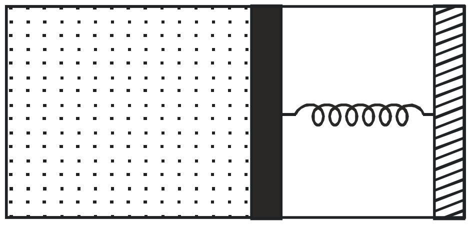
(A) The average energy per mole of the gas mixture is 2RT.

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

38. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are): **[JEE(Advanced) 2015]**



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

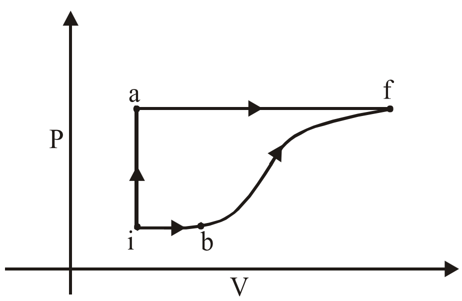
(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

39. Parallel rays of light of intensity $I = 912\ Wm^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to :- **[JEE(Advanced) 2014]**

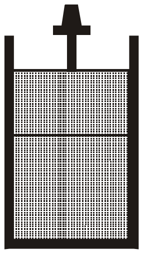
(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

40. A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio Q_{bf}/Q_{ib} is. **[JEE(Advanced) 2014]**



Paragraph for Questions No. 41 & 42

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$.



41. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be **[JEE(Advanced) 2014]**

(A) 550 K (B) 525 K (C) 513 K (D) 490 K

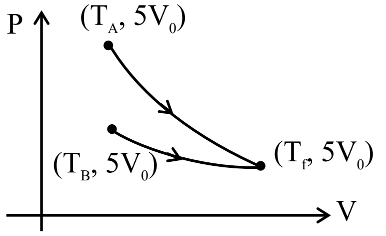
42. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be **[JEE(Advanced) 2014]**

(A) 250 R (B) 200 R (C) 100 R (D) –100 R

SOLUTIONS

1. Ans. (A)

Sol.



$T_A V_0^{\gamma-1} = T_f (5V_0)^{\gamma-1}$

$\frac{T_A}{T_f} = 5^{\gamma-1} = \frac{T_A}{T_B}$

2. Ans. (121)

Sol. At constant pressure

$W = nR\Delta T = 66$

$\Delta U = n(C_V)_{mix}\Delta T$

$(C_V)_{mix} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$(C_V)_{mix} = \frac{2\times\frac{3}{2}R + 1\times\frac{5}{2}R}{3}$

$(C_V)_{mix} = \frac{11}{6}R$

$\Delta U = \frac{11}{6}(nR\Delta T)$

$\Delta U = \frac{11}{6}\times 66 = 121J$

3. Ans. (C)

Sol. $\Rightarrow$ For option (P) temperature is minimum hence λm will be maximum $\beta = \frac{\lambda D}{d}$

$\Rightarrow \beta$ will also be maximum

$\Rightarrow$ For option (Q) T = 3000

$\lambda m = \frac{b}{T} = \frac{2.9\times10^{-3}}{30000}$

$\lambda m = \frac{2.9}{3}\times10^{-6}$

$= 0.96 \times 10^{-6} = 966.6$ nm

$P_{3000} = 6A\,(3000)^4$

$P_{6000} = 6A\,(6000)^4$

$\frac{P_{3000}}{P_{6000}} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$P_{3000} = \frac{1}{16}P_{6000}$

Q – 4

$\Rightarrow$ For (R) T = 5000 K

$\lambda m = \frac{2.9\times10^{-3}}{5\times10^{3}} = 0.58\times10^{-6}$

= 580 nm

Visible to human eyes R – 2

$\Rightarrow$ For (S) T = 10,000 $\rightarrow$ maximum

Hence (3) is wrong as it has minimum (λm)

4. Ans. (2)

Sol. $\frac{W_I}{W_{II}} = \frac{4P_0V_0 + 8P_0V_0\ell n2 - 6P_0V_0 - 0}{4P_0V_0\ell n2 - 0 - P_0V_0 + 0}$

$= \frac{8\ell n2 - 2}{4\ell n2 - 1} = 2$

5. Ans. (C)

Sol. (I) $\Delta U = \Delta Q - \Delta W$

$= \left\{(10^{-3}\times2250) - \frac{10^5(10^{-3}-10^{-6})}{10^3}\right\}$ kJ

= (2.25 – 0.0999) kJ

= (2.1501) kJ

(II) $\Delta U = nC_V\Delta T$

$= \frac{5}{2}nR\Delta T$

$= \frac{5}{2}\cdot(0.2)(8)(1500-500)$ J

= 4 kJ

(III) $P_1V_2^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow 2\left(\frac{1}{3}\right)^{5/3} = P_2\left(\frac{1}{24}\right)^{5/3}$

$\Rightarrow P_2 = 64$ kPa

$\Delta U = nC_V\Delta T = \frac{3}{2}\cdot(P_2V_2 - P_1V_1)$

$= \frac{3}{2}\left(64\times\frac{1}{24} - 2\times\frac{1}{3}\right)$kJ

= 3 kJ

(IV) $\Delta U = nC_V\Delta T$

$= n\cdot\frac{7}{2}R\Delta T$

$= \frac{7}{9}\Delta Q$ = 7 kJ

Ans. (C); I-P, II-R, III-T, IV-Q

6. Ans. (Dropped)

Sol. For adiabatic process (A $\rightarrow$ B)

$$P_A V_A^{\gamma} = P_B V_B^{\gamma}$$

$$10^5 \times (0.8)^{\frac{5}{3}} = 3\times 10^5 (V_B)^{\frac{5}{3}}$$

$$\Rightarrow V_B = 0.8\times\left(\frac{1}{3}\right)^{0.6} = 0.4$$

Work done in process A $\rightarrow$ B

$$W_{AB} = \frac{P_A V_A - P_B V_B}{\gamma - 1}$$

$$\Rightarrow W_{AB} = \frac{10^5\times 0.8 - 3\times 10^5 \times 0.4}{\frac{5}{3}-1}$$

$\Rightarrow W_{AB} = -60$ kJ $= \Rightarrow |W_{AB}| = 60$ kJ

Work done in process B $\rightarrow$ C (Isothermal process)

$$W_{BC} = nRT\ell n\frac{V_C}{V_B} = P_B V_B \ell n\frac{V_C}{V_B}$$

$$\Rightarrow W_{BC} = 3\times 10^5 \times 0.4 \ell n\frac{0.8}{0.4}$$

$\Rightarrow W_{BC} = 84$ kJ

Work done in process C $\rightarrow$ A

$W_{CA} = P\Delta V = 0 \quad (\because \Delta V = 0)$

So total work done in the process A $\rightarrow$ B $\rightarrow$ C

$W_{ABC} = W_{AB} + W_{BC} + W_{CA} = -60 + 84 + 0$

$W_{ABC} = 24$ kJ

So correct options are (B, C, D)

7. Ans. (9)

Sol. $\sigma A T^4 = -ms\frac{dT}{dt}$

$$\int_{200}^{100}\frac{dT}{T^4} = \int_0^{t_1} k\,dt$$

$$\left.\frac{1}{3T^3}\right|_{200}^{100} = kt_1$$

$$\frac{1}{3}\left(\frac{1}{100^3} - \frac{1}{200^3}\right) = kt_1$$

$$\left.\frac{1}{3T^3}\right|_{200}^{50} = kt_2$$

$$\frac{1}{3}\left(\frac{1}{50^3} - \frac{1}{200^3}\right) = kt_2$$

$$\frac{t_2}{t_1} = \left(\frac{200^3 - 50^3}{200^3 - 100^3}\right)\frac{100^3}{50^3} = 9$$

8. Ans. (C)

Sol. Process-1

P = constant, Volume increases and temperature also increases

$\Rightarrow$ W = positive , ΔU = positive

$\Rightarrow$ Heat is positive and supplied to gas

Process-2

V = constant, Pressure decrease

$\Rightarrow$ Temperature decreases

$W = \int p\,dV = 0$

ΔT is negative and $\Delta U = \frac{f}{2} nR\Delta T$

$\Rightarrow \Delta U$ in negative

$\Delta Q = \Delta U + W$

$\therefore \Delta Q \rightarrow$ Heat is negative and rejected by gas

Process-3

P = constant, Volume decreases

$\Rightarrow$ Temperature also decreases

$W = P\Delta V$ = negative

$\Delta U = \frac{f}{2} nR\Delta T$ = negative

$\Delta Q = W + \Delta U$ = negative

Heat is negative and rejected by gas.

Process-4

V = constant, Pressure increases

$W = \int p\,dV = 0$

PV = nRT $\Rightarrow$ Temperature increase

$\Rightarrow \Delta U = \frac{f}{2} nR\Delta T$ is positive

$\Delta Q = \Delta U + W$

= positive

Ans. (C) step 1 and step 4

9. Ans. (A)

10. Ans. (B)

Sol. Finally $V_L = \frac{3V_0}{2}, V_R = \frac{V_0}{2}$

$$C_V = \frac{R}{\gamma - 1} = 2R \Rightarrow \gamma - 1 = \frac{1}{2}$$

$$\gamma = \frac{3}{2}$$

$$T_0 V_0^{\gamma-1} = T_R\left(\frac{V_0}{2}\right)^{\gamma-1}$$

$$\frac{T_R}{T_0} = \sqrt{2}$$

$$\rho\left(\frac{V_0}{2}\right)^{\gamma} = P_0 V_0^{\gamma} \Rightarrow P = P_0 \times 2^{\frac{3}{2}}$$

$$\frac{PV}{T_L} = \frac{P_0V_0}{T_0} \Rightarrow T_L = 2^{\frac{3}{2}} \times \frac{3}{2}T_0 = 3\sqrt{2}T_0$$

$$Q = nC_V\Delta T_1 + nC_V\Delta T_2$$

$$= 1\times 2R\times\left(3\sqrt{2}-1\right)T_0 + 1\times 2R\times\left(\sqrt{2}-1\right)T_0$$

$$\frac{Q}{RT_0} = 2\left(3\sqrt{2}-1\right) + 2\left(\sqrt{2}-1\right) = 8\sqrt{2} - 4$$

11. Ans. (B, C, D)

Sol. $A = 64\ mm^2$, $T = 2500$ K (A = surface area of filament, T = temperature of filament, d is distance of bulb from observer, R_e = radius of pupil of eye)

Point source $d = 100$ m

$R_e = 3$ mm

(A) $P = \sigma A e T^4$

$= 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times 1 \times (2500)^4$

($e = 1$ black body)

$= 141.75$ w

Option (A) is wrong

(B) Power reaching to the eye

$$= \frac{P}{4\pi d^2}\times\left(\pi R_e^2\right)$$

$$= \frac{141.75}{4\pi\times(100)^2}\times\pi\times(3\times10^{-3})^2$$

$= 3.189375 \times 10^{-8}$ W

Option (B) is correct

(C) $\lambda_m T = b$

$\lambda_m \times 2500 = 2.9 \times 10^{-3}$

$\Rightarrow \lambda_m = 1.16 \times 10^{-6}$

$= 1160$ nm

Option (C) is correct

(D) Power received by one eye of observer =

$$\left(\frac{hc}{\lambda}\right)\times \dot{N}$$

$\dot{N}$ = Number of photons entering into eye per second

$\Rightarrow 3.189375 \times 10^{-8}$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{1740\times10^{-9}}\times \dot{N}$$

$\Rightarrow \dot{N} = 2.79 \times 10^{11}$

Option (D) is correct

12. Ans. (8.32 to 8.34)

Sol. $$\frac{dQ}{dt} = \sigma eA\left(T^4 - T_0^4\right) \quad \text{....(i)}$$

$$\frac{dQ}{Adt} = e\sigma\left(T_0 + \Delta T\right)^4 - T_0^4)$$

$$= \sigma T_0^4\left[\left(1+\frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$= e\sigma T_0^4\left[\left(1+4\frac{\Delta T}{T_0}\right) - 1\right]$$

$$\frac{dQ}{Adt} = \sigma e T_0^3 \cdot 4\Delta T \quad \text{....(ii)}$$

Now from equ. (i)

$$ms\frac{dT}{dt} = \sigma eT(T^4 - T_0^4)$$

$$\frac{dT}{dt} = \frac{\sigma eA}{ms}[(T_0+\Delta T)^4 - T_0^4]$$

$$= \frac{\sigma eA}{ms}T_0^4\times\left[\left(1+\frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$\frac{dT}{dt} = \frac{\sigma eA}{ms}T_0^4\cdot 4\Delta T$$

$$\frac{dT}{dt} = e\Delta T\ ;\ \left(K = \frac{4\sigma eAT_0^3}{ms}\right)$$

$$\Rightarrow 4\sigma eAT_0^3 = \frac{K}{A}(ms)$$

from equ. (i)

$$\frac{dQ}{Adt} = e\sigma T_0^3 \cdot 4\Delta T$$

700 = (K/A) (ms) ΔT

$$\therefore \quad \Delta T = \frac{700 \times 5 \times 10^{-2}}{10^{-3} \times 4200} = \frac{50}{6} = \frac{25}{3}$$

$\Delta T = 8.33$

13. Ans. (A, B, C)

Sol. $n_1 >> (n_1 - n_2) = \Delta n$

$$p_1 = \frac{n_1 RT}{N_A} \qquad p_2 = \frac{n_2 RT}{N_A}$$

$F = (n_1 - n_2)k_B TS = \Delta n k_B TS$ (A)

$$V = \frac{\Delta n\, k_B TS}{\beta}$$

Force balance $\Rightarrow$ Pressure × Area = Total number of molecules × βv

$$\Delta n\, k_B TS = \ell n_1 S \beta v$$

$\Rightarrow \quad n_1 \beta v \ell = \Delta n\, k_B T$ (B)

Total number of molecules/sec = $\dfrac{(n_1 v\, dt) S}{dt}$

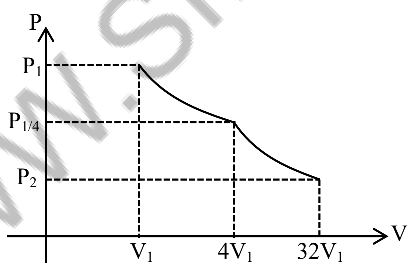
$$= n_1 v S = \frac{\Delta n\, k_B T v S}{\beta v \ell}$$

$$= \left(\frac{\Delta n}{\ell}\right)\left(\frac{k_B T}{\beta}\right) S \quad \text{(C)}$$

As Δn will decrease with time therefore rate of molecules getting transfer decreases with time.

14. Ans. (1.77 to 1.79)

Sol.



$$\frac{P_1}{4}(4V_1)^{5/3} = P_2(32v_1)^{5/3}$$

$$P_2 = \frac{P_1}{4}\left(\frac{1}{8}\right)^{5/3} = \frac{P_1}{128}$$

$$W_{adi} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1V_1 - \frac{P_1}{128}(32V_1)}{\frac{5}{3} - 1}$$

$$= \frac{P_1V_1(3/4)}{2/3} = \frac{9}{8}P_1V_1$$

$$W_{iso} = P_1V_1 \ \ln\left(\frac{4V_1}{V_1}\right) = 2P_1V_1 \ln 2$$

$$\frac{W_{iso}}{W_{adio}} = \frac{2P_1V_1 \ln 2}{\frac{9}{8}P_1V_1} = \frac{16}{9}\ln 2 = f \ln 2$$

$$f = \frac{16}{9} = 1.7778 \approx 1.78$$

15. Ans. (BONUS)

16. Ans. (2.05)

Sol. $W = (\Delta P)_{avg} \times 4\pi R^2 a$

$$\simeq \left|\frac{dP}{2}.4\pi R^2 a\right|$$

{for small change $(\Delta P)_{avg}$ <P> arithmetic mean}

$$= PV^\gamma = c \Rightarrow dP = -\gamma\frac{P}{V}dV = -\frac{\gamma P_0}{V}4\pi R^2 a$$

$$= \frac{\gamma P_0}{2V} \times 4\pi R^2 a \times 4\pi R^2 a$$

$$= \frac{\gamma P_0}{2 \times 4\pi R^3} 4\pi R^2 a \times 4\pi R^2 a$$

$= (4pRP \times a^2)\dfrac{3\gamma}{2}$

$\therefore x \simeq 2.05$

17. Ans. (A)

Sol. $P = \dfrac{dQ}{dt} \qquad T_{(t)} = T_0(1 + \beta t^{1/4})$

$$\frac{dQ}{dt} = \boxed{ms}\frac{dT}{dt} \Rightarrow S = \frac{P}{\left(\frac{dT}{dt}\right)}$$

$$\frac{dT}{dt} = T_0\left[0 + \beta\frac{1}{4} \cdot t^{-3/4}\right] = \frac{\beta T_0}{4}.t^{-3/4}$$

$$S = \frac{P}{(dT/dt)} = \frac{4P}{\beta T_0}.\ t^{3/4}$$

$$S = \frac{4P}{\beta}\left[\frac{t^{3/4}}{T_0}\right]$$

$$\frac{T(t)}{T_0} = (1+\beta t^{1/4})$$

$$\beta t^{1/4} = \frac{T(t)}{T_0} - 1 = \frac{T(t)-T_0}{T_0}$$

$$t^{3/4} = \left(\frac{T(t)-T_0}{\beta.\, T_0}\right)^3$$

$$\Rightarrow \quad S = \frac{4P}{T_0\beta}\left[\frac{T(t)-T_0}{\beta.T_0}\right]^3 = \frac{4P}{\beta^4 T_0^4}\left[T(t)-T_0\right]^3$$

18. **Ans. (270.00)**

Sol. Let m = mass of calorimeter,

x = specific heat of calorimeter

s = specifc heat of liquid

L = latent heat of liquid

First 5 g of liquid at 30° is poured to calorimter at 110°C

$\therefore$ m × x × (110 – 80) = 5 × s × (80 × 30) + 5 L

$\Rightarrow$ mx × 30 = 250 s + 5 L ...(i)

Now, 80 g of liquid at 30° is poured into calorimeter at 80°C, the equilibrium temperature reaches to 50°C.

$\therefore$ m × x × (80 – 30) = 80 × s × (50 – 30)

$\Rightarrow$ mx × 30 = 1600 s ...(ii)

From (i) & (ii)

250 s + 5 L = 1600 s $\Rightarrow$ 5L = 1350 s

$$\Rightarrow \frac{L}{s} = 270$$

19. **Ans. (A, B)**

Sol. From graph

Process 1 $\to$ 2 is isobaric with $P = \frac{RT_0}{V_0}$

Process 2 $\to$ 3 is isochoric with $V = 2V_0$

Process 3 $\to$ 4 is isobaric with $P = \frac{RT_0}{2V_0}$

Process 4 $\to$ 1 is isochoric with $V = V_0$

Work in cycle = $\frac{RT_0}{V_0}.V_0 - \frac{RT_0}{2V_0}.V_0 = \frac{RT_0}{2}$

$Q_{1-2} = nC_P\Delta T = n.\frac{5R}{2}.T_0$

$Q_{2-3} = nC_V\Delta T = n.\frac{3R}{2}.T_0$

$$\therefore \quad \left|\frac{Q_{1-2}}{Q_{2-3}}\right| = \frac{5}{3}$$

$Q_{3-4} = nC_P\Delta T = n.\frac{5R}{2}.\frac{T_0}{2}$

$$\therefore \quad \left|\frac{Q_{1-2}}{Q_{3-4}}\right| = 2$$

20. **Ans. (A, C, D)**

Sol. $n_1 = 5$ moles $C_{V1} = \frac{3R}{2} P_0V_0T_0$

$n_2 = 1$ mole $C_{V2} = \frac{5R}{2}$

$$(C_V)_m = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1+n_2} = \frac{5\times\frac{3R}{2} + 1\times\frac{5R}{2}}{6}$$

$$= \frac{5R}{3}$$

$$\gamma_m = \frac{(c_P)_m}{(c_V)_m} = \frac{8}{5}$$

$\therefore$ Option (D) is correct

$$(C_P)_m = \frac{5R}{3} + R = \frac{8R}{3}$$

(A) $P_0V_0^\gamma = P\left(\frac{V_0}{4}\right)^\gamma \Rightarrow P = P_0(4)^{8/5} = 9.2\,P_0$

which is between $9P_0$ and $10P_0$

(B) Average K.E. = $5 \times \frac{3}{2}RT + 1 \times \frac{5RT}{2}$

= 10RT

To calculate T

$$\frac{P_0V_0}{T_0} = 9.2P_0 \times \frac{V_0}{4\times T}$$

so, $T = \frac{9.2}{4}T_0$

Now average KE = $10\,R \times 9.2\,\frac{T_0}{4} = 23RT_0$

(C) $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$$= \frac{P_0V_0 - 9.2P_0\times\frac{V_0}{4}}{3/5} = -13RT_0$$

21. Ans. (C)

Sol. (I) Degree of freedom f = 3

Work done in any process = Area under P – V graph

$\Rightarrow$ Work done in $1 \to 2 \to 3 = P_0V_0$

$$= \frac{RT_0}{3} \Rightarrow (Q)$$

(II) Change in internal energy $1 \to 2 \to 3$

$$\Delta U = nC_V\Delta T$$

$$= \frac{f}{2}nR\Delta T$$

$$= \frac{f}{2}\left(P_fV_f - P_iV_i\right)$$

$$= \frac{3}{2}\left(\frac{3P_0}{2}2V_0 - P_0V_0\right)$$

$$= 3P_0V_0$$

$$\Delta U = RT_0 \Rightarrow (R)$$

(III) Heat absorbed in $1 \to 2 \to 3$

for any process, I[st] law of thermodynamics

$$\Delta Q = \Delta W + \omega$$

$$\Delta Q = RT_0 + \frac{RT_0}{3}$$

$$\Delta Q = \frac{4RT_0}{3} \Rightarrow (S)$$

(IV) Heat absorbed in process $1 \to 2$

$$\Delta Q = \Delta U + W$$

$$= \frac{f}{2}\left(P_fV_f - P_iV_i\right) + W$$

$$= \frac{3}{2}\left(P_0 2V_0 - P_0V_0\right) + P_0V_0$$

$$= \frac{5}{2}P_0V_0$$

$$= \frac{5}{2}\left(\frac{RT_0}{3}\right)$$

$$\boxed{\Delta Q = \frac{5RT_0}{6}} \Rightarrow (U)$$

22. Ans. (D)

Sol. Process-1$\to$ 2 is isothermal (temperature constant)

Process-2 $\to$ 3 is isochoric (volume constant)

(I) Work done in $1 \to 2 \to 3$

$$W = W_{1\to2} + W_{2\to3}$$

$$= nRT \ln\left(\frac{V_f}{V_i}\right) + W_{2\to3}$$

$$= \frac{RT_0}{3}\ln\left(\frac{2V_0}{V_0}\right) + 0$$

$$W = \frac{RT_0}{3}\ln 2 \Rightarrow (P)$$

(II) ΔU in $1 \to 2 \to 3$

$$\Delta U = \frac{f}{2}nR(T_f - T_i)$$

$$= \frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right)$$

$$= \frac{3}{2}R\left(\frac{2T_0}{3}\right)$$

$$\boxed{\Delta U = RT_0} \Rightarrow (R)$$

(III) For any system, first law of thermodynamics for $1 \to 2 \to 3$

$$\Delta Q = \Delta U + W$$

$$\Delta Q = RT_0 + \frac{RT_0}{3}\ln 2$$

$$\Delta Q = \frac{RT_0}{3}(3 + \ln 2) \Rightarrow (T)$$

(IV) For process $1 \to 2$ (isothermal)

$$\Delta Q = \Delta U + W$$

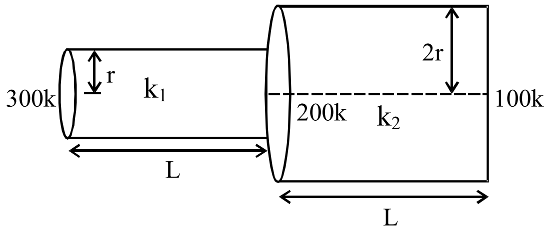
$$= \frac{f}{2}nR(T_f - T_i) + nRT\ln(V_f / V_i)$$

$$= 0 + R\left(\frac{T_0}{3}\right)\ln\left(\frac{2v_0}{v_0}\right)$$

$$\boxed{\Delta Q = \frac{RT_0}{3}\ln 2} \Rightarrow (P)$$

23. Ans. (4.00)

Sol.



We have in steady state,

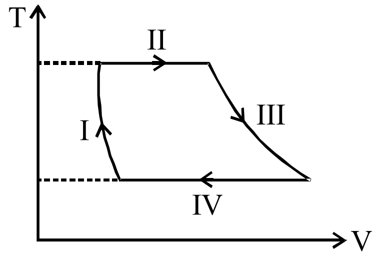
$$\left(\frac{\frac{200-300}{L}}{k_1\pi r^2}\right)+\left(\frac{\frac{200-100}{L}}{k_2\pi(2r)^2}\right)=0$$

$$\Rightarrow \frac{k_1\pi r^2\times 100}{L}=\frac{100k_2\pi\times 4r^2}{L}$$

$$\Rightarrow \frac{k_1}{k_2}=4$$

24. Ans. (B, C, D)

Sol.



(A) Process-I is not isochoric, V is decreasing.

(B) Process-II is isothermal expansion

$\Delta U = 0, W > 0$

$\Delta Q > 0$

(C) Process-IV is isothermal compression,

$\Delta U = 0, W < 0$

$\Delta Q < 0$

(D) Process-I and III are NOT isobaric because in isobaric process $T \propto V$ hence isobaric T–V graph will be linear.

25. Ans. (900)

Sol. $v_i = v$

$v_F = 8v$

For adiabatic process ($\gamma = \frac{5}{3}$ for monoatomic gas)

$$T_1V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$$

$$100(v)^{2/3} = T_2(8v)^{2/3}$$

$T_2 = 25$ k

$$\Delta U = nc_V\Delta T = 1\left(\frac{FR}{2}\right)[100-25]$$

$= 12 \times 75 = 900$ Joule

26. Ans. (C)

Sol. Process – I is an adiabatic process

$\Delta Q = \Delta U + W \qquad \Delta Q = 0$

$W = -\Delta U$

Volume of gas is decreasing $\Rightarrow W < 0$

$\Delta U > 0$

$\Rightarrow$ Temperature of gas increases.

$\Rightarrow$ No heat is exchanged between the gas and surrounding.

Process – II is an isobaric process

(Pressure remain constant)

$W = P\,\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$

Process - III is an isochoric process

(Volume remain constant)

$\Delta Q = \Delta U + W$

$W = 0$

$\Delta Q = \Delta U$

Process – IV is an isothermal process

(Temperature remains constant)

$\Delta Q = \Delta U + W$

$\Delta U = 0$

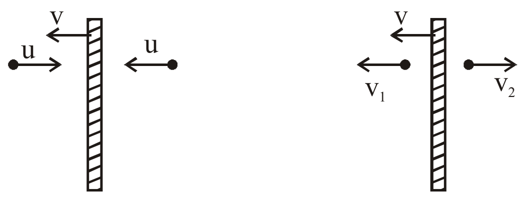
27. Ans. (C)

Sol. $P = \sigma A\,(T^4 - T_0^4)$

Surface area decrease $\Rightarrow$ Energy radiation decreases

28. Ans. (A, B, D)

Sol.



Just before the collision Just after the collision

$v_1 = u + 2v \qquad v_2 = (u - 2v)$

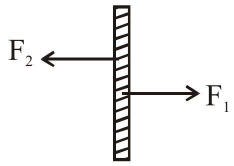
$\Delta v_1 = (2u + 2v) \qquad \Delta v_2 = (2u - 2v)$

$$F_1 = \frac{dp_1}{dt} = \rho A(u+v)\,(2u+2v) \quad F_2 = \frac{dp_2}{dt}$$

$= \rho A(u - v)\,(2u - 2v)$

$= 2\rho A\,(u + v)^2$

$= 2\rho A\,(u - v)^2$



$\Delta F = F_1 - F_2$(ΔF net force due to the air molecules on the plate)

$= 2\rho A\,(4uv) = 8\rho Auv$

$$P = \frac{\Delta F}{A} = 8\rho(uv)$$

$F_{net} = (F - \Delta F) = ma$ (m is mass of the plate)

$F - (8\rho Au)v = ma$

29. Ans. (B)

Sol. Work (Column-I), process (Column-II) & corresponding graph (Column-III) are in this sequence.

I ⟶ IV ⟶ Q

II ⟶ III ⟶ P

III ⟶ II ⟶ S

IV ⟶ I ⟶ R

Only "B" option follow the sequence.

30. Ans. (A)

Sol. Only option "A" follow the sequence.

31. Ans. (D)

Sol. It is for an adiabatic process. Only option (D).

32. Ans. (B)

Sol. $3000 - P = (120 \times 1)(4.2 \times 10^3)\dfrac{dT}{dt}$

$$\frac{dT}{dt} = \frac{20}{60\times 60\times 3}$$

P = 2067 W

33. Ans. (9)

Sol. $P = eA\sigma T^4$ where T is in kelvin

$$\log_2 \frac{eA\sigma(487+273)^4}{P_0} = 1 \quad ...(i)$$

$$\log_2 \frac{eA\sigma(2767+273)^4}{P_0} = x \quad ...(ii)$$

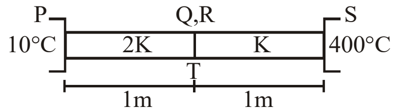
(ii) – (i)

$$\log_2\left(\frac{3040}{760}\right)^4 = x - 1$$

$\therefore x = 9$

34. Ans. (A)

Sol.



Heat flow from P to Q

$$\frac{dQ}{dt} = \frac{2KA(T-10)}{1}$$

Heat flow from Q to S

$$\frac{dQ}{dt} = \frac{KA(400-T)}{1}$$

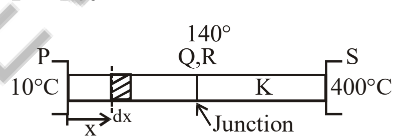
At steady state heat flow is same in whole combination

$$\frac{2KA(T-10)}{1} = KA(400-T)$$

$2T - 20 = 400 - T$

$3T = 420$

$T = 140°$



Temp of junction is 140°C

Temp at a distance x from end P

is $T_x = (130x + 10°)$

Change in length dx is dy

$dy = \alpha dx(T_x - 10)$

$$\int_0^{\Delta y} dy = \int_0^1 \alpha dx(130x + 10 - 10)$$

$$\Delta y = \left[\frac{\alpha x^2}{2}\times 130\right]_0^1$$

$\Delta y = 1.2 \times 10^{-5} \times 65$

$\Delta y = 78.0 \times 10^{-5}$ m = 0.78 mm

35. Ans. (C)

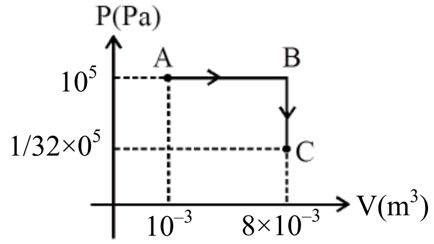
Sol. In adiabatic process

P^3V^5 = constant

$\Rightarrow PV^{5/3}$ = constant

$$\Rightarrow \gamma = \frac{5}{3} \Rightarrow C_V = \frac{3}{2}R \text{ and } C_P = \frac{5}{2}R$$

In another process



$\Delta Q = nC_P\Delta T + nC_V\Delta T$

$= \frac{5}{2}nR(T_B - T_A) + \frac{3}{2}nR(T_C - T_B)$

$\Delta Q = \frac{5}{2}(P_BV_B - P_AV_A) + \frac{3}{2}(P_CV_C - P_BV_B)$

Putting values

$\Delta Q = 587.5\ J \approx 588J$

36. **Ans. (2)**

Sol. $P = e\sigma AT^4$

λT = constant

$= \frac{P_A}{P_B} = \frac{e\sigma A_A T_A^4}{e\sigma A_B T_B^4} = \frac{A_A}{A_B}\frac{T_A^4}{T_B^4} = \frac{r_A^2}{r_B^2}.\frac{\lambda_B^4}{\lambda_A^4}$

$10^4 = 400^2.\left(\frac{\lambda_B}{\lambda_A}\right)^4$

$\frac{10^4}{16\times10^4} = \left(\frac{\lambda_B}{\lambda_A}\right)^4$

$\frac{\lambda_A}{\lambda_B} = \left(\frac{16}{1}\right)^{\frac{1}{4}} = 2$

37. **Ans. (A, B, D)**

Sol. $C_{V(mix)} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}R\right)}{2} = 2R$

$C_{P(mix)} = 3R \qquad \gamma_{mix} = \frac{3}{2} \Rightarrow f = 4$

Average energy/mole = $f\frac{1}{2}RT = 2RT$

$\frac{(V_{sound})_{mixture}}{(V_{sound})_{He}} = \frac{\sqrt{\frac{RT}{2}}}{\sqrt{\frac{5RT}{12}}} = \sqrt{\frac{6}{5}}$

$\frac{(V_{rms})_{He}}{(V_{rms})_{H_2}} = \frac{\sqrt{\frac{3RT}{4}}}{\sqrt{\frac{3RT}{2}}} = \frac{1}{\sqrt{2}}$

$\therefore$ (A,B,D)

38. **Ans. (B)**

Sol. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

if $V_2 = 2V_1$ & $T_2 = 3T_1$

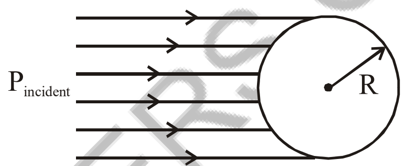
$\Rightarrow P_2 = \frac{3}{2}P_1$

$\Delta U = \frac{f}{2}(nR\Delta T) = \frac{3}{2}(P_2V_2 - P_1V_1)$

$= \frac{3}{2}\times 2P_1V_1 = 3P_1V_1$

39. **Ans. (A)**

Sol.



Energy incident = $I\pi R^2 = 912 \times \pi R^2$

Energy emitted, assuming temp of the sphere at steady state to be T,

$\sigma \times 4\pi R^2 (T^4 - 300^4)$

At quilibrium,

$\sigma \times 4\pi R^2 (T^4 - 300^4) = 912 \times \pi R^2$

$\Rightarrow T^4 - 300^4 = \frac{912}{5.7\times10^{-8}\times4} = 40 \times 10^8$

$\therefore T^4 = (40 + 81) \times 10^8$

$\therefore T = 331.66 \approx 330$ K

40. **Ans. (2)**

Sol. $Q_{iaf} = W_{iaf} + \Delta U_{iaf}$

$500 = 200 + \Delta U_{iaf}$

$\Delta U_{iaf} = 300$

$U_f = 400$

$Q_{ib} = W_{ib} + \Delta U_{ib}$

$= 50 + 100$

$Q_{ib} = 150$

$Q_{bf} = W_{bf} + \Delta U_{bf}$

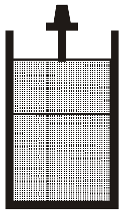
$= 100 + 200$

$Q_{bf} = 300$

$\frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$ ans

41. Ans. (D)

Sol. $\theta_A = \theta_B$



Process in A is isobaric and

Process in B is isochoric

$nC_p\ (T - 400) = nC_V\ (700 - T)$

$2\frac{7}{2}R\ (T - 400) = 2 \times \frac{5}{2}R\ (700 - T)$

$7T - 2800 = 2100 - 3T$

$T = 490$

42. Ans. (D)

Sol. $\theta_A = \theta_B$

$n\frac{7}{2}R\ (T - 400) = n\frac{5}{2}R\ (700 - T)$

$7T - 2800 = 3500 - 5T$

$12T = 6300$

$T = 525$

$W = nR\Delta T + nR\Delta T_1$

$W = -100R$