GRAVITATION

1. Two satellites P and Q are moving in different circular orbits around the Earth (radius R). The heights of P and Q from the Earth surface are h_P and h_Q, respectively, where $h_p = R/3$. The accelerations of P and Q due to Earth's gravity are g_P and g_Q, respectively. If $g_P / g_Q = 36/25$, what is the value of h_Q?

[JEE(Advanced) 2023]

(A) 3R/5 (B) R/6 (C) 6R/5 (D) 5R/6

2. Two spherical stars A and B have densities ρ_A and ρ_B, respectively. A and B have the same radius, and their masses M_A and M_B are related by $M_B = 2M_A$. Due to an interaction process, star A loses some of its mass, so that its radius is halved, while its spherical shape is retained, and its density remains ρ_A. The entire mass lost by A is deposited as a thick spherical shell on B with the density of the shell being ρ_A. If ν_A and ν_B are the escape velocities from A and B after the interaction process, the ratio $\frac{\nu_B}{\nu_A} = \sqrt{\frac{10n}{15^{1/3}}}$.

The value of n is _______. **[JEE(Advanced) 2022]**

3. The distance between two stars of masses $3M_S$ and $6M_S$ is 9R. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period nT, where T is the period of Earth's revolution around the Sun. The value of n is _______. **[JEE(Advanced) 2021]**

4. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is :

[G is universal gravitational constant] **[JEE(Advanced) 2019]**

(A) $\frac{K}{\pi r^2 m^2 G}$ (B) $\frac{K}{6\pi r^2 m^2 G}$ (C) $\frac{3K}{\pi r^2 m^2 G}$ (D) $\frac{K}{2\pi r^2 m^2 G}$

5. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **[JEE(Advanced) 2018]**

List–I	List–II
P. $\frac{v_1}{v_2}$	(1) $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	(2) 1
R. $\frac{K_1}{K_2}$	(3) 2
S. $\frac{T_1}{T_2}$	(4) 8

(A) P → 4 ; Q → 2 ; R → 1 ; S → 3 (B) P → 3 ; Q → 2 ; R → 4 ; S → 1

(C) P → 2 ; Q → 3 ; R → 1 ; S → 4 (D) P → 2 ; Q → 3 ; R → 4 ; S → 1

6. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2$ km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet)

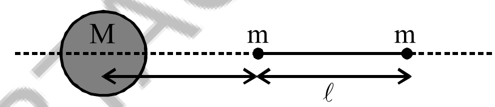
[JEE(Advanced) 2017]

(A) $v_s = 22$ km s^{-1} (B) $v_s = 72$ km s^{-1}

(C) $v_s = 42$ km s^{-1} (D) $v_s = 62$ km s^{-1}

7. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{th}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **[JEE(Advanced) 2015]**

8. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is: **[JEE(Advanced) 2015]**



9. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at r(r < R), then the correct option(s) is(are) :- **[JEE(Advanced) 2015]**

(A) $P(r = 0) = 0$ (B) $\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{63}{80}$

(C) $\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{16}{21}$ (D) $\frac{P(r = R/2)}{P(r = R/3)} = \frac{20}{27}$

10. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-3} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity on Earth is 10 ms^{-2})

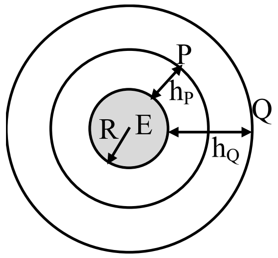
[JEE(Advanced) 2014]

(A) 96 N (B) 108 N (C) 120 N (D) 150 N

SOLUTIONS

1. Ans. (A)

Sol.



$$\frac{g_P}{g_Q}=\frac{\frac{GM}{r_P^2}}{\frac{GM}{r_Q^2}}=\left(\frac{r_Q}{r_P}\right)^2$$

$$\frac{36}{25}=\left(\frac{r_Q}{r_P}\right)^2;\ \frac{r_Q}{r_P}=\frac{6}{5}$$

$$r_Q=\frac{6}{5}r_P$$

$$R+h_Q=\frac{6}{5}\left(R+\frac{R}{3}\right)$$

$$h_Q=\frac{24}{15}R-R=\frac{9}{15}R=\frac{3}{5}R$$

2. Ans. (2.2 - 2.4)

Sol. Given $R_A = R_B = R$

$M_B = 2M_A$

Calculation of escape velocity for A:

Radius of remaining star $=\frac{R_A}{2}$.

Mass of remaining star $=\rho_A\frac{4}{3}\pi\frac{R_A^3}{8}=\frac{M_A}{8}$

$$\frac{-GM_{A/B}}{R_{A/2}}+\frac{1}{2}mv_A^2=0$$

$$\Rightarrow v_A=\sqrt{\frac{2GM_{A/B}}{R_{A/2}}}=\sqrt{\frac{GM_A}{2R}}$$

Calculation of escape velocity for B

Mass collected over $B=\frac{7}{8}M_A$

Let the radius of B becomes r.

$$\therefore\ \frac{4}{3}\pi\left(r^3-R_B^3\right)\rho_A=\frac{7}{8}\rho_A\frac{4}{3}\pi R_A^3$$

$$\Rightarrow\ \pi^3=\frac{7}{8}R_R^3+R_B^3=\frac{(15)^{1/3}R}{2}$$

$$\therefore\ \frac{V_B^2}{2}=\frac{23GM_A}{8\times15^{1/3}\frac{R}{2}}=\frac{23GM_A}{4\times15^{1/3}R}$$

$$\therefore\ V_B=\sqrt{\frac{23GM_A}{2\times15^{1/3}R}}$$

$$\therefore\ \frac{V_B}{V_A}=\sqrt{\frac{23}{15^{1/3}}}=\sqrt{\frac{10\times2.30}{15^{1/3}}}$$

$n = 2.30$

3. Ans. (9)

Sol. Circular orbits

$$T=2\pi\sqrt{\frac{R^2}{GM_S}}$$

Binary stars

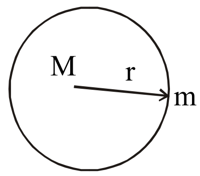
$$nT=2\pi\sqrt{\frac{(9R)^3}{G\left(3M_S+6M_S\right)}}$$

$$n\times2\pi\sqrt{\frac{R^3}{GM_S}}=9\times2\pi\sqrt{\frac{R^3}{GM_S}}$$

$n = 9$

4. Ans. (D)

Sol. Let total mass included in a sphere of radius r be M.



For a particle of mass m,

$$\frac{GMm}{r^2}=\frac{mv^2}{r}$$

$$\Rightarrow\ \frac{GMm}{r}=2K$$

$$\Rightarrow\ M=\frac{2Kr}{Gm}$$

$$\therefore\ dM=\frac{2Kdr}{Gm}$$

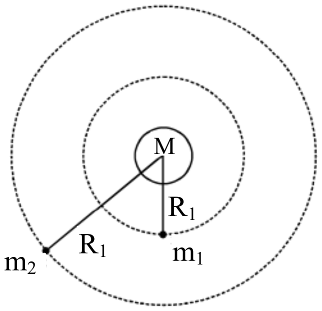
$$\Rightarrow\ (4\pi r^2dr)\rho=\frac{2Kdr}{Gm}$$

$$\Rightarrow\ \rho=\frac{K}{2\pi r^2Gm}$$

$$\therefore\ n=\frac{\rho}{m}=\frac{K}{2\pi r^2m^2G}$$

5. Ans. (B)

Sol.



$$\left.\begin{aligned}\frac{m_1}{m_2}=2\\ \frac{R_1}{R_2}=\frac{1}{4}\end{aligned}\right\}\text{given}$$

$$\frac{GMm_1}{R_1^2}=\frac{m_1v_1^2}{R_1}$$

$$v_1^2=\frac{GM}{R_1},\ v_2^2=\frac{GM}{R_2}$$

$$\frac{v_1^2}{v_2^2}=\frac{R_2}{R_1}=4$$

(P) $\frac{v_1}{v_2}=2$

(Q) $L = mvR$

$$\frac{L_1}{L_2}=\frac{m_1v_1R_1}{m_2v_2R_2}=2\times2\times\frac{1}{4}=1$$

(R) $K=\frac{1}{2}mv^2$

$$\frac{K_1}{K_2}=\frac{m_1v_1^2}{m_2v_2^2}=2\times(2)^2=8$$

(S) $T = 2\pi R/V$

$$\frac{T_1}{T_2}=\frac{R_1}{v_1}\times\frac{v_2}{R_2}=\frac{R_1}{R_2}\times\frac{v_2}{v_1}=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

6. Ans. (C)

Sol. Given $v_e=11.2\text{km/sec}=\sqrt{\frac{2GM_e}{R_e}}$

From energy conservation

$$\frac{1}{2}mv_s^2-\frac{GM_sm}{r}-\frac{GM_em}{R_e}=0-0$$

where, r = distance of rocket from Sun

$$\Rightarrow v_s=\sqrt{\frac{2GM_e}{R_e}+\frac{2GM_s}{r}}$$

given : $M_s = 3\times10^5\ M_e$ & $r = 2.5\times10^4\ R_e$

$$\Rightarrow v_s=\sqrt{\frac{2GM_e}{R_e}+\frac{2G3\times10^5M_e}{2.5\times10^4R_e}}$$

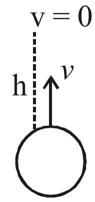
$$=\sqrt{\frac{2GM_e}{R_e}\left(1+\frac{3\times10^5}{2.5\times10^4}\right)}$$

$$=\sqrt{\frac{2GM_e}{R_e}\times13}$$

$$\Rightarrow v_s\simeq42\text{ km/s}$$

7. Ans. (2)

Sol. $g'=\frac{g}{\left(1+\frac{h}{R}\right)^2}=\frac{g}{4}$



$$1+\frac{h}{R}=2$$

$\boxed{h=R}$ ...(i)

So velocity of particle becomes zero at h = R

given $v_{esc}=v\sqrt{N}$

so $\sqrt{\frac{2GM}{R}}=v\sqrt{N}$ ...(ii)

Applying conservation of energy

$$\frac{-GMm}{R}+\frac{1}{2}mv^2=-\frac{GMm}{2R}+0$$

on solving

$$v^2=\frac{GM}{R}$$

so $v=\sqrt{\frac{GM}{R}}$ putting in equation (ii)

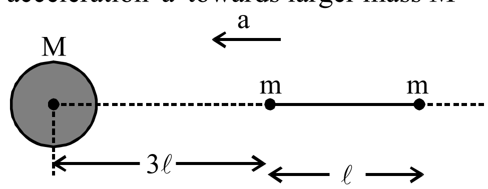
$$\sqrt{\frac{2GM}{R}}=\sqrt{\frac{GM}{R}}\sqrt{N}$$

comparing N = 2

8. Ans. (7)

Sol. Due to gravitational interaction connected masses have some acceleration.

Let both small masses are moving with acceleration 'a' towards larger mass M



Force eq. for mass nearer to larger mass

$$\frac{GMm}{(3\ell)^2}-\frac{Gm^2}{\ell^2}=ma \quad \text{... (i)}$$

Force eq. for mass away from larger mass

$$\frac{GMm}{(4\ell)^2}+\frac{Gm^2}{\ell^2}=ma \quad \text{... (ii)}$$

from equation (i) & (ii)

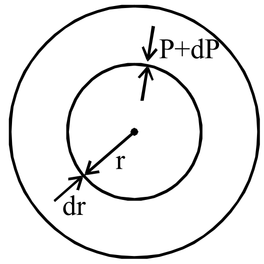
$$\frac{GM}{9\ell^2}-\frac{Gm}{\ell^2}=\frac{GM}{16\ell^2}+\frac{Gm}{\ell^2}$$

$$\Rightarrow\frac{M}{9}-\frac{M}{16}=m+m \qquad \Rightarrow\frac{7M}{144}=2m$$

$$\Rightarrow m=\frac{7M}{288}=k\left(\frac{M}{288}\right) \qquad \Rightarrow K=7$$

9. Ans. (B, C)

Sol.



dF = [P – (P + dP)]A

$$\Rightarrow \frac{Gm}{r^2}dm = -(dP)A$$

$$\Rightarrow \int_0^P dP = -\int_R^r \frac{G\frac{Mr^3}{R^3}4ar^2 dr\rho}{r^2(4\pi r^2)}$$

$$\therefore\ P = \frac{G\rho M}{2R}\left(1 - \frac{r^2}{R^2}\right)$$

10. Ans. (B)

Sol. $E_G = \frac{4\pi G r\rho}{3}$

$dF = E_G \lambda dr$

$$F = \int_{\frac{4R}{5}}^{R} \frac{4\pi G\rho\lambda}{3} r\,dr = \frac{4\pi G\rho\lambda}{3}\left[\frac{r^2}{2}\right]_{\frac{4R}{5}}^{R}$$

$$= \frac{4\pi G\rho\lambda}{3\times 2}\left[R^2 - \frac{16R^2}{25}\right] = \frac{4\pi}{6}G\rho\lambda \times \frac{9}{25}R^2$$

$$F = \frac{4\pi}{6}G \times \frac{M}{\frac{4\pi}{3}R_e^3} \times \lambda \times \frac{9}{25} \times \frac{R_e^2}{100}$$

After solving [F ⇒ 108 N]