ERRORS

1. In an experiment for determination of the focal length of a thin convex lens, the distance of the object from the lens is 10 ± 0.1 cm and the distance of its real image from the lens is 20 ± 0.2 cm. The error in the determination of focal length of the lens is n %. The value of n is _______.

[JEE(Advanced) 2023]

2. Area of the cross-section of a wire is measured using a screw gauge. The pitch of the main scale is 0.5 mm. The circular scale has 100 divisions and for one full rotation of the circular scale, the main scale shifts by two divisions. The measured readings are listed below. **[JEE(Advanced) 2022]**

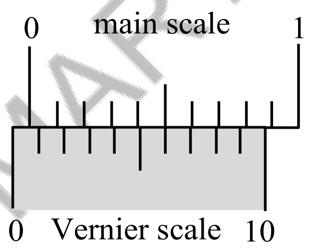
Measurement condition	Main scale reading	Circular scale reading
Two arms of gauge touching each other without wire	0 division	4 division
Attempt-1: With wire	4 divisions	20 divisions
Attempt-2: With wire	4 divisions	16 divisions

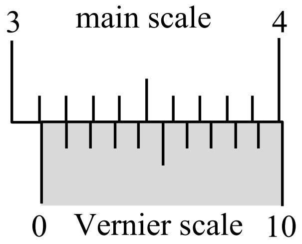
What are the diameter and cross-sectional area of the wire measured using the screw gauge?

(A) 2.22 ± 0.02 mm, $\pi(1.23 \pm 0.02)$ mm^2 (B) 2.22 ± 0.01 mm, $\pi(1.23 \pm 0.01)$ mm^2

(C) 2.14 ± 0.02 mm, $\pi(1.14 \pm 0.02)$ mm^2 (D) 2.14 ± 0.01 mm, $\pi(1.14 \pm 0.01)$ mm^2

3. The smallest division on the main scale of a Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calipers with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is **[JEE(Advanced) 2021]**





(A) 3.07 cm (B) 3.11 cm (C) 3.15 cm (D) 3.17 cm

4. Two capacitors with capacitance values $C_1 = 2000 \pm 10$ pF and $C_2 = 3000 \pm 15$ pF are connected in series. The voltage applied across this combination is $V = 5.00 \pm 0.02$ V. The percentage error in the calculation of the energy stored in this combination of capacitors is _______. **[JEE(Advanced) 2020]**

5. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is________. **[JEE(Advanced) 2019]**

Paragraph for Q.6 & Q.7

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation z = x/y. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in Δy/y, is $1 \mp (\Delta y / y)$. The relative errors in independent variables are always added. So the error in z will be $\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)$.

The above derivation makes the assumption that $\frac{\Delta x}{x} << 1, \frac{\Delta y}{y} << 1$. Therefore, the higher powers of these quantities are neglected.

6. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa(Δa/a << 1), then what is the error Δr in determining r ? **[JEE(Advanced) 2018]**

(A) $\frac{\Delta a}{(1+a)^2}$ (B) $\frac{2\Delta a}{(1+a)^2}$ (C) $\frac{2\Delta a}{(1-a^2)}$ (D) $\frac{2a\Delta a}{(1-a^2)}$

7. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| << 1$, In (1 + x) = x up to first power in x. The error Δλ, in the determination of the decay constant λ, in s^{-1} is :- **[JEE(Advanced) 2018]**

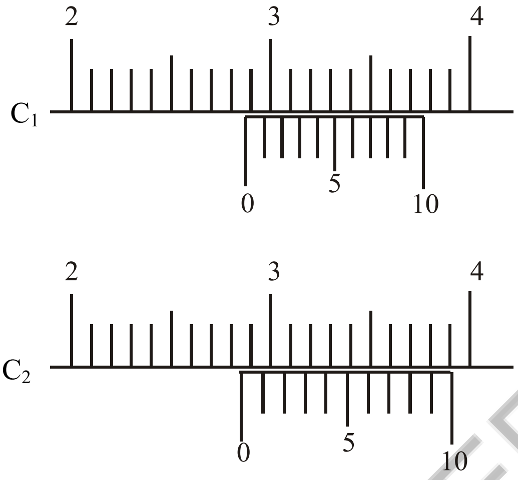
(A) 0.04 (B) 0.03 (C) 0.02 (D) 0.01

8. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} N m^{-2} carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg, the vernier scale division which coincides with a main scale division is_______.
(Take g = 10 ms^{-2} and π = 3.2) **[JEE(Advanced) 2018]**

9. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is δT = 0.01 second and he measures the depth of the well to be L = 20 meters. Take the acceleration due to gravity g = 10 ms^{-2} and the velocity of sound is 300 ms^{-1}. Then the fractional error in the measurement, δL/L, is closest to **[JEE(Advanced) 2017]**

(A) 0.2% (B) 5% (C) 3% (D) 1%

10. There are two vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2 respectively, are **[JEE(Advanced) 2016]**



(A) 2.87 and 2.86 (B) 2.87 and 2.87 (C) 2.87 and 2.83 (D) 2.85 and 2.82

11. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57s, 0.54s and 0.59s. The least count of the watch used for the measurement of time period is 0.01s. Which of the following statement(s) is(are) true ? **[JEE(Advanced) 2016]**

(A) The error in the measurement of r is 10% (B) The error in the measurement of T is 3.57 %

(C) The error in the measurement of T is 2% (D) The error in the determined value of g is 11%

12. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then : **[JEE(Advanced) 2015]**

(A) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(B) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(D) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

13. The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2 s^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of E(t) at t = 5 s is. **[JEE(Advanced) 2015]**

14. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45^{th} division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is. **[JEE(Advanced) 2014]**

SOLUTIONS

1. Ans. (1)

Sol. u = 10 ± 0.1 cm, v = 20 ± 0.2 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v^2}dv+\frac{1}{u^2}du=-\frac{1}{f^2}df$$

$$\frac{1}{20}+\frac{1}{10}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{3}{20}\Rightarrow f=\frac{20}{3}cm$$

$$\Rightarrow\frac{1}{(20)^2}(0.2)+\frac{1}{(10)^2}(0.1)=\frac{9}{400}df$$

$$df=\frac{1}{9}\left(\frac{400}{400}\times0.2+\frac{400}{100}\times0.1\right)$$

$$df=\frac{1}{9}(0.2+0.4)\Rightarrow df=\frac{0.6}{9}$$

$$\frac{df}{f}=\frac{0.6}{9}\times\frac{3}{20}=\frac{1}{100}$$

% error = 1 %

Alternate Solution

$$\frac{1}{V}-\frac{1}{U}=\frac{1}{f};\qquad +\frac{1}{20}+\frac{1}{10}=\frac{1}{f}$$

$$-\frac{1}{V^2}dv+\frac{dU}{u^2}=-\frac{df}{f^2}\qquad \frac{1+2}{20}=\frac{1}{f};\ f=\frac{20}{3}$$

$$\frac{0.1}{100}+\frac{0.2}{400}=\frac{f\%}{f}$$

$$\frac{0.4+0.2}{400}=\frac{\Delta f}{f\left(\frac{20}{3}\right)}$$

$$\frac{0.6\times20}{400\times3}=\frac{\Delta f}{f}$$

$$\frac{1}{100}=\frac{\Delta f}{f}$$

% change in f is 1%

2. Ans. (Dropped)

Sol. $LC=\frac{0.1}{100}=0.001mm$

Zero error = 4 × 0.001 = 0.004 mm

Reading 1 = 0.5 × 4 + 20 × 0.001 – 0.004 = 2.16 mm

Reading 2 = 0.5 × 4 + 16 × 0.001 – 0.004 = 2.12 mm

Mean value = 2.14 mm

Mean absolute error = $\frac{0.02+0.02}{2}=0.02$

Diameter = 2.14 ± 0.02

Area = $\frac{\pi}{4}d^2$

3. Ans. (C)

Sol. Given 10 VSD = 9 MSD Here

MSD → Main Scale division

$$1\text{ VSD}=\frac{9}{10}\text{MSD}$$

VSD→ Vernier Scale division

Least count = 1 MSD – 1 VSD

$=\left(1-\frac{9}{10}\right)$ MSD

= 0.1 MSD

= 0.1 × 0.1 cm

= 0.01 cm

As '0' of V.S. lie before '0' of M.S.

Zero error = –[10 – 6]L.C.

= – 4 × 0.01 cm

= –0.04 cm

Reading = 3.1 cm + 1 × LC

= 3.4 cm + 1 × 0.01 cm

= 3.11 cm

True diameter = Reading – Zero error

= 3.11 – (–0.04) cm = 3.15 cm

4. Ans. (1.30)

Sol. $U=\frac{1}{2}\frac{C_1C_2}{C_1+C_2}V^2$

Let $C_{eq}=\frac{C_1C_2}{C_1+C_2}$

$$\frac{1}{C_{eq}\pm\Delta C_{eq}}=\frac{1}{C_1\pm\Delta C_1}+\frac{1}{C_2\pm\Delta C_2}$$

$$\Rightarrow C_{eq}\pm\Delta C_{eq}\simeq\frac{C_1C_2+C_1\Delta C_2+C_2\Delta C_1}{C_1+C_2+\Delta C_1+\Delta C_2}$$

$$=\frac{1200\left(1\pm\frac{12}{1200}\right)}{\left(1\pm\frac{25}{5000}\right)}$$

$$=1200\left[1\pm\left(\frac{1}{100}-\frac{1}{200}\right)\right]$$

$$\frac{\Delta U}{U}\times100=\frac{\Delta C_{eq}}{C_{eq}}\times100+\frac{2\Delta V}{V}\times100$$

$$=\frac{1}{200}\times100+2\times\frac{0.02}{5}\times100$$

= 1.3 %

5. Ans. (1.35 to 1.45)

Sol. For the given lens

u = –30 cm

v = 60 cm

& $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ on solving : f = 20 cm

also $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

on differentiation

$$\frac{df}{f^2}=\frac{dv}{v^2}+\frac{du}{u^2}$$

$$\frac{df}{f}=f\left[\frac{dv}{v^2}+\frac{du}{u^2}\right]$$

& $\frac{df}{f}\times100=f\left[\frac{dv}{v^2}+\frac{du}{u^2}\right]\times100\%$ = 1.38

6. Ans. (B)

Sol. $r=\left(\frac{1-a}{1+a}\right)$

$$\frac{\Delta r}{r}=\frac{\Delta(1-a)}{(1-a)}+\frac{\Delta(1+a)}{(1+a)}$$

$$=\frac{\Delta a}{(1-a)}+\frac{\Delta a}{(1+a)}$$

$$=\frac{\Delta a(1+a+1-a)}{(1-a)(1+a)}$$

$$\therefore \Delta r=\frac{2\Delta a}{(1-a)(1+a)}\frac{(1-a)}{(1+a)}=\frac{2\Delta a}{(1+a)^2}$$

7. Ans. (C)

Sol. $N = N_0e^{-\lambda t}$

$\ell nN = \ell nN_0 - \lambda t$

$$\frac{dN}{N}=-d\lambda t$$

Converting to error,

$$\frac{\Delta N}{N}=\Delta\lambda t$$

$$\therefore \Delta\lambda=\frac{40}{2000\times L}=0.02$$

(N is number of nuclei left undecayed)

8. Ans. (3.00)

Sol. d = 0.5 mm Y = 2 × 10^{11} ℓ = 1m

$$\Delta\ell=\frac{F\ell}{Ay}=\frac{mg\ell}{\frac{\pi d^2}{4}y}$$

$$=\frac{1.2\times10\times1}{\frac{\pi}{4}\times(5\times10^{-4})^2\times2\times10^{11}}$$

$$\Delta\ell=\frac{1.2\times10}{\frac{3.2}{4}\times25\times10^{-8}\times2\times10^{11}}$$

$$=\frac{12}{0.8\times25\times2\times10^3}=\frac{12}{40\times10^3}=0.3\text{mm}$$

so 3^{rd} division of vernier scale will coincide with main scale.

9. Ans. (D)

Sol. Total time taken

$$T=\sqrt{\frac{2L}{g}}+\frac{L}{c}$$

Now, for an error δL in L,

We have an error δT in T

$$\text{So, } T+\delta T=\sqrt{\frac{2(L+\delta L)}{g}}+\frac{(L+\delta L)}{c}$$

$$=\sqrt{\frac{2L}{g}\left(1+\frac{\delta L}{L}\right)}+\frac{L}{c}\left(1+\frac{\delta L}{L}\right)$$

Since, $\frac{\delta T}{T}$ is very small, hence $\frac{\delta L}{L}$ is also small, so taking binomial approximation

$$T+\delta T=\sqrt{\frac{2L}{g}}\left(1+\frac{1}{2}\frac{\delta L}{L}\right)+\frac{L}{c}\left(1+\frac{\delta L}{L}\right)$$

$$T+\delta T=\left(\sqrt{\frac{2L}{g}}\right)+\sqrt{\frac{2L}{g}}\left(\frac{1}{2}\frac{\delta L}{L}\right)+\left(\frac{L}{c}\right)+\frac{L}{c}\left(\frac{\delta L}{L}\right)$$

$$=\left(\sqrt{\frac{2L}{g}}+\frac{L}{c}\right)+\left(\frac{1}{2}\sqrt{\frac{2L}{g}}+\frac{L}{c}\right)\left(\frac{\delta L}{L}\right)$$

$$=T+\left(\frac{1}{2}\sqrt{\frac{2\times20}{10}}+\frac{20}{300}\right)\frac{\delta L}{L}$$

$$\Rightarrow \delta T=\left(1+\frac{1}{15}\right)\frac{\delta L}{L} \qquad \Rightarrow \frac{\delta L}{L}=\left(\frac{15}{16}\right)\delta T$$

$$=\left(\frac{15}{16}\right)\left(\frac{1}{100}\right)$$

$$\%\text{ error}=\left(\frac{\delta L}{L}\right)\times100\% = \frac{15}{16}\% \approx 1\%$$

10. Ans. (C)

Sol. For caliper C_1

10 VSD = 9MSD

LC = 1MSD – 1VSD

LC = 0.01 cm

Measured value =

Main scale reading + vernier scale reading

= (2.8 + 7 × 0.01) cm

= 2.87 cm

For Caliper C_2

10 VSD = 11 MSD

LC = 0.01 cm

Measured value = {2.8 + (10–7) × 0.01}cm

= 2.83 cm

11. Ans. (A, B, D)

Sol.

	T	Absolute error
1	0.52	–0.04
2	0.56	00
3	0.57	+0.01
4	0.54	–0.02
5	0.59	+0.03
	$T_{avg} = 0.556$ $= 0.56$	

Avg. absolute error

$$= \frac{.04 + 00 + .01 + .02 + .03}{5} = .02$$

$$\Rightarrow \frac{\Delta T}{T} \times 100\% = \frac{.02}{.56} \times 100\% \approx 3.57\% \text{ (B)}$$

$$\Rightarrow \frac{\Delta r}{r} \times 100\% = \frac{1}{10} \times 100 = 10\% \text{ (A)}$$

also $\frac{\Delta g}{g} = \frac{\Delta R + \Delta r}{R - r} + \frac{2\Delta T}{T}$

$$\frac{\Delta g}{g} \times 100\% = \frac{1+1}{50} \times 100\% + 2(3.57)\%$$

≈ 11% (D)

12. Ans. (B, C)

Sol. 1 main scale division (M.S.D.) = $\frac{1}{8}$ cm

5 veriner scale division (V.S.D.) = 4 M.S.

1 V.S.D. = $\frac{4}{5}$ M.S.D.

Least count of vernier scale (L.C.)

= 1 M.S.D. – 1 V.S.D.

$= 1\text{M.S.D.} - \frac{4}{5}\text{M.S.D.}$

$$(\text{L.C}) = \frac{1 \text{ M.S.D.}}{5} = \frac{1}{40}\text{cm}$$

For option A and B

If the pitch of the screw gauge is twice the least count of the vernier callipers then

pitch = 2 × L.C of vernier scale = $\frac{1}{20}$cm

hence least count of screw gauge = $\frac{\text{Pitch}}{100}$

= 0.005 m

For option C and D

Least count of linear scale of screw gauge

$= 2 \times \frac{1}{40} = \frac{1}{20}\text{cm}$

Pitch = 2 × $\frac{1}{20} = \frac{1}{10}\text{cm} = 1\text{mm}$

Least count of screw gauge

$= \frac{1\text{mm}}{100} = 0.01\text{mm}$

13. Ans. (4)

Sol. Energy $E = A^2e^{-\alpha t}$

for small % errors, we can, do differentiation

$dE = 2A(dA)e^{-\alpha t} + A^2(-\alpha e^{-\alpha t}\, dt)$

fractional error = $\frac{dE}{E}$

$$= \frac{2Ae^{-\alpha t}(dA) + (-\alpha A^2 e^{-\alpha t})dt}{A^2 e^{-\alpha t}}$$

$$= 2\left(\frac{dA}{A}\right) + \left(-\alpha\frac{dt}{t}\right)t$$

% error = 2(1.25%) + (0.2 × 1.5%) × 5

= 4% (errors always add up)

Alternate Solution

$E = A^2e^{-\alpha t}$

taking natural logarithm on both sides,

$\ell n(E) = \ell n(A^2) + (-\alpha t)$

differentiating

$$\frac{dE}{E}=2\left(\frac{dA}{A}\right)+(-\alpha dt)$$

for small fractional erros, errors always add up

$$\left|\frac{dE}{E}\right|=2\left|\frac{dA}{A}\right|+\alpha\left(\frac{dt}{t}\right)\times t$$

= 2(1.25%) + (0.2)(1.5%)5 = 4%

14. Ans. (4)

Sol. Using searle's method young modules is calculated

$$y=\frac{F/A}{\frac{\Delta\ell}{\ell}}$$

$$\frac{dy}{y}=\frac{dF}{F}+\frac{dA}{A}+\frac{d\ell}{\ell}+\frac{d(\Delta\ell)}{\Delta\ell}$$

Only $\Delta\ell$ calculations have error

% error of y = $\frac{dy}{y}\times 100=\frac{d\Delta\ell}{\Delta\ell}\times 100$

$$=\frac{1\times 10^{-5}}{25\times 10^{-5}}\times 100=4\%$$