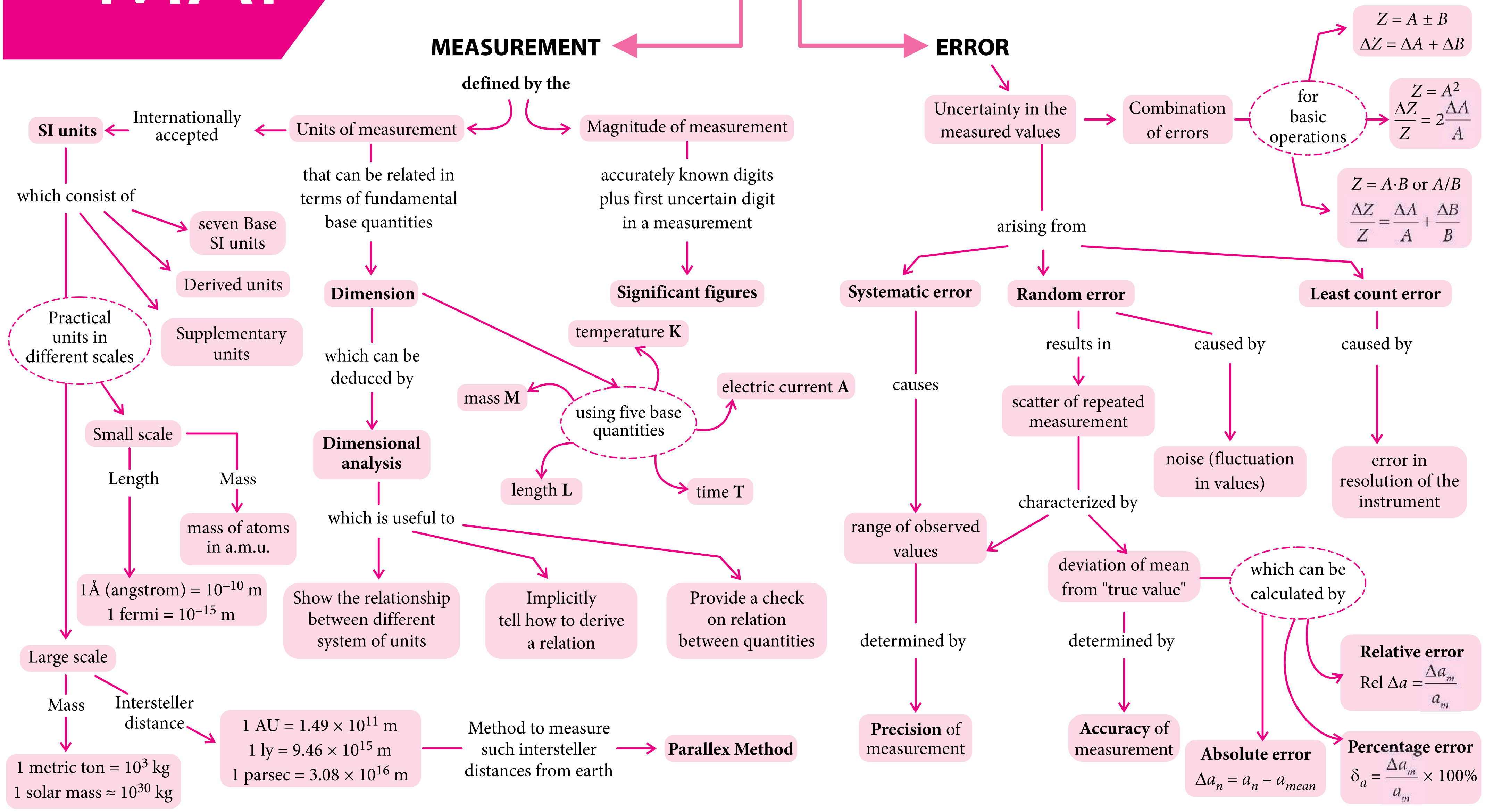
BRAIN MAP

CLASS XI

UNITS AND MEASUREMENTS



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CLASS XII

BRAIN MAP

CLASS XI

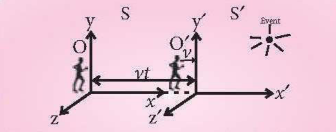
MOTION IN A STRAIGHT LINE

Motion

If a body changes its position as time passes w.r.t. frame of reference, it is said to be in motion.

Frame of Reference

A system consisting a set of coordinates and with reference to which observer describes any event.



The observers are moving in a particular direction x at a relative velocity of v and each observer has their own set of coordinates (x, y, z) and (x', y', z').

Distance

The actual path length covered by moving particle.

Displacement

The change in position vector.

Speed

The rate of distance covered with time is called speed,

$$v = \frac{\text{distance}}{\text{total time}} = \frac{d}{t}$$

Average Speed

$$v_{av} = \frac{\text{total distance}}{\text{total time}}$$

Instantaneous Speed

$$v = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta d}{\Delta t}$$

differentiating with respect to time

Velocity

The rate of change of position per unit time,

$$\vec{v} = \frac{\text{displacement}}{\text{time}} = \frac{\Delta \vec{x}}{\Delta t}$$

Average Velocity

$$\vec{v}_{av} = \frac{\Delta \vec{x}}{\Delta t}$$

Instantaneous Velocity

$$\vec{v} = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

differentiating with respect to time

Acceleration

The time rate of change of velocity, $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

Average Acceleration

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous Acceleration

$$\vec{a} = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Uniform Acceleration

Magnitude of velocity changes by equal amounts in equal intervals of time.

Non-uniform Acceleration

Acceleration changes with time.

Kinematic Equations

A mathematical treatment to describe the motion of a body in 1-dimension.

Constant Acceleration

For Uniformly Accelerated Motion

- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$
- $S_n = u + \frac{a}{2}(2n - 1)$

Acceleration changes with time

For Motion with Variable Acceleration

If $a = f(t) \longrightarrow$ a function of time

- $v = u + \int_0^t f(t)dt$
- $s = ut + \int_0^t (f(t)dt)dt$

For Motion Under Gravity

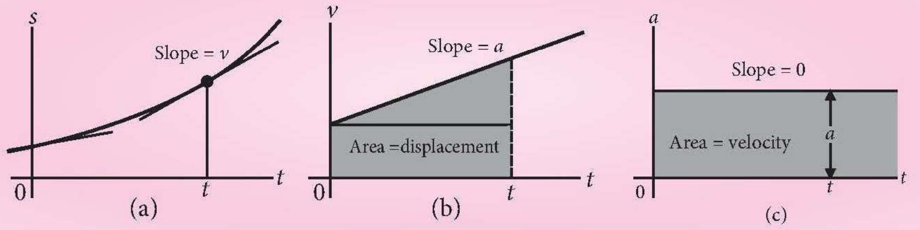
Vertically downward motion (Free fall case) $u = 0, a = g$

- $v = gt$
- $h = 1/2\, gt^2$
- $v^2 = 2gh$

Vertically upward motion $v = 0$, acceleration $a = -g$

- $u = gt$
- $h = ut - 1/2\, gt^2$
- $u = \sqrt{2gh}$

Graphical Representation of Uniformly Accelerated Motion



(a) (b) (c)

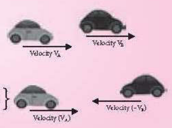
Relative Velocity

The velocity with which an object moves with respect to another object is called relative velocity

$V_{AB} = (V_A - V_B)$

$V_{AB} = \{V_A - (-V_B)\}$

$V_{AB} = (V_A + V_B)$



BRAIN MAP

MOTION IN A PLANE

CLASS XI

VECTOR

Physical quantities having both magnitude and direction

Addition of Vectors

Triangle Law

If two vectors are represented by two sides of a triangle in same order, the resultant will be the third side but in opposite order. $\vec{C} = \vec{A} + \vec{B}$

Parallelogram Law

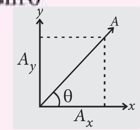
For two coinitial vectors represented by the two adjacent side of a parallelogram, the diagonal of the parallelogram so formed will be the resultant. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Resolution of Vector

Horizontal component of $\vec{A}$, $A_x = A\cos\theta$

Vertical component of $\vec{A}$, $A_y = A\sin\theta$

$A = \sqrt{A_x^2 + A_y^2}$, $\tan\theta = \left(\dfrac{A_y}{A_x}\right)$



Multiplication by a Scalar Quantity

Multiplication of a vector by a scalar ϕ changes its magnitude by a factor of ϕ.

KINEMATICS IN TWO DIMENSIONS

Kinematic Equations in 2-Dimensions

- $\vec{v} = \vec{u} + \vec{a}t$
  - $v_x = u_x + a_x t$ (along x-axis)
  - $v_y = u_y + a_y t$ (along y-axis)
- $\vec{r} = \vec{r}_0 + \vec{u}_0 t + (1/2)\vec{a}t^2$
  - $x = x_0 + u_x t + (1/2)a_x t^2$ (along x-axis)
  - $y = y_0 + u_y t + (1/2)a_y t^2$ (along y-axis)

Relative Velocity

According to vector law of addition

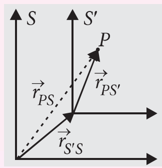
$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$

$\dfrac{d\vec{r}_{PS}}{dt} = \dfrac{d\vec{r}_{PS'}}{dt} + \dfrac{d\vec{r}_{S'S}}{dt}$

So, $\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$

i.e., $\vec{v}_{actual} = \vec{v}_{relative} + \vec{v}_{reference}$

$\vec{v}_{relative} = \vec{v}_{actual} - \vec{v}_{reference}$



Projectile Motion

Projectile Fired Horizontally

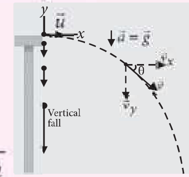
Equation of trajectory: $y = \dfrac{g}{2u^2}.x^2$

Velocity after time t: $v = \sqrt{u^2 + g^2t^2}$

$\beta = \tan^{-1}\dfrac{gt}{u}$

Time of flight: $T = \sqrt{\dfrac{2h}{g}}$

Horizontal range: $R = u\sqrt{\dfrac{2h}{g}}$



Projectile Fired at an Angle with Horizontal

Position after time t:

$x = (u\cos\theta)t,\ y = (u\sin\theta)t - \dfrac{1}{2}gt^2$

Equation of trajectory:

$y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta}.x^2$

Maximum height: $H = \dfrac{u^2\sin^2\theta}{2g}$

Time of flight: $T = \dfrac{2u\sin\theta}{g}$

Horizontal range: $R = \dfrac{u^2\sin 2\theta}{g}$

Circular Motion

Uniform Circular Motion

A particle moves along a circular path with a constant speed.

Angular displacement, $\theta = \dfrac{s}{r}$

Angular velocity, $\omega = \dfrac{\theta}{t}$

Also, $\omega = \dfrac{2\pi}{T} = 2\pi\upsilon$

Linear velocity, $v = r\omega$

Centripetal acceleration, $a = \dfrac{v^2}{r} = r\omega^2$

Linear acceleration, $a = r\alpha$

Non Uniform Circular Motion

Circular motion with variable speed

Resultant acceleration of the particle

$a = \sqrt{a_t^2 + a_r^2}$

$a_r = \dfrac{v^2}{r}$, a_t = tangential acceleration

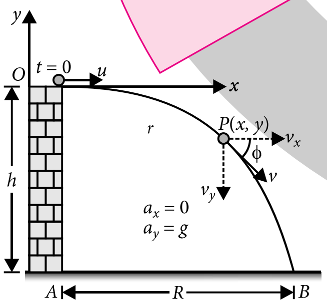
BRAIN MAP
CLASS XI

PROJECTILE MOTION

PROJECTILE Motion

A body which is in flight through the atmosphere under the effect of gravity alone and is not being propelled by any fuel is called projectile and its motion is called projectile motion.

Horizontal Projectile Motion



- **Equation of Trajectory**

$$y = \frac{1}{2}\frac{gx^2}{u^2}$$

- **Time of Descent**

$$T = \sqrt{\frac{2h}{g}}$$

- **Horizontal Range**

$$R = u\sqrt{\frac{2h}{g}}$$

- **Instantaneous Velocity**

$$v = \sqrt{u^2 + 2gy} = \sqrt{u^2 + g^2t^2}$$

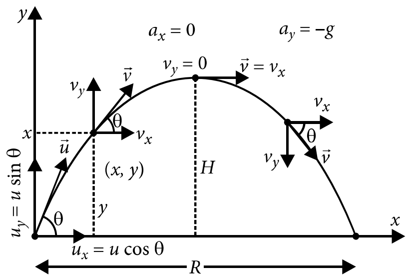
$$\tan\phi = \frac{v_y}{v_x} = \tan^{-1}\left(\frac{gt}{u}\right)$$

Oblique Projectile Motion

- **Equation of Trajectory**

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

This represents the parabolic path.



- **Maximum Height**

$$H = \frac{u^2\sin^2\theta}{2g}$$

- **Time of Flight**

$$T = \frac{2u\sin\theta}{g}$$

- **Horizontal Range**

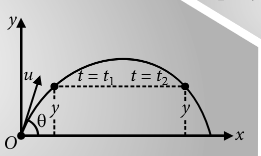
$$R = \frac{u^2\sin 2\theta}{g}$$

- Projectile passing through two different points on same height at time t_1 and t_2

$$y = \frac{gt_1t_2}{2}$$

$$t_2 = \frac{u\sin\theta}{g}\left[1 + \sqrt{1 - \left(\frac{2gy}{u\sin\theta}\right)^2}\right]$$

$$t_1 = \frac{u\sin\theta}{g}\left[1 - \sqrt{1 - \left(\frac{2gy}{u\sin\theta}\right)^2}\right]$$



- Ratio of time of ights for projectiles at complimentary angles θ and 90 – θ

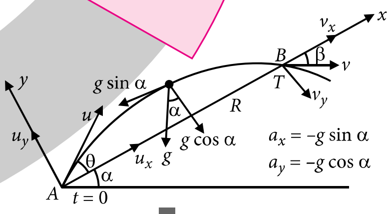
$$\frac{T_\theta}{T_{90-\theta}} = \tan\theta$$

- Range R is n times the maximum height H
  $R = nH; \theta = \tan^{-1}[4/n]$

- If $R = H$ then $\theta = \tan^{-1}(4)$ or $\theta = 76°$
- If $R = 4H$ then $\theta = \tan^{-1}(1)$ or $\theta = 45°$

- For complimentary angles θ and (90 – θ) range remains unchanged
- Relation between horizontal range and maximum height
  $R = 4H\cot\theta$

Projectile Motion on an Inclined Plane



- **Time of Flight**

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

- **Maximum Height**

$$H = \frac{u^2\sin^2\theta}{2g\cos\alpha}$$

- **Horizontal Range**

$$R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$$

- Maximum range occurs when
  $\theta = \frac{\pi}{4} + \frac{\alpha}{2}$
- Maximum range along the incline when projectile is thrown upwards
  $R_{max} = \frac{u^2}{g(1+\sin\alpha)}$
- Maximum range along incline when the projectile thrown downwards
  $R_{max} = \frac{u^2}{g(1-\sin\alpha)}$

BRAIN MAP
CLASS XI

CIRCULAR MOTION

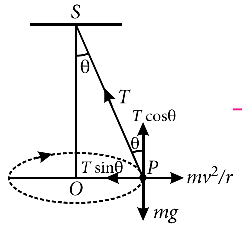
CIRCULAR MOTION

Dynamics of Circular Motion

Time period $T = 2\pi\sqrt{\dfrac{l\cos\theta}{g}}$

Linear velocity $v = \sqrt{gr\tan\theta}$

Angular velocity $\omega = \sqrt{\dfrac{g}{l\cos\theta}}$



Conical Pendulum : Tension in the string

$$T = mg\sqrt{1+\left(\frac{v^2}{rg}\right)^2}$$

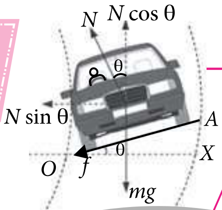
- As $F = \dfrac{mv^2}{r} \neq 0$, Particle is not in equilibrium
- $\tau_{net} = 0$ as central force is involved. *i.e.*, $p \neq$ constant but L = constant

Maximum safe speed on a banked frictional road

$$v = \sqrt{\frac{rg(\mu+\tan\theta)}{1-\mu\tan\theta}}$$

Banking of a road : Optimum speed of a vehicle on a banked road.

$v = \sqrt{rg\tan\theta}$



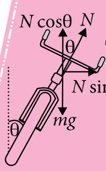
Forces

Centripetal force

$F_c = ma_c = \dfrac{mv^2}{r} = mr\omega^2$

Tangential force, $F_t = ma_t$

Net force, $F_{net} = m\sqrt{a_c^2 + a_t^2}$



Bending of a Cyclist : The cyclist should bend through an angle

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Kinematics of Uniform Circular Motion

Uniform Circular Motion

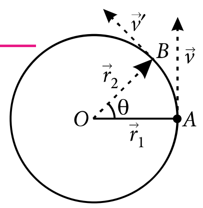
A particle moves in a circle at a constant speed.

- **Angular displacement :** It is denoted by θ. SI unit : rad or degree
- **Angular velocity (ω) :**

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relations between :

- s and $\theta \Rightarrow s = r\,\theta$
- v and $\omega \Rightarrow \vec{v} = \vec{\omega}\times\vec{r}$



Kinematics of Non-uniform Circular Motion

Tangential acceleration : $\vec{a}_t = \vec{\alpha}\times\vec{r}$

Centripetal acceleration : $\vec{a}_c = \vec{\omega}\times\vec{v}$

Non-uniform Circular Motion

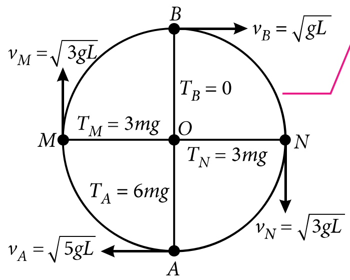
Speed of the particle in a horizontal circular motion changes with respect to time.

Motion in a Vertical Circle

Body is under the influence of gravity of earth and total mechanical energy is conserved.

Tension at any point on vertical loop :

$$T - mg\cos\theta = \frac{mv^2}{L}$$



Condition of Leaving the Circle

Tension in the string becomes zero before reaching the highest point if $\sqrt{2gL} < u < \sqrt{5gL}$

Condition of Oscillations

Particle will oscillate if velocity becomes zero the tension in the string is not zero.

$0 < u \leq \sqrt{2gL}$

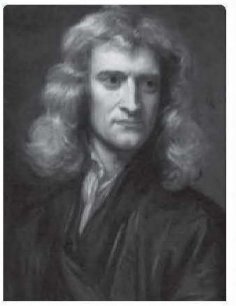
Condition of Looping the Loop

The particle will complete the circle if $u \geq \sqrt{5gL}$

BRAIN MAP

CLASS XI

LAWS OF MOTION


SIR ISSAC NEWTON
(1643-1727)

LAWS OF MOTION AND THEIR CONSEQUENCES

Newton's 1st Law
A body continues its state of rest or motion until unless an external force is acted on it.

Inertia of Direction
The property due to which a body cannot change its direction of motion by itself.

Inertia of Motion
The tendency of a body to remain in a state of uniform motion in a straight line.

Inertia of Rest
The property of a body due to which it cannot change its state of rest by itself.

Newton's 2nd Law
The rate of change of linear momentum of a body is directly proportional to the external force applied on the body in the direction of force.

Introduces the Concept of Force

Force = mass × acceleration

$F = ma$, SI unit ≡ N

Newton's 3rd Law
To every action there is always an equal and opposite reaction

$\vec{F}_{AB} = -\vec{F}_{BA}$

Impulse
A large force acts for a very short duration of time producing a finite change in momentum.

$J = F \times t = \Delta p$

Conservation of Linear Momentum
Total momentum of an isolated system of interacting particles is conserved if there is no external force acting on it. $P_{\text{initial}} = P_{\text{final}}$

Friction
Resistance offered to the relative motion between the surface of the two bodies in contact.

- Acceleration of a body sliding down a rough inclined plane, $a = g(\sin\theta - \mu\cos\theta)$
- Angle of friction, $\theta = \tan^{-1}(\mu_s)$.
- Angle of repose, $\alpha = \tan^{-1}(\mu_s)$.
- Numerically, $\alpha = \theta$

Types of Friction
(a) Static friction : acts when a body is at rest on application of a force, $f_s = F_{\text{ext}}$

(b) Limiting friction : acts when a body is just at the verge of movement, $f_l = \mu_s N$

(c) Kinetic friction : acts when a body is actually sliding, $f_l = \mu_s N$

CIRCULAR MOTION IN HORIZONTAL PLANE

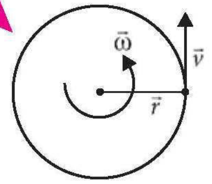
Circular Motion
A body moving in a circular path is called circular motion.

Centripetal force $F_e = \dfrac{mv^2}{r}$

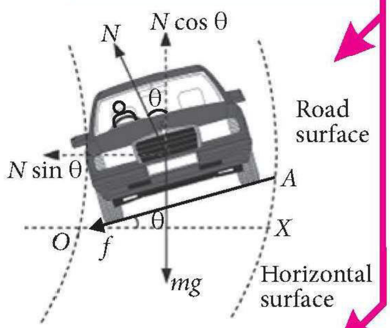
Non-uniform circular motion

Acceleration, $\vec{a} = \vec{a}_T + \vec{a}_C = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$

$a = \sqrt{a_T^2 + a_C^T}$



Motion of a Car on Banked Road
- Maximum velocity on banked road to avoid skidding
  $v_{\max} = \sqrt{\dfrac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}}$
- Optimum speed on a smooth banked road
  $v_0 = \sqrt{rg\tan\theta}$

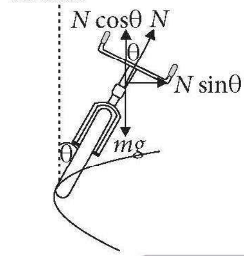


Motion of a Car on Level Road
Maximum velocity on a curved road to avoid skidding

$v_{\max} = \sqrt{\mu_s rg}$

Bending of a Cyclist on a circular turn
Angle of bending

$\theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right)$



Problem Solving Techniques in Mechanics
- Make free body diagram of each element showing all external forces acting on it.
- Analyse the magnitude and direction of pseudo forces if there is any.
- For equilibrium of concurrent forces use
  $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n = 0$
  $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$ (Lami's theorem)
- For horse cart type system
  $a = \dfrac{F_x - f}{M_H + M_{\text{cart}}}$ F_x = horizontal component of reaction, f = frictional force

BRAIN MAP
CLASS XI

NEWTON'S LAWS OF MOTION

Problem Solving Strategies

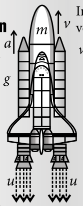
- Identify the unknown forces and accelerations.
- Draw FBD of bodies in the system.
- Resolve forces into their components.
- Apply $\Sigma\vec{F} = M\vec{a}$ in the direction of motion.
- Apply $\Sigma\vec{F} = 0$ in the direction of equilibrium.
- Write constraint relation if exists.
- Solve equations $\Sigma\vec{F} = M\vec{a}$ and $\Sigma\vec{F} = 0$.

Newton's 1st Law

A body continues its state of rest or motion until unless an external force is acted on it.

- Inertia of rest
- Inertia of motion
- Inertia of direction

Rocket Propulsion



Acceleration $a = \dfrac{u}{m}\dfrac{dm}{dt} - g$

Instantaneous velocity $v = u\log_e\left(\dfrac{m_0}{m_r}\right) - gt$

Burn out speed $v_{max} = u\log_e\left(\dfrac{m_0}{m_r}\right)$

Thrust $F = -u\dfrac{dm}{dt}$

Pseudo Force

$\vec{F}_{ext} + \vec{F}_{Pseudo} = M\vec{a}$

$\vec{F}_{pseudo} = -M\vec{a}_{frame}$

For non-inertial frame of reference

LAWS OF MOTION AND THEIR CONSEQUENCES

Newton's 2nd Law

The rate of change of linear momentum of a body is directly proportional to the external force applied on the body in the direction of force.

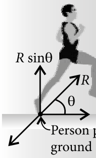
$F = \dfrac{dp}{dt} = ma$

Newton's 3rd Law

To every action there is always an equal and opposite reaction.

$\vec{F}_{AB} = -\vec{F}_{BA}$

Walking



Horse Cart Type System

For horse cart type system

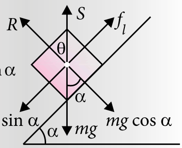
$a = \dfrac{F_x - f}{M_H + M_{cart}}$ $\left\{\begin{array}{l} F_x = \text{horizontal component of reaction force} \\ f = \text{frictional force}\end{array}\right.$

Angle of Friction (θ) and Angle of Repose (α)

$S = \sqrt{R^2 + f_l^2}$

$\tan\theta = \dfrac{f_l}{R} = \mu_s = \tan\alpha$

∴ Numerically, $\theta = \alpha$



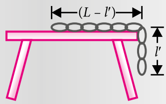
The motion resisted by a bonding between the body and the surface in contact represented by single force called

FRICTION

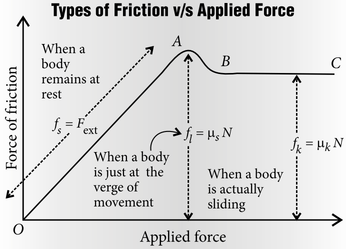
Maximum Length of Hanging Chain

Length of a chain hanging in air

$l' = \dfrac{\mu L}{1+\mu}$

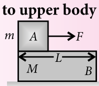


Types of Friction v/s Applied Force

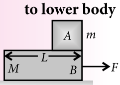


Motion of Two Bodies One Resting on the Other

Force F applied to upper body



Force F applied to lower body



Force F applied to upper body

- **When there is no friction**
  - $a_A = F/m;\ a_B = 0$
  - A will fall from B after time $t = \sqrt{\dfrac{2L}{a}} = \sqrt{\dfrac{2mL}{F}}$
- **Friction present between A and B ($F < f_l$)**
  - Combined system will move together with $a = F/(M+m)$
- **Friction present between A and B ($F > f_l$)**
  - Relative acceleration $a = a_A - a_B = \dfrac{MF - \mu_k mg(m+M)}{mM}$
  - A will fall from B after time $t = \sqrt{\dfrac{2L}{a}} = \sqrt{\dfrac{2mML}{MF - \mu_k mg(m+M)}}$

Force F applied to lower body

- **When there is no friction**
  - $a_B = F/M$ and $a_A = 0$
  - A will fall from B (backward) after time t ∴ $t = \sqrt{\dfrac{2L}{a}} = \sqrt{\dfrac{2ML}{F}}$
- **Friction present between A and B ($F < f_l$)**
  - Both the bodies will move together $a = \dfrac{F}{M+m}$ and $f_l = \mu_s mg$
  - Pseudo force on the body A, $F' = ma = \dfrac{mF}{m+M}$
- **Friction present between A and B ($F > f_l$)**
  - Relative acceleration $a = a_A - a_B = -\left[\dfrac{F - \mu_k g(m+M)}{M}\right]$
  - A will fall from B (backward) after time $t = \sqrt{\dfrac{2L}{a}} = \sqrt{\dfrac{2ML}{F - \mu_k g(m+M)}}$

BRAIN MAP

CLASS XI

WORK, ENERGY AND POWER

WORK

Work is said to be done whenever a force acts on a body and the body moves through some distance.

$W = \vec{F}\cdot\vec{S} = FS\cos\theta$ (where θ is the angle between force applied $\vec{F}$ and displacement vector $\vec{S}$.)

The SI unit of work is joule (J).

Nature of Work Done

If $\theta = 0°$, $W = FS$ *i.e.*, work done is maximum.

If $\theta = 90°$, $W = 0$ *i.e.*, work done is zero.

Work Done by a Variable Force

The work done by a variable force in changing the displacement from S_1 to S_2 is $W = \int_{S_1}^{S_2} \vec{F}\cdot d\vec{S}$ = Area under the force-displacement graph

ENERGY

It is defined as the ability of a body to do work. It is measured by the amount of work that a body can do. The unit of energy used at the atomic level is electron volt (eV) and SI unit is J.

Kinetic Energy

It is the energy possessed by a body by virtue of its motion. The K.E. of a body of mass m moving with speed v is

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Potential Energy

It is the energy possessed by a body by virtue of its position (in a field) or configuration (shape or size). For a conservative force in one dimension, the potential energy function $U(x)$ may be defined as

$$F(x) = -\frac{dU(x)}{dx} \text{ or } \Delta U = U_f - U_i = -\int_{x_i}^{x_f} F(x)dx$$

Power

The rate of doing work is called power.

Average Power:

It is defined as the ratio of the small amount of work done W to the time taken t to perform the work.

$$P = \frac{W}{t}$$

The SI unit of power is watt (W).

Work Energy Theorem

The work done by the net force acting on a body is equal to the change in kinetic energy of the body.

W = Change in kinetic energy

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \Rightarrow W = \Delta\text{K.E.}$$

The work energy theorem may be regarded as the scalar form of Newton's second law of motion.

Potential Energy of a Spring

According to Hooke's law, when a spring is stretched through a distance x, the restoring force F is such that

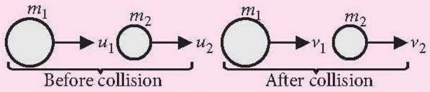
$F \propto x$ (where k is the spring constant or $F = -kx$ and its unit is N m^{-1}.)

The work done is stored as potential energy U of the spring

$$W = \int_0^x kx\,dx = \frac{1}{2}kx^2 \Rightarrow U = \frac{1}{2}kx^2$$

Head-on Collision or One-Dimensional Collision

It is a collision in which the colliding bodies move along the same straight line path before and after the collision.



Velocity of approach = Velocity of separation

or $u_1 - u_2 = v_2 - v_1$

Also, $v_1 = \frac{m_1 - m_2}{m_1 + m_2}\cdot u_1 + \frac{2m_2}{m_1 + m_2}\cdot u_2$ and

$v_2 = \frac{2m_1}{m_1 + m_2}\cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2}\cdot u_2$

COLLISION

A collision between two bodies is said to occur if either they physically collide against each other or the path of the motion of one body is influenced by the other.

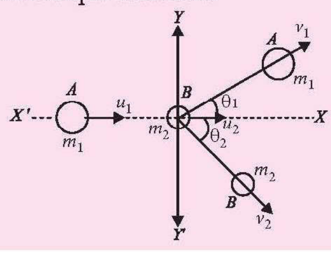
Types of Collision

Elastic collision : Both the momentum and kinetic energy of the system remain conserved.

Inelastic collision : Only the momentum of the system is conserved but kinetic energy is not conserved.

Oblique Collision

If the two bodies do not move along the same straight line path before and after the collision, the collision is said to be oblique collision.

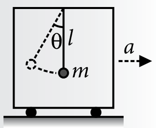


BRAIN MAP
CLASS XI

WORK AND ENERGY

Pendulum Suspended in an Accelerating Trolley

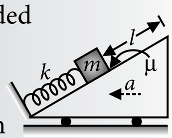
- For a pendulum suspended from the ceiling of a trolley moving with acceleration a, the maximum deflection θ of the pendulum from the vertical is $\theta = 2\tan^{-1}\left(\frac{a}{g}\right)$



Work Energy Theorem for Non-inertial Frames

For a block of mass m welded with light spring (relaxed) When the wedge fitted moves with an acceleration a, block slides through maximum distance l relative to wedge,

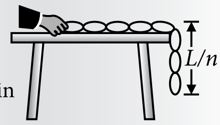
$$l = \frac{2m}{k}[a(\cos\theta - \mu\sin\theta) - g(\sin\theta + \mu\cos\theta)]$$



Work Done in Pulling the Chain

$$W = \frac{MgL}{2n^2}$$

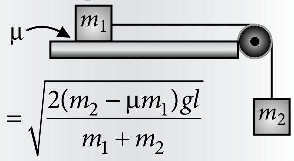
{M = Mass of chain
L = Length
n = Fraction of chain hanged}



Motion of Blocks Connected with Pulley

- Two blocks connected by a string, as shown. If they are released from rest. After they have moved a distance l, their common speed is

$$v = \sqrt{\frac{2(m_2 - \mu m_1)gl}{m_1 + m_2}}$$



Nature of Work Done

- Positive work ($0° \le \theta < 90°$)
  Component of force is parallel to displacement
- Negative work ($90° < \theta \le 180°$)
  Component of force is opposite to displacement
- Zero work ($\theta = 90°$)
  Force is perpendicular to displacement

Different cases explained using work energy theorem

Work Energy Theorem

Work done by a force acting on a body is equal to the change in the kinetic energy of the body. It is valid for a system in presence of all types of forces.

$W_{total} = \Delta K$

Work

Work done by a force (F) is equal to the scalar product of the force and the displacement (S) of the body.

$W = \vec{F}\cdot\vec{S}$ (θ is the angle between F and S)

$W = FS\cos\theta$

Energy

The energy of a body is defined as its capacity for doing work. Energy is a scalar quantity.

Unit and dimensions for both energy and work are same

Dimensions : $[ML^2T^{-2}]$

S.I. Unit : joule (J)

Work Depends on Frame of Reference

With change of the frame of reference (inertial), force does not change while displacement may change. So the work done by a force will vary in different frames.

Work Done by Friction

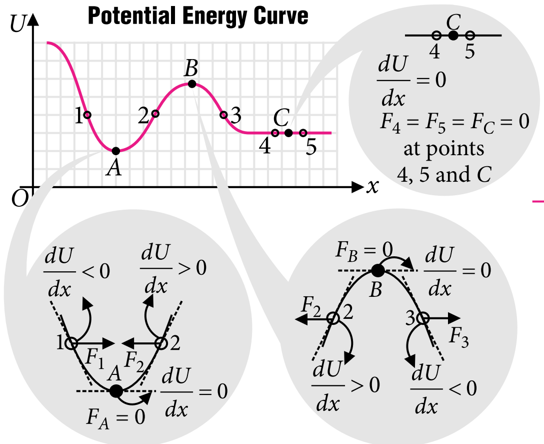
- Work done by static friction is always zero.
- Work done by kinetic friction on the system is always negative.

Work Done by a Spring Force

- Work done for a displacement from x_i to x_f

$$W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

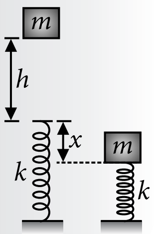
Potential Energy Curve



An Application of Conservation of Energy

- A block of mass m, falling from height h, on a mass less spring of stiffness k.
  - The maximum compression in the spring will be

$$x = \frac{mg}{k}\left[1 + \sqrt{1 + \frac{2kh}{mg}}\right]$$



- If block is released slowly ($h = 0$), maximum compression, $x = \frac{2mg}{k}$
- Work done in bringing the block to stable equilibrium, $W_{ext} = -\frac{m^2g^2}{2k}$

Potential Energy

It is the ability of doing work by a conservative force. It arises from the configuration of the system or position of the particles in the system.

Relation between Conservative Force and Potential Energy

Negative gradient of the potential energy gives force.

$$F = -\frac{dU}{dr}$$

BRAIN MAP

CLASS XI

COLLISION

CLASSIFICATION OF COLLISON

On the basis of line of impact

HEAD-ON COLLISION

The velocities of the particles are along the same line before and after the collision.

OBLIQUE COLLISION

The velocities of the particles are along different lines before and after the collision.

On the basis of kinetic energy

INELASTIC COLLISION

If the kinetic energy after and before collision are not equal, the collision is said to be inelastic.

PERFECTLY INELASTIC COLLISION

If velocity of separation just after collision becomes zero, then the collision is perfectly inelastic.

ELASTIC COLLISION

If the kinetic energy after and before collision is same, the collision is said to be perfectly elastic.

By substituting $e = 1$, we get $\Delta K = 0$

Head on Inelastic Collision

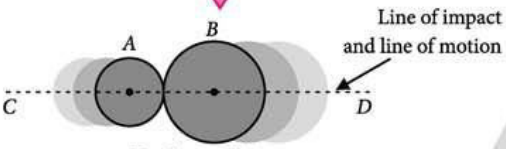
Velocity after collision :

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

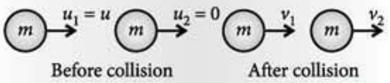
Loss in kinetic energy :

$$(\Delta K) = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(1-e^2)(u_1 - u_2)^2$$



Head-on collision

- **Velocities after inelastic collision :**



$$\therefore \frac{v_1}{v_2} = \frac{1-e}{1+e}$$

- **Coefficient of Restitution (e)**

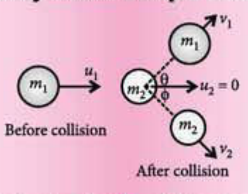
$$e = \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

Rebounding of Ball After Collision

- **After first rebound**
  - **Speed :** $v_1 = ev_0 = e\sqrt{2gh_0}$
  - **Height :** $h_1 = e^2 h_0$
- **After n^{th} rebound :**
  - **Speed :** $v_n = e^n v_0$
  - **Height :** $h_n = e^{2n} h_0$
- **Total distance travelled by the ball before it stops bouncing :**

  $H = h_0[(1+e^2)/(1-e^2)]$

Perfectly Elastic Oblique Collison



After perfectly elastic oblique collision of two bodies of equal masses, the scattering angle $(\theta + \phi)$ would be 90°.

In Case of Smooth Surfaces

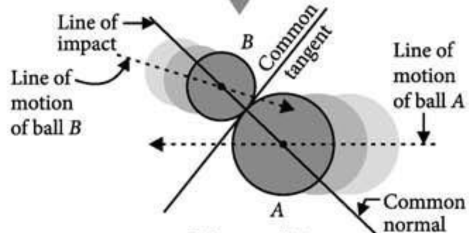
Common normal :

Force is exerted in common normal direction only. Momentum changes in common normal direction.

Common tangent :

$F = 0$

Neither momentum nor velocity changes in common tangent direction.

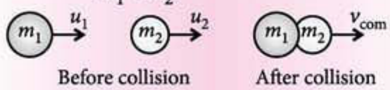


Oblique collision

Perfectly Inelastic Collision

- **When the colliding bodies are moving in the same direction :**

$$v_{com} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$



Loss in kinetic energy

$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

- **When the colliding bodies are moving in the opposite direction :**

$$\therefore v_{com} = \frac{m_1 u_1 - m_2 u_2}{m_1 + m_2}$$

Loss in kinetic energy

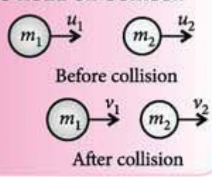
$$\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

Elastic or Perfectly Elastic Head on Collison

Velocity after collision :

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$$



Special Cases

If projectile and target are of same mass

For $m_1 = m_2 \Rightarrow v_1 = u_1$ and $v_2 = u_1$

i.e., their velocities get interchanged.

If massive projectile collides with a light target

For $m_1 >> m_2 \Rightarrow v_1 = u_1$ and $v_2 = 2u_1 - u_2$

Sub case : For $u_2 = 0$, *i.e.*, target is at rest

$v_1 = u_1$ and $v_2 = 2u_1$

If light projectile collides with a heavy target

For $m_1 << m_2 \Rightarrow v_1 = -u_1 + 2u_2$ and $v_2 = u_2$

Sub case : For $u_2 = 0$, *i.e.*, target is at rest

$v_1 = -u_1$ and $v_2 = 0$, the ball rebounds with same speed.

If $m_2 = nm_1$ and $u_2 = 0$

The fractional kinetic energy transferred by projectile

$$\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$$

Fractional kinetic energy retained by the projectile

$$\left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 - \text{fractional kinetic energy transferred by projectile}$$