

CHAPTER-2

Polynomials

KEY POINTS

Definition

A polynomial $p(x)$ in one variable x of degree n is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$, where

- (i) $a_0, a_1, a_2, \dots, a_n$ are constants and $a_n \neq 0$
- (ii) $a_0, a_1, a_2, \dots, a_n$ are respectively the coefficients of $x^0, x^1, x^2, \dots, x^n$ terms of the polynomial.
- (iii) Each of $a_n x^n, a_{n-1} x^{n-1}, a_{n-2} x^{n-2}, \dots, a_2 x^2, a_1 x, a_0$ are called terms of the polynomial.
- (iv) n is called the degree of the polynomial where n is a non-negative integer.

Zeros of Polynomial

For a polynomial $p(x)$ if $p(a) = 0$, where a is a real number we say that ' a ' is a zero of the polynomial.

Facts about Polynomial:

1. A polynomial having four or more than four terms does not have particular name. These are simply called polynomials.
2. A polynomial of degree four or more than four does not have any particular name. Such a polynomial is usually called a polynomial of degree four or five or ... etc.
3. The degree of zero polynomial is not defined or we can not determine the degree of zero polynomial.
4. A polynomial of degree ' n ' can have at most n zeroes.
5. A non-zero constant polynomial has no-zero.
6. Every real number is a zero of the zero polynomial.

Very Short Answer type Questions (1 Mark)

1. The coefficient of x^2 in the polynomial $4x^3 - 7x^2 + 2x + 1$ is :-
(a) 4
(b) 7
(c) -4
(d) -7

2. Which of the following is not a polynomial?
- (a) $x + 1$ (b) $\sqrt{x} + 1$
(c) $x^2 + 1$ (d) $\left(\frac{1}{x} + 1\right)x^2$
3. If $x = -1$ is a zero of $x^3 - 2x^2 + 3ax + 5$, then value of a is :-
- (a) 2 (b) $\frac{2}{3}$
(c) $\frac{3}{2}$ (d) -5
4. If $(x + 2)$ is a factor of $x^2 - kx + 14$, then find the value of k :-
- (a) -9 (b) 9
(c) -2 (d) 14
5. When $p(x) x^3 - 6x^2 + 2x - 4$ is divided by $x - 2$ then remainder is :-
- (a) 16 (b) 24
(c) -16 (d) -24
6. If the side of a square is $(x + 2y - z)$ units, then its area is _____.
7. The polynomial $x^2 - a^2$ has _____ zeroes.
8. A quadratic polynomial can have at most _____ terms.
9. $(49)^3 - (30)^3 + \underline{\hspace{2cm}} = 3 \times 49 \times 30 \times 19$
10. $x^3 - 64$ is a polynomial of degree _____ having _____ terms.
11. Check whether $x = 3$ is a zero of the polynomial $x^3 - 3x^2 + x - 3$
12. If $p + q + r = 9$, then find the value of $(3 - p)^3 + (3 - q)^3 + (3 - r)^3$.
13. Find the remainder when $x^3 + 3x^2 + 2x$ is divided by x .
14. If $f(x) = x^2 - 3$, then find $f(1) + f(-1)$
15. Find the sum of coefficient of x^2 and coefficient of x in the polynomial $3x^3 - 4x^2 + 5x + 2$

Short Answer Type-I Questions (2 Marks)

16. Check whether $q(x)$ is a multiple of $r(x)$ or not.
Where $q(x) = 2x^3 - 11x^2 - 4x + 5$, $r(x) = 2x + 1$.
17. Show that $(x - 5)$ is a factor of $x^3 - 3x^2 - 4x - 30$.
18. Evaluate by using suitable identity: $(997)^2$
19. Find the zeroes of the polynomial $p(x) = x(x - 2)(x + 3)$
20. Find the remainder when $3x^2 - 7x - 6$ is divided by $(x - 3)$
21. Factorise : $8x^3 + \sqrt{27} y^3$
22. If $p(x) = x + 9$, then find $p(x) + p(-x)$
23. Find the product without multiplying directly 106×94
24. The factors of $x^2 - 18x + 9$ are $(x + b)$ and $(x + b)$. Find the values of a and b .
25. Find $p(1) + p(-1) + p(10)$ if $p(x) = x^2 - 3x + 2$
26. Find $(x - y)^2$ if $\frac{x}{y} + \frac{y}{x} = 2$
27. Show that -1 is a zero of $3x^4 - x^3 + 3x - 1$.
28. Multiply $(x + 1)(x - y)$

Short Answer Type-II Questions (3 Marks)

29. Factorise: $64a^2 + 96ab + 36b^2$
30. Factorise: $x^3 + 6x^2 + 11x + 6$
31. If $x^2 + y^2 = 49$ and $x - y = 3$, then find the value of $x^3 - y^3$.
32. Simplify: $(5a - 2b)(25a^2 + 10ab + 4ab^2) - (2a + 5b)(4a^2 - 10ab + 25b^2)$
33. Find the sum of remainders when $x^3 - 3x^2 + 4x - 4$ is divided by $(x - 1)$ and $(x + 2)$.
34. Find the product of $\left(p - \frac{1}{p}\right)\left(p + \frac{1}{p}\right)\left(p^2 + \frac{1}{p^2}\right)\left(p^4 + \frac{1}{p^4}\right)$
35. Factorise: $7\sqrt{2}k^2 - 10k - 4\sqrt{2}$
36. Simplify: $(3x - 4y)^3 - (3x + 4y)^3$
37. Simplify: $(x + y + z)^2 - (x - y - z)^2$.

38. Factorise: $125x^3 + 8y^3 + z^3 - 30xyz$
39. $x + 2$ is a factor of polynomial $ax^2 + bx^2 + x - 2$ and the remainder 4 is obtained on dividing this polynomial by $(x - 2)$. Find the value of a and b .
40. If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x - 3)$, find the value of a .
41. If $\left(\frac{9}{10}\right)^3 - \left(\frac{2}{5}\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{x}{50}$, find x
42. If $(x - 3)$ and $\left(x - \frac{1}{3}\right)$ are factors of the polynomial $px^2 + 3x + r$, show that $p = r$.

Long Answer type Questions (5 Marks)

43. A literacy campaign was organised by Class IX girl students under NSS. Students made $(x - 5)$ rows and $(3x - 4)$ columns for the rally. Write the total number of students in the form of a polynomial.
44. (i) Using identity, find the value of $(-7)^3 + (5)^3 + (2)^3$.
(ii) Find dimensions of cuboid whose volume is given by the expression $4x^2 + 14x + 6$.
45. If $a + b + c = 0$, find the value of $\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab}$
46. Simplify: $\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$
47. Factorize $(2a - b - c)^3 + (2b - c - a)^3 + (2c - a - b)^3$
48. If the polynomial $4x^3 - 16x^2 + ax + 7$ is exactly divisible by $x - 1$, then find the value of a . Hence factorise the polynomial.
49. If $\frac{x}{y} + \frac{y}{x} = -1$ where $x \neq 0, y \neq 0$ then find the value of $x^3 - y^3$
50. Simplify: $\frac{155 \times 155 + 155 \times 55 + 55 \times 55}{155 \times 155 \times 155 - 55 \times 55 \times 55}$

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Answer

1. (d) -7
2. (b) $\sqrt{x} + 1$
3. (b) $\frac{2}{3}$
4. (a) -9
5. (c) -16
6. $x^2 + 4y^2 + z^2 + 4xy - 4yz - 2xz$
7. Two
8. Three
9. $(-19)^3$
10. 3, 2
11. Yes
12. $p + q + r = 9$
 $(3 - p) + (3 - q) + (3 - r) = 0$
 $\therefore (3 - p)^3 + (3 - q)^3 + (3 - r)^3$
 $= 3(3 - p)(3 - q)(3 - r)$
13. 0
14. $f(1) + f(-1)$
 $= (-2) + (-2) = -4$
15. $(-4) + (5) = 1$
16. Since, $q\left(\frac{-1}{2}\right) = 4 \neq 0$
 $\therefore r(x)$ is not a multiple of $q(x)$.

17. Put $x = 5$ in given polynomial
18. 994009
19. 0, 2, -3
20. 0
21. $(2x + \sqrt{3}y)(4x^2 - 2\sqrt{3}xy + 3y^2)$
22. 18
23. $(100 + 6)(100 - 6) = 9964$
24. $a = 5, b = -3$
25. 78
26. 0
28. $x^2 - xy + x - y$
29. $(8a + 6b)^2$
30. $(x + 1)(x + 2)(x + 3)$
31. 207
32. $117a^3 - 133b^3$
33. -34
34. $p^8 - \frac{1}{p^8}$
35. $(k - \sqrt{2})(7\sqrt{2}k + 4)$
36. $-128y^3 - 216x^2y$
37. $4xy + 4xz$
38. $(5x + 2y + z)(25x^2 + 4y^2 + z^2 - 10xy - 2yz - 5zx)$

39. $a = 0, b = 1$

40. $a = -1$

41. $x = 27$, {use, if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$ }

43. $3x^2 - 19x + 20$

44. (i) -210 , (ii) $2, (x + 3), (2x + 1)$

45. 3

46. $(a + b)(b + c)(c + a)$

47. $3(2a - b - c)(2b - c - a)(2c - a - b)$

48. $a = 5, (x - 1)(2x + 1)(2x - 7)$

49. 0

50.
$$\frac{(155)^2 + 155 \times 55 + (55)^2}{(155)^3 - (55)^3} = \frac{(155)^2 + 155 \times 55 + (55)^2}{(155 - 55)((155)^2 + 155 \times 55 + (55)^2)}$$
$$= \frac{1}{100} = 0.01$$

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POLYNOMIALS
PRACTICE TEST

Time: 1 hr.

M.M. 20

1. Show that $x = 1$ is a zero of the polynomial $3x^3 - 4x^2 + 8x - 7$. (1)
2. Find the value of the polynomial $2x + 5$ at $x = -3$. (1)
3. Find the zeroes of the polynomial $x^2 - 4x + 3$. (2)
4. If $x + y + z = 6$, $xy + yz + zx = 11$. Find the value of $x^2 + y^2 + z^2$. (2)
5. If $3x - 4$ is a factor of the polynomial $p(x) = 2x^3 - 11x^2 + kx - 20$, find the value of k . (3)
6. Factorise: $a^2 + b^2 + 2(ab + bc + ca)$ (3)
7. Factorise: $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$ (3)
8. Factorise: (5)
 - (i) $4x^2 + 20x + 25$
 - (ii) $6x^2 + 7x - 3$