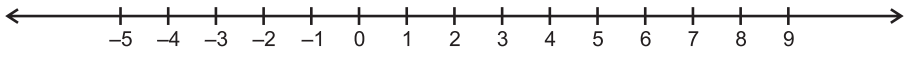
CHAPTER-1

NUMBER SYSTEMS

KEY POINTS



- 1, 2, 3, ... are natural numbers which are represented by N.
- 0, 1, 2, 3, ... are whole numbers which are represented by W.
- ..... –3. –2, –1, 0, 1, 2, 3 ..... are integers which are represented by Z or I.
- A number is a rational if

  (a) it can be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  OR

  (b) its decimal expansion is terminating (e.g. $\frac{2}{5} = 0.4$)

  OR

  (c) its decimal expansion is non-terminating recurring (repeating) (e.g. $0.\overline{1234} = 0.1234234.....$)

- A number is irrational number if

  (a) it can not be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  OR

  (b) its decimal expansion is non-terminating non-recurring (e.g. 0.1010010001.....)

- All rational and irrational numbers collectively form real numbers.
- There are infinite rational numbers between any two rational numbers.
- There is a unique real number corresponding to every point on the number line. Also, corresponding to each real number, there is a unique point on the number line.
- Rationalisation of a denominator means to change the Irrational denominator to rational form.

- To rationalise the denominator of $\frac{1}{\sqrt{a} \pm b}$, we multiply this by $\frac{\sqrt{a} \mp b}{\sqrt{a} \mp b}$, where a is a natural number and b is an integer.
- If r is rational and s is irrational then $r + s$, $r - s$, $r \cdot s$ are always irrational numbers but $\frac{r}{s}$ may be rational or irrational. For $r \neq 0$, $r \cdot s$ and $\frac{r}{s}$ are always irrational.
- Law of Exponents: Let $a > 0$ be a real number and m and n are rational numbers, then

  (1) $a^m \, a^n = a^{m+n}$ (2) $a^m \div a^n = a^{m-n}$

  (3) $(a^m)^n = a^{mn}$ (4) $a^m \, . \, b^m = (ab)^m$

  (5) $a^0 = 1$ (6) $a^{-m} = \frac{1}{a^m}$

- For positive real numbers a and b, the following identities hold

  (1) $\sqrt{a} \, . \sqrt{b} = \sqrt{ab}$ (2) $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

  (3) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$ (4) $\left(\sqrt{a} + \sqrt{b}\right)^2 = a + 2\sqrt{ab} + b$

  (5) $\left(a + \sqrt{b}\right)\left(a - \sqrt{b}\right) = a^2 - b$

All natural numbers, whole numbers and integers are rational

Prime Numbers: All natural numbers that have exactly two factors (i.e., 1 and itself) are called prime numbers, e.g., 2, 3, 5, 7, 11, 13, 17, 19, 23, ... etc.

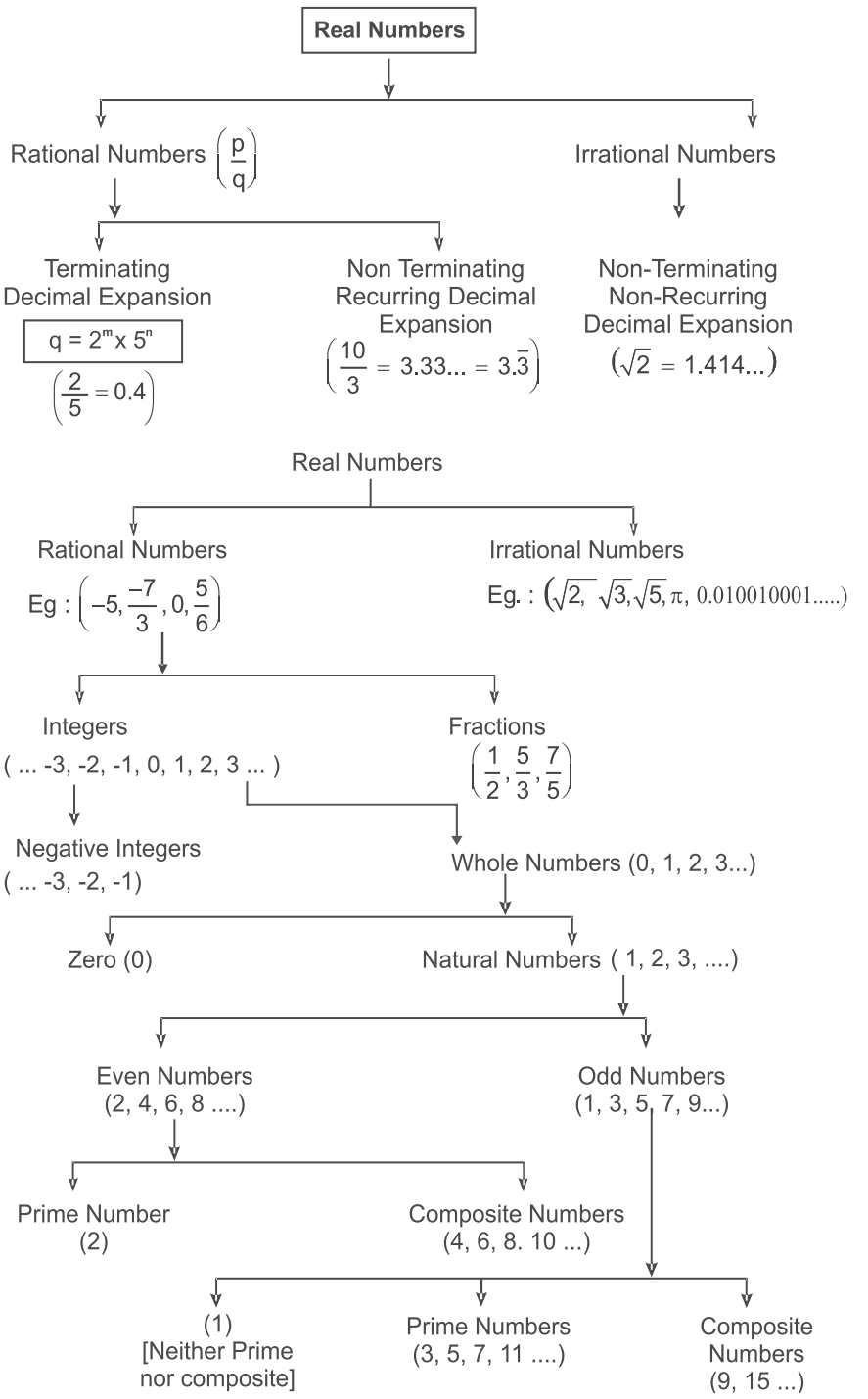
Composite Numbers: Those natural numbers which have more than two factors are known as composite numbers. e.g., 4, 6, 8, 9, 10, 12, ...

1 is neither prime nor composite.

$\sqrt[n]{a} = a^{1/n}$ where 'a' is positive real number and n is a positive integer

$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ where a is positive real number m and n are co-prime integers and $n > 0$

Types of Numbers



Very Short Answer Questions (1 mark)

1. Which of the following is a rational number?

(a) $1 + \sqrt{5}$ (b) $2\sqrt{3}$

(c) 0 (d) π

2. Which of the following is irrational?

(a) $\sqrt{\frac{4}{9}}$ (b) $\frac{\sqrt{12}}{\sqrt{3}}$

(c) $\sqrt{81}$ (d) $\sqrt{5}$

3. If $x = 2 + \sqrt{3}$ then $(1/x)$ is equal to

(a) $2 + \frac{1}{\sqrt{3}}$ (b) $\frac{1}{2-\sqrt{3}}$

(c) $2 - \sqrt{3}$ (d) $\frac{1}{2} + \sqrt{3}$

4. An irrational number between $\sqrt{2}$ and $\sqrt{3}$ is

(a) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (b) $\frac{-\sqrt{2}+\sqrt{3}}{2}$

(c) $\sqrt{2} \times \sqrt{3}$ (d) $\sqrt{5}$

5. If $5^{2y} = 25$ then 5^{-y} is equal to

(a) $\frac{-1}{5}$ (b) $\frac{1}{50}$

(c) $\frac{1}{625}$ (d) $\frac{1}{5}$

Fill in the blanks:

6. $\sqrt{6} \times \sqrt{8}$ = _____

7. The decimal expansion of the number $\sqrt{3}$ is _____ and _____

8. _____ is a whole number but not a natural number.

9. $\sqrt[2]{(81)^{0.50}}$ = _____

10. Between two distinct rational number there lie _____ rational numbers.

11. The sum and difference of rational and irrational number is always _____ numbers.

12. Every rational number is a _____ number.

13. Find a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$.

14. Express $0.\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

15. Find the value of $0.\overline{23} + 0.\overline{22}$ in the form $\frac{p}{q}$, where p & q are integres and $q \neq 0$.

16. Find the value of x, if $5^{x-3} . 3^{2x-8} = 225$

17. Find the value of $[(4 - 5(4 - 5)^4]^3$

18. Write first five whole numbers in $\frac{p}{q}$ form, where p and q are integers and $q \neq 0$.

19. Find two irrational numbers between $\sqrt{25}$ and $\sqrt{27}$.

20. Write two numbers whose decimal expansions are terminating.

21. Find the value of $(256)^{0.16} \times (256)^{0.09}$

22. Evaluate $\left(\frac{3}{5}\right)^3 \times \left(\frac{5}{3}\right)^5$

23. What can be the maximum number of digits in the repeating block of digits in the decimal expansion of $\frac{5}{7}$.

Short Answer Type-I Questions (2 Marks)

24. Represent following on number line

(a) $\frac{-7}{5}$ (b) $\sqrt{3}$

25. Find the value of x, $\sqrt[3]{2x+3} = 5$

26. Express the mixed recurring decimal $1.\overline{27}$ in the form $\frac{p}{q}$.

27. Simplify $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{80}+\sqrt{48}-\sqrt{45}-\sqrt{27}}$

28. Which of the following rational numbers will have a terminating decimal expansion or a non-terminating repeating (recurring) decimal expansion?

(a) $\frac{135}{50}$ (b) $\frac{4}{11}$

(c) $\frac{5^2 \times 3^3}{2 \times 5^3 \times 27}$ (b) $\frac{55}{9}$

29. Classify the numbers as terminating decimal or non-terminating recurring decimal or non-terminating non-recurring decimals.

(a) 0.1666 (b) 0.27696

(c) 2.142857142857..... (d) 2.502500250002.....

(e) $4.\overline{123456789}$

Also classify these numbers as rational and irrational numbers.

30. Classify the following numbers as rational or irrational numbers.

(a) $\frac{7\sqrt{7}}{\sqrt{343}}$ (b) $5+2\sqrt{23}-\left(\sqrt{25}+\sqrt{92}\right)$

(c) $\sqrt{360}$ (d) $\frac{22}{7}$

(e) π

31. Solve

(a) Add $\sqrt{125} + 2\sqrt{27}$ and $-5\sqrt{5} - \sqrt{3}$

(b) Multiply $\left(-3+\sqrt{5}\right)$ and $\left(7+\sqrt{3}\right)$

(c) Divide $2\sqrt{216} - 3\sqrt{27}$ by 3

Short Answer Type-II Questions (3 Marks)

32. If $\dfrac{3+2\sqrt{5}}{3-2\sqrt{5}} = p + q\sqrt{5}$, then find the value of 11 $(p + q)$

33. Simplify $\dfrac{(25)^{5/2} \times (81)^{1/4}}{(125)^{2/3} \times (27)^{2/3} \times 8^{4/3}}$

34. If $32^{2x-5} = 4 \times 8^{x-5}$ then find the value of x.

35. Evaluate

(a) $\dfrac{2^{38} + 2^{37} + 2^{36}}{2^{39} + 2^{38} + 2^{37}}$

(b) $\left(9 + \sqrt{2} - \sqrt{3}\right)^2$

(c) $\left[5\left(8^{1/3} + 27^{1/3}\right)^7\right]^{1/4}$

(d) $\left(6-\sqrt{2}\right)\left(2+\sqrt{3}\right)$

36. If $5^{2x-1} - (25)^{x-1} = 2500$ then find the value of x?

37. If $x = 3 - 2\sqrt{2}$, show that $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right) = \pm 2$

38. If xyz = 1 then simplify

$$\left(1 + x + y^{-1}\right)^{-1} + \left(1 + y + z^{-1}\right)^{-1} + \left(1 + z + x^{-1}\right)^{-1}$$

39. Find the value of x if

(a) $25^{2x-3} = 5^{2x+3}$

(b) $(4)^{2x-1} - (16)^{x-1} = 384$

40. Solve

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$$

41. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Long Answer type Questions (5 marks)

42. Evaluate $\frac{64^{\frac{a}{6}}}{4^a} \times \frac{2^{2a+1}}{2^{a-1}}$

43. Simplify $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}$

44. Simplify $\left(\frac{x^a}{x^{-b}}\right)^{a-b} \times \left(\frac{x^b}{x^{-c}}\right)^{b-c} \times \left(\frac{x^c}{x^{-a}}\right)^{c-a}$

45. Show that $\frac{7\sqrt{3}}{\left(\sqrt{10}+\sqrt{3}\right)}-\frac{2\sqrt{5}}{\left(\sqrt{6}+\sqrt{5}\right)}-\frac{3\sqrt{2}}{\left(\sqrt{15}+3\sqrt{2}\right)}=1$

46. Show that $a=\frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}}$ and $b=\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}$, then find the value of $a^2 + b^2 + ab$

47. If $x = 9 - 4\sqrt{5}$ then find

(a) $x+\frac{1}{x}$ (b) $x-\frac{1}{x}$

(c) $x^2+\frac{1}{x^2}$ (d) $x^2-\frac{1}{x^2}$

(e) $x^3+\frac{1}{x^3}$ (f) $x^3-\frac{1}{x^3}$

(g) $\sqrt{x}+\frac{1}{\sqrt{x}}$

(h) $\sqrt{x}-\frac{1}{\sqrt{x}}$

(i) $x+\frac{14}{x}$

48. If $P = 5-2\sqrt{6}$ find

(a) $P^2+\frac{1}{P^2}$

(b) $P^2-\frac{1}{P^2}$

(c) $P^4+\frac{1}{P^4}$

49. Find the value of $\frac{4}{(216)^{-2/3}}+\frac{1}{(256)^{-3/4}}+\frac{2}{(243)^{-1/5}}$

50. If $\frac{9^n\times 3^2\times\left(3^{-n/2}\right)^{-2}-(27)^n}{3^{3m}\times 2^3}=\frac{1}{729}$ then prove that $m-n=2$

51. If $x = 2^y$ and $\frac{9\times 3^{2x}-3^x\times 3^{x-2}}{2}=360$. Find the value of y.

52. If $a = 2$, $b= 3$ then find the values of the following

(a) $(a^b+b^a)^{-1}$

(b) $(a^a+b^b)^{-1}$

53. If $ab + bc + ca = 0$, find the value of $\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab}$

CHAPTER-1
NUMBER SYSTEM
ANSWERS

1. (c) 0

2. (d) $\sqrt{5}$

3. (c) $2-\sqrt{3}$

4. (a) $\frac{\sqrt{2}+\sqrt{3}}{2}$

5. (d) $\frac{1}{5}$

6. $4\sqrt{3}$

7. Non-terminating and non-repeating

8. 0

9. 3

10. Infinite

11. Irrational

12. Real

13. Hint: $\frac{a+b}{2}$ or make denominators equal

$\frac{1}{12}$: (other answers are also possible)

14. $\frac{7}{9}$

15. $\frac{5}{11}$

16. Hint: Compare powers

$x = 5$

17. –1

18. $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}$

19. $\sqrt{25} = 5$

$\sqrt{27} = 3\sqrt{3} = 3 \times 1.732 = 5.196$

Two irrational No. 5.012301234012345.....

5.1378424134876.....

(other answers are also possible)

20. $\frac{17}{5}, \frac{43}{10}$ (other answers are also possible)

21. 4

22. $\left(\frac{5}{3}\right)^2$

23. 6

25. Hint: cubing on both sides

$$\left(\sqrt[3]{2x+3}\right)^3 = 5^3$$

$$x + 3 \qquad 2 = 125$$

$$x = 61$$

26. $\frac{14}{11}$

27. 1

28. (a) Terminating decimal

(b) Non-terminating but recurring decimal

(c) Hint: simplify it first

Terminating decimal

(d) Non-terminating but recurring decimal

29. (a) Terminating decimal/Rational number

(b) Terminating decimal/Rational number

(c) Non-terminating but repeating/Rational number

(d) Non-terminating non-Repeating/Irrational number

(e) Non-terminating but Repeating/Rational number.

30. (a) Rational

(b) Rational

(c) Irrational

(d) Rational

(e) Irrational

31. (a) $5\sqrt{3}$

(b) $-21-3\sqrt{3}+7\sqrt{5}+\sqrt{15}$

(c) $4-3\sqrt{3}$

32. Hint: Rationalise the denominator

$$p=\frac{-29}{11},$$

$$q=\frac{-12}{11}$$

-41

33. Hint: $\dfrac{\left(5^2\right)^{5/2}\times\left(3^4\right)^{1/4}}{5^2\times3^2\times2^4}=\dfrac{5^3}{3\times2^4}=\dfrac{125}{48}$

34. Hint:

$$2^{5(2x-5)} = 2^2 \times 2^{3(x-5)}$$

$$2^{10x-25} = 2^{3x-15+2}$$

$x-25$ $\qquad$ 10 $\qquad$ $= 3x-13$

$$x = \frac{12}{7}$$

35. (a) Hint : $\dfrac{2^{36}\left(2^2+2^1+1\right)}{2^{37}\left(2^2+2^1+1\right)}=\dfrac{1}{2}$

(b) Hint: $(9)^2+\left(\sqrt{2}-\sqrt{3}\right)^2+2\times 9\left(\sqrt{2}-\sqrt{3}\right)=2\left(43-\sqrt{6}+9\sqrt{2}-9\sqrt{3}\right)$

(c) 25

(d) $12+6\sqrt{3}-2\sqrt{2}-\sqrt{6}$

36. Hint:

$$5^{2x-1}-5^{2(x-1)}=5^4\times 2^2$$

$$5^{2x-1}\frac{-5^{2x-1}}{5}=5^4\times 2^2$$

$$x=3$$

37. Hint:

$$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2=x+\frac{1}{x}-2=4$$

$$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)=\pm 2$$

38. Hint: replace

$$y=\frac{1}{xz}$$

$$=(1+x+xz)^{-1}+\left(1+\frac{1}{xz}+\frac{1}{z}\right)^{-1}+\left(1+z+\frac{1}{x}\right)^{-1}$$

$$=\frac{1}{1+x+xz}+\left(\frac{xz+1+x}{x^2}\right)^{-1}+\left(\frac{x+xz+1}{x}\right)^{-1}$$

$$=\frac{1}{1+x+xz}+\frac{xz}{1+x+xz}+\frac{x}{1+x+xz}$$

$$=\frac{1+zx+x}{1+x+xz}=1$$

39. (a) Hint:

$$5^{2(2x-3)} = 5^{2x+3}$$

$$x = \frac{9}{2}$$

(b) Hint:

$$2^{2(2x-1)} - 2^{4(x-1)} = 2^7 \times 3$$

$$2^{4x-2} - 2^{4x-4} = 2^7 \times 3$$

$$2^{4x-2}(1 - 2^{-2}) = 2^7 \times 3$$

$$x = \frac{11}{4}$$

40. Hint:

$$\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{1-2} = -(1-\sqrt{2})$$

$$= \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \sqrt{5} - \sqrt{4} + \sqrt{6} - \sqrt{5} + \sqrt{7} - \sqrt{6} + \sqrt{8} - \sqrt{7} + \sqrt{9} - \sqrt{8}$$

$$= \sqrt{9} - 1 = 3 - 1 = 2$$

41. $\frac{167}{90}$

42. 4

43. 1

44. 1

45. 1

46. Hint:

$$a = 13 - 2\sqrt{42}$$

$$b = 13 + 2\sqrt{42}$$

$$(a+b)^2 - ab = a^2 + b^2 + ab$$

$$a^2 + b^2 + ab = \left(13 - 2\sqrt{42} + 13 + 2\sqrt{42}\right)^2 - \left(13 - 2\sqrt{42}\right)\left(13 + 2\sqrt{42}\right)$$

$$a^2 + b^2 + ab = (26)^2 - (169 - 168)$$

$$= 676 - 1 = 675$$

47. (a) 18

(b) $-8\sqrt{5}$

(c) 322

(d) $-144\sqrt{5}$

(e) Hint:

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)$$
$$=18^3-3\times 18=5778$$

(f) Hint:

$$x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)$$
$$=(-8\sqrt{5})^3+3\times-8\sqrt{5}$$
$$=-2584\sqrt{5}$$

(g) $2\sqrt{5}$

(h) 4

(i) $135+52\sqrt{5}$

48. (a) 98

(b) Hint: $P^2-\frac{1}{P^2}=\left(P+\frac{1}{P}\right)\left(P-\frac{1}{P}\right)=-40\sqrt{6}$

(c) Hint: $P^4+\frac{1}{P^4}=\left(P^2+\frac{1}{P^2}\right)^2-2=9602$

49. 214

50. Hint:

$$\frac{3^{2n}\times 3^2\times 3^{\frac{-n}{2}\times -2}-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{729}$$

$$\frac{3^{2n+2+n}-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{729}$$

$$3^{3n-3m}=3^{-6}$$

$$n-m=-2$$

$$\therefore \quad m-n=2$$

51. Hint:

$$\frac{3^2\times 3^{2x}-3^x\times 3^{x-2}}{2}=360$$

$$\frac{3^{2x}\left(3^2-3^{-2}\right)}{2}=360$$

$$3^{2x}=81$$

$$x=2$$

$$y=1$$

52. (a) $\frac{1}{17}$

(b) $\frac{1}{31}$

53. Hint: $ab=-(bc+ca)$; $bc=-(ca+ab)$; $ca=-(ab+bc)$

$$=\frac{1}{a^2+ac+ab}+\frac{1}{b^2+ab+bc}+\frac{1}{c^2+bc+ca}$$

$$=\frac{1}{a(a+b+c)}+\frac{1}{b(a+b+c)}+\frac{1}{c(a+b+c)}$$

$$=0$$

CHAPTER-1
NUMBER SYSTEM
PRACTICE TEST

Time: 1 hr **M.M: 20**

1. Write one rational number and one irrational number. (1)

2. If $p = 5 - 2\sqrt{6}$ then find the value of $\frac{1}{p}$. (1)

3. Simplify $4\sqrt{3} + 3\sqrt{48} - \frac{5}{2}\sqrt{12}$ (2)

4. If $(5)^{2x-1} - (25)^{x-1} = 2500$ then find the value of x. (2)

5. Find the value of x and y

$$\frac{\sqrt{11} - \sqrt{7}}{\sqrt{11} + \sqrt{7}} = x - y\sqrt{77}$$

(3)

6. Represent $\left(2 + \sqrt{3}\right)$ on number line (3)

7. Simplify:

$$\frac{16 \times 2^{a+1} - 4 \times 2^{a}}{16 \times 2^{a+2} - 2 \times 2^{a+2}}$$

(3)

8. Express the following in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$

$0.\overline{4} + 0.1\overline{8} + 0.\overline{2}$ (5)