CHAPTER – 2

RELATIONS AND FUNCTIONS

KEY POINTS

- **Ordered Pair:** An ordered pair consists of two objects or elements in a given fixed order.

  Remarks: An ordered pair is not a set consisting of two elements. The ordering of two elements in an ordered pair is important and the two elements need not be distinct.

- **Equality of Ordered Pair:** Two ordered pairs (x_1, y_1) & (x_2, y_2) are equal if $x_1 = x_2$ and $y_1 = y_2$.

  i.e. $(x_1, y_1) = (x_2, y_2) \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$

- **Cartesian product of two sets:** Cartesian product of two non-empty sets A and B is given by $A \times B$ and $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$.

- **Cartesian product of three sets:** Let A, B and C be three sets, then $A \times B \times C$ is the set of all ordered triplet having first element from set A, 2nd element from set B and 3rd element from set C.

  i.e., $A \times B \times C = \{(x, y, z) : x \in A, y \in B \text{ and } z \in C\}$.

- **Number of elements in the Cartesian product of two sets:** If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

- **Relation:** Let A and B be two non-empty sets. Then a relation from set A to set B is a subset of $A \times B$.

- **No. of relations:** If n(A) = p, n(B) = q then no. of relations from set A to set B is given by 2^{pq}.
- **Domain of a relation:** Domain of R = {a : (a,b) $\in$ R}
- **Range of a relation:** Range of R = { b : (a,b) $\in$ R}
- Co-domain of R from set A to set B = set B.
- Range $\subseteq$ Co-domain
- **Relation on a set:** Let A be non-empty set. Then a relation from A to A itself. i.e., a subset of A × A, is called a relation on a set.
- **Inverse of a relation:** Let A, B be two sets and Let R be a relations from set A to set B.

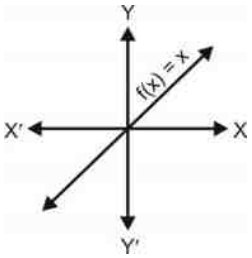
  Then the inverse of R denoted R^{-1} is a relation from set B to A and is defined by R^{-1} = {(b, a) : (a, b) $\in$ R}
- **Function:** Let A and B be two non-empty sets. A relation from set A to set B is called a function (or a mapping or a map) if each element of set A has a unique image in set B.

  Remark: If (a, b) $\in$ f then 'b' is called the image of 'a' under f and 'a' is called pre-image of 'b'.
- **Domain and range of a function:** If a function 'f' is expressed as the set of ordered pairs, the domain of 'f' is the set of all the first components of members of f and range of 'f' is the set of second components of member of 'f'.

  i.e., D_f = {a : (a, b) $\in$ f} and R_f = {b : (a, b) $\in$f}
- **No. of functions:** Let A and B be two non-empty finite sets such that n(A) = p and n(B) = q then number of functions from A to B = q^p.
- **Real valued function:** A function f : A $\rightarrow$ B is called a real valued function if B is a subset of R (real numbers).

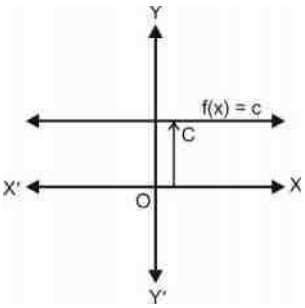
- **Identity function:** f : R→R given by f(x) = x ∀ x ∈ R (real number)

  Here, D_f = R and R_f = R



- **Constant function:** f : R → R given by f(x) = c for all x ∈ R where c is any constant

  Here, D_f = R and R_f = {c}

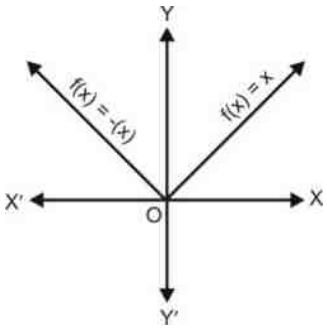


- **Modulus function:** f : R → R given by f(x) = |x| ∀ x ∈ R

  Here, D_f = R and R_f = [0, ∞)

  Remarks : $\sqrt{x^2} = |x|$

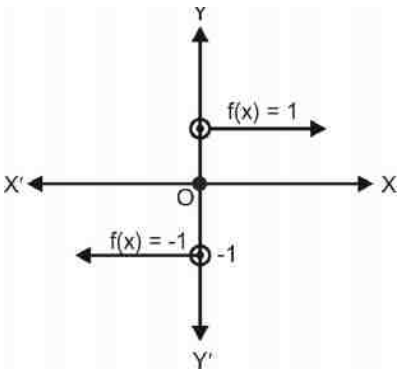
  or $f(x) = |x| = \begin{bmatrix} x : x \geq 0 \\ -x : x < 0 \end{bmatrix}$



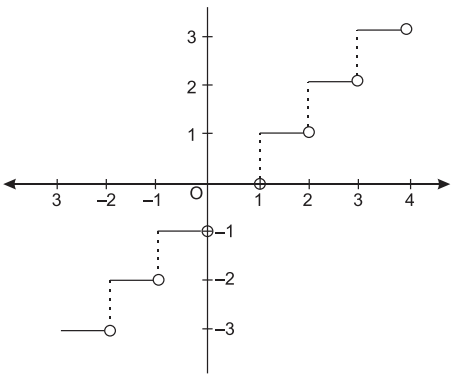
- **Signum function:** f : R $\to$ R defined by $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Or

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

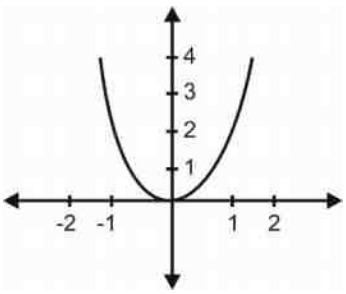


- **Greatest Integer function:** f : R $\to$ R defined by f(x) = [x], x $\in$ R assumes the value of the greatest integer, less than or equal to x.Here, D_f = R and R_f = Z



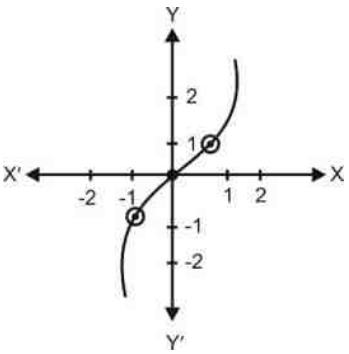
- Graph for f : R $\rightarrow$ R, defined by f(x) = x^2

  Here, D_f = R and R_f = [0, ∞)

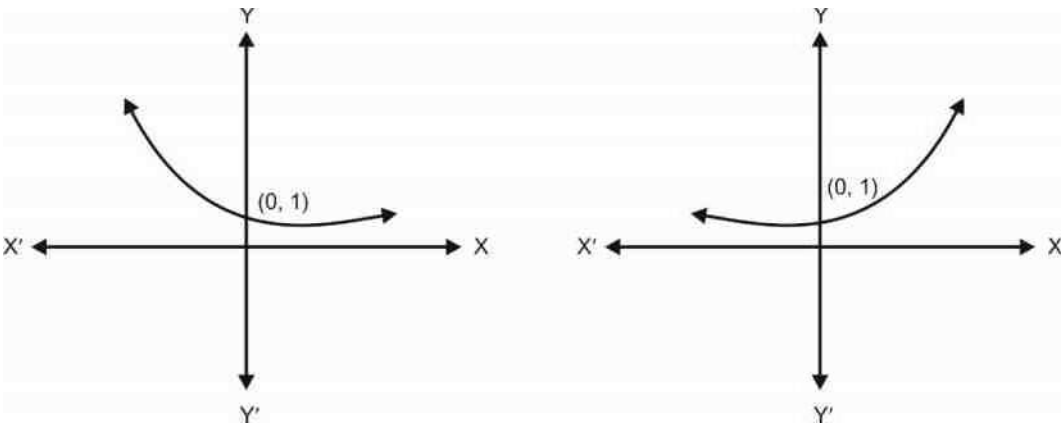


- Graph for f : R $\rightarrow$ R, defined by f(x) = x^3

  Here D_f = R and R_f = R



- **Exponential function:** f : R $\rightarrow$ R, defined by f(x) = a^x, a > 0, a $\neq$ 1



When $0 < a < 1$

$$f(x) = a^x \begin{cases} > 1 & \text{for } x < 0 \\ = 1 & \text{for } x = 0 \\ < 1 & \text{for } x > 0 \end{cases}$$

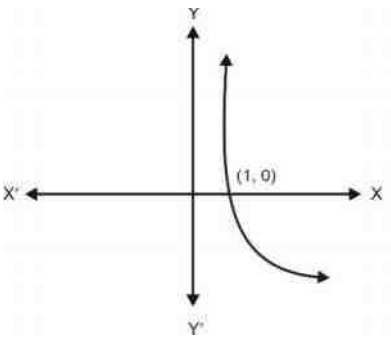
When $a > 1$

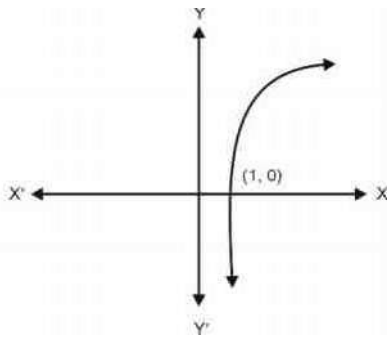
$$f(x) = a^x \begin{cases} < 1 & \text{for } x < 0 \\ = 1 & \text{for } x = 0 \\ > 1 & \text{for } x > 0 \end{cases}$$

- Natural exponential function, $f(x) = e^x$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots \infty, \quad 2 < e < 3$$

- Logarithmic functions, $f : (0, \infty) \to R$; $f(x) \log_a x$, $a > 0$, $a \neq 1$





When, $0 < a < 1$

$D_f = (0, \infty)$

$R_f = R$

When, $a > 1$

$D_f = (0, \infty)$

$R_f = R$

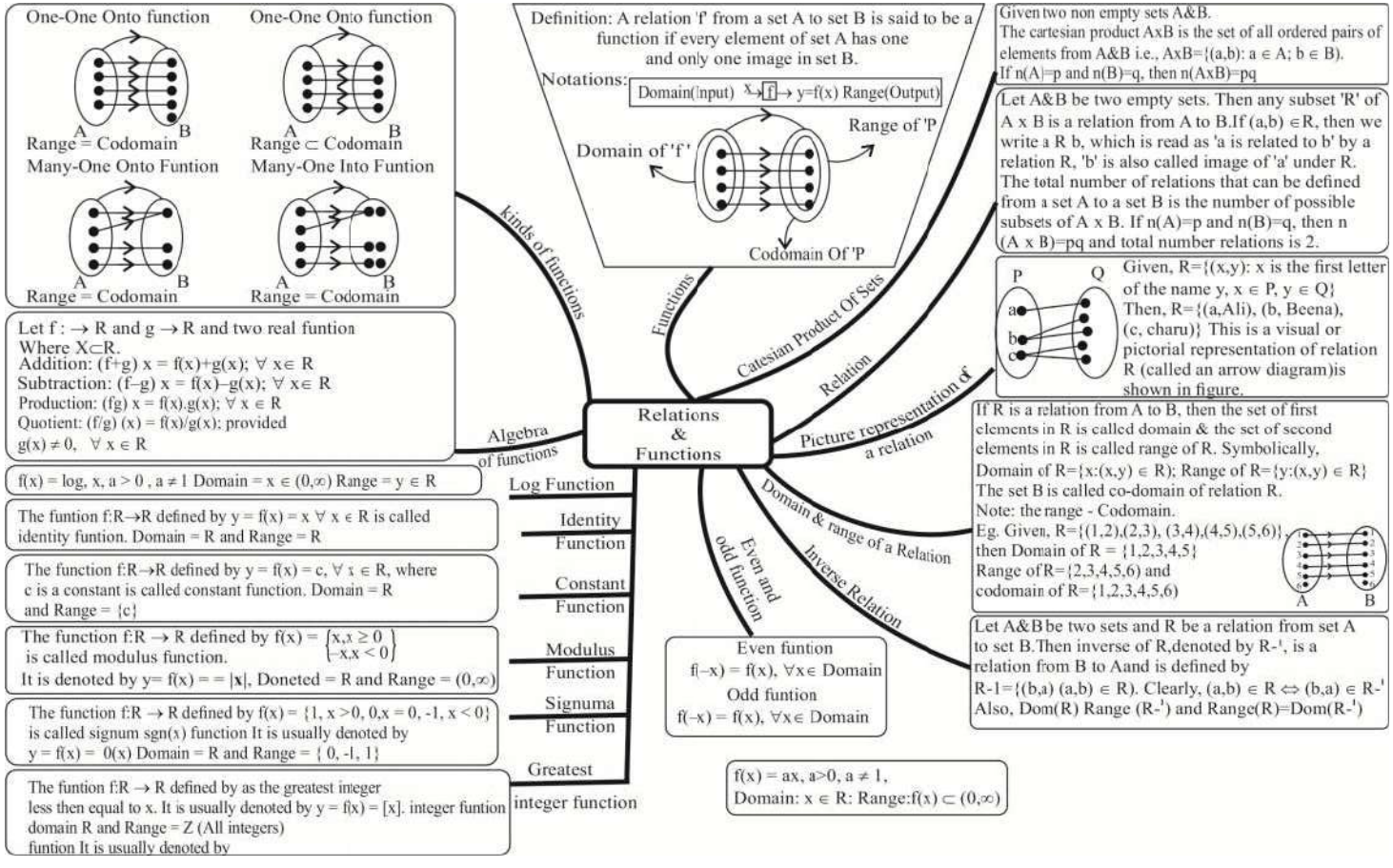
- **Natural logarithm function:** $f(x) = \log_e x$ or $\ln(x)$.
- Let $f : X \to R$ and $g : X \to R$ be any two real functions where $x \subset R$ then

$(f \pm g)(x) = f(x) \pm g(x) \; \forall \; x \in X$

$(fg)(x) = f(x)\, g(x) \; \forall \; x \in X$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \forall \; x \in X \text{ provided } g(x) \neq 0$$

MIND MAP


Relations & Functions
kinds of functions
One-One Onto function
Range = Codomain
Many-One Onto Funtion
Range = Codomain
One-One Onto function
Range ⊂ Codomain
Many-One Into Funtion
Range = Codomain
Let f : → R and g → R and two real funtion
Where X⊂R.
Addition: (f+g) x = f(x)+g(x); ∀ x∈ R
Subtraction: (f–g) x = f(x)–g(x); ∀ x∈ R
Production: (fg) x = f(x).g(x); ∀ x ∈ R
Quotient: (f/g) (x) = f(x)/g(x); provided
g(x) ≠ 0, ∀ x ∈ R
Algebra of functions
f(x) = log, x, a > 0 , a ≠ 1 Domain = x ∈ (0,∞) Range = y ∈ R
Log Function
The funtion f:R→R defined by y = f(x) = x ∀ x ∈ R is called identity funtion. Domain = R and Range = R
Identity Function
The function f:R→R defined by y = f(x) = c, ∀ x ∈ R, where c is a constant is called constant function. Domain = R and Range = {c}
Constant Function
The function f:R → R defined by f(x) = {x, x ≥ 0; –x, x < 0} is called modulus function. It is denoted by y= f(x) = = |x|, Doneted = R and Range = (0,∞)
Modulus Function
The function f:R → R defined by f(x) = {1, x >0, 0,x = 0, -1, x < 0} is called signum sgn(x) function It is usually denoted by y = f(x) = 0(x) Domain = R and Range = { 0, -1, 1}
Signuma Function
The funtion f:R → R defined by as the greatest integer less then equal to x. It is usually denoted by y = f(x) = [x]. integer funtion domain R and Range = Z (All integers) funtion It is usually denoted by
Greatest integer function
Functions
Definition: A relation 'f' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.
Notations: Domain(Input) x → f → y=f(x) Range(Output)
Domain of 'f'
Range of 'P
Codomain Of 'P
Even and odd function
Even funtion
f(–x) = f(x), ∀x∈ Domain
Odd funtion
f(–x) = f(x), ∀x∈ Domain
f(x) = ax, a>0, a ≠ 1,
Domain: x ∈ R: Range:f(x) ⊂ (0,∞)
Cartesian Product Of Sets
Given two non empty sets A&B.
The cartesian product AxB is the set of all ordered pairs of elements from A&B i.e., AxB={(a,b): a ∈ A; b ∈ B).
If n(A)=p and n(B)=q, then n(AxB)=pq
Relation
Let A&B be two empty sets. Then any subset 'R' of A x B is a relation from A to B.If (a,b) ∈R, then we write a R b, which is read as 'a is related to b' by a relation R, 'b' is also called image of 'a' under R.
The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A x B. If n(A)=p and n(B)=q, then n (A x B)=pq and total number relations is 2.
Picture representation of a relation
P
Q
a
b
c
Given, R={(x,y): x is the first letter of the name y, x ∈ P, y ∈ Q}
Then, R={(a,Ali), (b, Beena), (c, charu)} This is a visual or pictorial representation of relation R (called an arrow diagram)is shown in figure.
Domain & range of a Relation
If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. Symbolically,
Domain of R={x:(x,y) ∈ R); Range of R={y:(x,y) ∈ R}
The set B is called co-domain of relation R.
Note: the range - Codomain.
Eg. Given, R={(1,2),(2,3), (3,4),(4,5),(5,6)}
then Domain of R = {1,2,3,4,5}
Range of R={2,3,4,5,6) and
codomain of R={1,2,3,4,5,6)
A
B
Inverse Relation
Let A&B be two sets and R be a relation from set A to set B.Then inverse of R,denoted by R-1, is a relation from B to Aand is defined by
R-1={(b,a) (a,b) ∈ R). Clearly, (a,b) ∈ R ⇔ (b,a) ∈ R-1
Also, Dom(R) Range (R-1) and Range(R)=Dom(R-1)

Very Short Answer Type Question

1. Find a and b if (a – 1, b + 5) = (2, 3)

   If A = {1,3,5}, B = {2,3}, find : (Question- 2, 3)

2. A × B

3. B × A

   Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4, 5)

4. A × (B $\cap$ C)

5. A × (B $\cup$ C)

6. If P = {1,3}, Q = {2,3,5}, find the number of relations from P to Q

7. If R = {(x,y): x,y $\in$ Z, $x^2 + y^2 = 64$}, then,

   Write R in roster form

   Which of the following relations are functions? Give reason. (Questions 18 to 20)

8. R = { (1,1), (2,2), (3,3), (4,4), (4,5)}

9. R = { (2,1), (2,2), (2,3), (2,4)}

10. R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)}

SHORT ANSWER TYPE QUESTIONS

11. If A and B are finite sets such that n(A) = 5 and n(B) = 7, then find the number of functions from A to B.

12. If $f(x) = x^2 – 3x + 1$ find x $\in$ R such that f (2x) = f (x)

Let f and g be two real valued functions, defined by, f(x) = x, g(x) = |x|.Find: (Question 13 to 16)

13. f + g

14. f – g

15. fg

16. $\frac{f}{g}$

17. If $f(x) = x^3$, find the value of, $\frac{f(5)-f(1)}{5-1}$

18. Find the domain of the real function, $f(x)=\sqrt{x^2-4}$

19. Find the domain of the function, $f(x)=\frac{x^2+2x+3}{x^2-5x+6}$

Find the range of the following functions. (Question- 20, 21)

20. $f(x)=\frac{1}{4-x^2}$

21. $f(x) = x^2 + 2$

22. Find the domain of the relation,
$R = \{(x, y): x, y \in Z, xy = 4\}$

Find the range of the following relations: (Question-23, 24)

23. $R = \{(a,b) : a, b \in N \text{ and } 2a + b = 10\}$

24. $R=\left\{\left(x,\frac{1}{x}\right): x\in Z, 0<x<6\right\}$

25. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

R = {(x, y): $x \in A$, $y \in B$ and $y = x^2$}

(a) Depict this relation using arrow diagram.
(b) Find domain of R.
(c) Find range of R.
(d) Write co-domain of R.

26. If A = {2,4,6,9}, B = {4,6,18,27,54} and a relation R from A to B is defined by R = {(a,b): $a \in A$, $b \in B$, a is a factor of b and a < b}, then write R in Roster form. Also find its domain and range.

27. Find the domain and range of,

$f(x) = |2x - 3| - 3$

28. Draw the graph of the Constant function $f : R \to R$; $f(x) = 2 \ \forall \ x \in R$. Also find its domain and range.

29. Draw the graph of the function $|x - 2|$

Find the domain and range of the following real functions (Question 30-35)

30. $f(x) = \sqrt{x^2 + 4}$

31. $f(x) = \dfrac{x+1}{x-2}$

32. $f(x) = \dfrac{|x+1|}{x+1}$

33. $f(x) = \dfrac{x^2 - 9}{x-3}$

34. $f(x) = 1 - |x-3|$

35. $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$

36. Determine a quadratic function f defined by $f(x) = ax^2 + bx + c$. If $f(0) = 6$; $f(2) = 11$, $f(-3) = 6$

37. Draw the graph of the function $f(x) = \begin{cases} 1+2x & x < 0 \\ 3+5x & x \geq 0 \end{cases}$ also find its range.

38. Draw the graph of following function

$$f(x) = \begin{cases} \dfrac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Also find its range.

Find the domain of the following function.

39. $f(x) = \dfrac{1}{\sqrt{x + |x|}}$

40. $f(x) = \dfrac{1}{\sqrt{x - |x|}}$

41. $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

42. $f(x) = \sqrt{4 - x} + \dfrac{1}{\sqrt{x^2 - 1}}$

43. Find the domain for which the following functions:
$f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

44. If $f(x) = x - \dfrac{1}{x}$ prove that $[f(x)]^3 = f(x^3) + 3f\left(\dfrac{1}{x}\right)$.

45. If [x] denotes the greatest integer function. Find the solution set of equation, $[x]^2 + 5[x] + 6 = 0$.

46. If $f(x) = \frac{ax - b}{bx - a} = y$. Find the value of f(y).

Long Answer Type Questions

47. Draw the graph of following function and find range (R_f) of $f(x) = |x - 2| + |2 + x| \quad \forall \quad -3 \le x \le 3$.

48. Find domain and range $f(x) = \frac{1}{2\sin 3x}$

CASE STUDY TYPE QUESTIONS

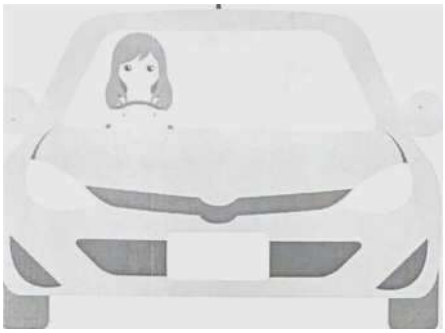
49. To make himself self-dependent and to earn his living, a person decided to setup a small scale business of manufacturing hand sanitizers. He estimated a fixed cost of Rs. 15000 per month and a cost of Rs. 30 per unit to manufacture.

i. If x units of hand sanitizers are manufactured per month. What is the cost function?

ii. If each unit is sold for Rs. 45. What is the selling (revenue) function?

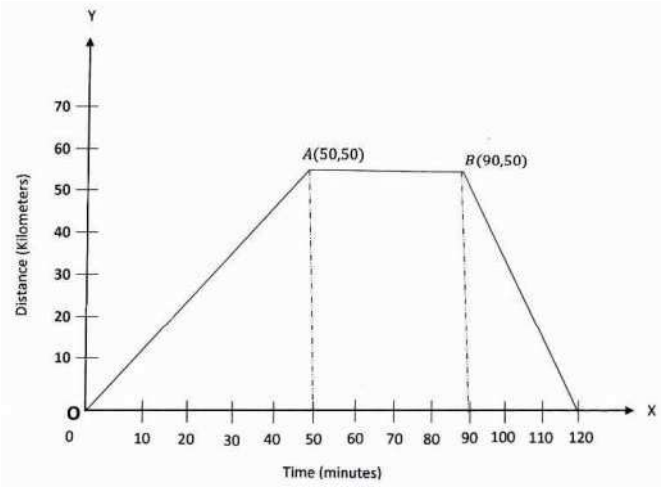
iii. What is the profit function?

iv. For Break even (No Profit, no loss situation) in a month, how many units should be manufactured and sold?



50. This is a graph showing how far the distances have been travelled by Sunita (in her car) in a given time.

She drove, stopped, does her work and returned back.



i. The line OA of the graph represents the function:

(a) $x=10y$ (b) $y=10x$

(c) $y=x$ (d) $y=\frac{x}{2}$

ii. At what distance from the starting point Sunita stopped to do her work?

(a) 30m (b) 40m

(c) 50m (d) 60m

iii. How much time Sunita took to complete her work?

(a) 30 min (b) 40 min

(c) 50 min (d) 60 min

iv. Line AB represents the constant function:

(a) $y = 50$ (b) $x = 50$

(c) $y = 10$ (d) $x = 9$

v. How much time Sunita took to reach at a distance of 40 km. from the initial point?

(a) 30 min (b) 40 min

(c) 50 min (d) 1 hour

Multiple Choice Questions

51. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5} then (A – B) × (B – C)

(a) {(1, 2), (1, 5), (2, 5)} (b) {1, 4}

(c) {1, 4} (d) None of these.

52. If R is a relation on set A = {1, 2, 3, 4, 5, 6, 7, 8} given by xRy $\Leftrightarrow$ y = 3x, then R = ?

(a) {(3, 1), (6, 2), (8, 2), (9, 3)} (b) {(3, 1), (6, 2), (9, 3)}

(c) {(3, 1), (2, 6), (3, 9)} (d) None of these.

53. Let A = {1, 2, 3}, B = {4, 6, 9} is relation R from A to B defined by R = {(x, y) x is greater than y}, the range of R is -

(a) {1, 4, 6, 9} (b) {4, 6, 9}

(c) {1} (d) None of these.

54. If R be a relation from a set A to a set B then -

(a) $R = A \cup B$ (b) $R = A \cap B$

(c) $R \subseteq A \times B$ (d) $R \subseteq B \times A$.

55. If $2f(x)-3f\left(\frac{1}{x}\right)=x^2$ $(x \neq 0)$, then f(2) is equal to -

(a) $\frac{-7}{4}$ (b) $\frac{5}{2}$

(c) -1 (d) None of these.

56. Range of the function f(x) = cos[x] for $\frac{-\pi}{2} < x < \frac{\pi}{2}$ is -

(a) {–1, 1, 0} (b) {cos1, cos2, 1}

(c) {cos1, –cos1, 1} (d) {–1, 1}.

57. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$ then f{g(x)} is equal to -

(a) f(3x) (b) $\{f(x)\}^3$

(c) 3f(x) (d) –f(x).

58. If f(x) = cos(logx) then value of $f(x).f(y)-\frac{1}{2}\left\{f\left(\frac{x}{y}\right)+f(xy)\right\}$ is -

(a) 1 (b) –1

(c) 0 (d) ±1.

59. Doman of $f(x) = \sqrt{4x-x^2}$ is -

(a) R – [0, 4] (b) R – (0, 4)

(c) (0, 4) (d) [0, 4].

60. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function then -

(a) $x \in [3, 4]$ (b) $x \in (2, 3]$

(c) $x \in [2, 3]$ (d) $x \in [2, 4)$.

61. If A = {2, 3, 5}, B = {2, 5, 6}, then (A – B) × (A ∩ B) is

(a) {(3, 2), (3, 3), (3, 5)} (b) {(3, 2), (3, 5), (3, 6)}

(c) {(3, 2), (3, 5)} (d) None of these

62. The relation R defined on the set of natural numbers as {(a, b), a – b = 3}, is given by

(a) {(1, 4), (2, 5), (3, 6), ...} (b) {(4, 1), (5, 2), (6, 3), ...}

(c) {(1, 3), (2, 6), (3, 9), ...} (d) None of these

63. If R = {(x, y)| x, y ∈ Z, $x^2 + y^2 < 4$} is a relation is Z, then domain of R is

(a) {0, 1, 2} (b) {0, – 1, – 2}

(c) {–2, –1, 0, 1, 2} (d) None of these

64. Let n(A) = n. Then the number of all relations on A is

(a) 2^n (b) $2^{n!}$

(c) 2^{n^2} (d) None of these

65. If n(A) = 4, n(B) = 3, n(A × B × C) = 24, then n(C) =

(a) 288 (b) 1

(c) 12 (d) 2

66. If $f(x) = \frac{x}{x-1}$, then $\frac{f(a)}{f(a+1)} =$

(a) f(–a) (b) $f\left(\frac{1}{a}\right)$

(c) $f(a^2)$ (d) $f\left(\frac{-a}{a-1}\right)$

Direction: Each of these questions contains two statements Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes. (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct: reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct: reason is not a correct explanation for assertion.
(c) Assertion is correct, reason is incorrect.
(d) Assertion is incorrect, reasoni is correct.

67. Let A = {a, b, c, d, e, f, g, h} and R = {(a, b), (b, b), (a, g), (b, a), (b, g), (g, a), (g, b), (g, g), (b, b)}
Consider the following statements:
Assertion: $R \subset A \times A$.
Reason: R is not a relation on A.

68. Let A = {1, 2, 3, 4, 6}. If R is the relation on A defined by {(a, b) : a, b $\in$ A. b is exactly divisible by a}.
Assertion: The relation R in Roster form is {(6, 3), (6, 2), (4, 2)}.
Reason: The Domain and Range of R is {1, 2, 3, 4, 6}

69. **Assertion:** If (x + 1, y – 2) = (3, 1), then x = 2 and y = 3.
Reason: Two ordered pairs are equal, if their corresponding elements are equal.

70. **Assertion:** If $f(x) = \frac{1}{x-2}$, $x \neq 2$ and $g(x) = (x-2)^2$, then $(f+g)(x) = \frac{1+(x-2)^3}{x-2}$, $x \neq 2$.
Reason: If f and g are two functions, then their sum is defined by (f + g) (x) = f(x) + g(x) $\forall$ x $\in$ $D_1 \cap D_2$, where D_1 and D_2 are domains of f and g, respectively.

ANSWERS

1. a = 3, b = –2

2. A × B = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)}

3. B × A = { (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)}

4. {(1,4), (2,4)}

5. {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)}

6. $2^6 = 64$

7. R = { (0,8), (0,–8), (8,0), (–8,0)}

8. Not a function because 4 has two images.

9. Not a function because 2 does not have a unique image.

10. Function because every element in the domain has its unique image.

11. 7^5

12. 0,1

13. $f+g=\begin{cases}2x & x\geq 0\\ 0 & x<0\end{cases}$

14. $f-g=\begin{cases}0 & x\geq 0\\ 2x & x<0\end{cases}$

15. $fg=\begin{cases}x^2 & x\geq 0\\ -x^2 & x<0\end{cases}$

16. $\frac{f}{g}=\begin{cases}1 & x>0\\ -1 & x<0\end{cases}$ and Note:-$\frac{f}{g}$ is not defined at x = 0

17. 31

18. $(-\infty, -2] \cup [2, \infty)$

[Hint: Put $x^2 - 4 \geq 0$]

19. $R - \{2,3\}$

20. $(-\infty, 0) \cup [1/4, \infty)$

21. $[2,\infty)$

22. $\{-4, -2, -1,1,2,4\}$

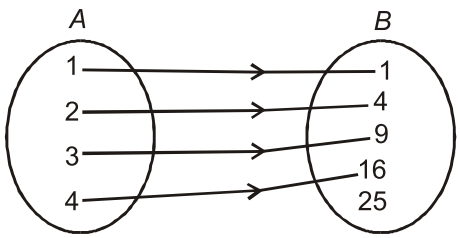
23. {2,4,6,8} [Hint: Use roster Form]

24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

23. {2,4,6,8} [Hint: Use roster Form]

24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

25. (a)



(b) {1,2,3,4}

(c) {1,4,9,16}

(d) {1,4,9,16,25}

26. R = { (2,4) (2,6) (2,18) (2,54) (6,18) (6,54) (9,18) (9,27) (9,54) }

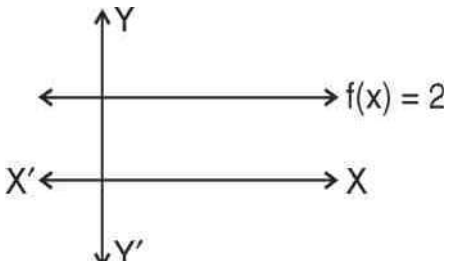
Domain is R = {2,6,9}

Range of R = { 4, 6, 18, 27, 54}

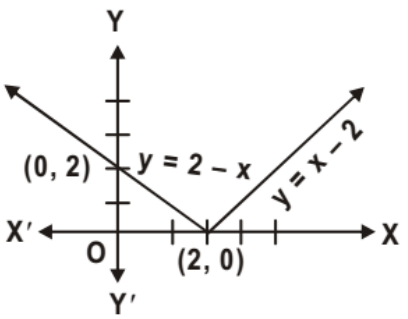
27. Domain is R

Range is $[-3, \infty)$

28. Domain = R,Range = {2}



29. Hint: $|x-2| = \begin{cases} x-2 : x \geq 2 \\ 2-x : x < 2 \end{cases}$



30. Domain = R,

Range =[2, ∞)

31. Domain = R – {2}

Range = R – {1}

32. Domain = R – {–1}

Range = {1, –1}

33. Domain = R – {3}

Range = R – {6}

34. Domain = R

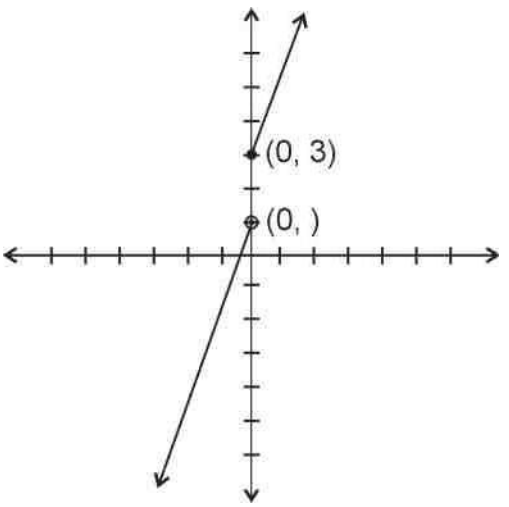
Range =(–∞, 1]

35. Domain = $(-3, 3)$

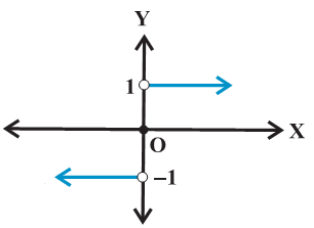
Range = $\left[\frac{1}{3}, \infty\right)$

36. $\frac{1}{2}x^2 + \frac{3}{2}x + 6$

37. $(-\infty, 1) \cup [3, \infty)$



38. Range of f = {–1,0,1}



39. $(0, \infty)$

40. ϕ(given function is not defined)

41. $(-\infty, -2) \cup (4, \infty)$

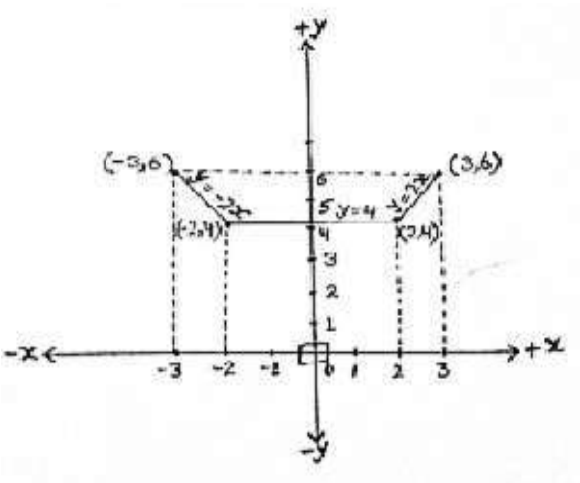
42. $(-\infty, -1) \cup (1, 4]$

43. $\left\{-2, \frac{1}{2}\right\}$

45. [–3,–1)

46. x

47. R_f = [4, 6] and graph is



48. Domain = R
Range = [1/3, 1]

49. i. 15000 + 30x ii. 45x iii. 15(x – 1000) iv. 1000

50. i. (c) ii. (c) iii. (b) iv. (a) v. (b)

51.	(b)	52.	(d)	53.	(c)
54.	(c)	55.	(a)	56.	(b)
57.	(c)	58.	(c)	59.	(d)
60.	(d)	61.	(c)	62.	(b)
63.	(c)	64.	(c)	65.	(d)
66.	(c)	67.	(c)	68.	(d)
69.	(a)	70.	(a)		