

CHAPTER - 1

SETS AND FUNCTIONS

KEY POINTS

- Definition of Set : Set is well defined collection of objects.
- Objects in Set are called elements of Set.
- Elements are said to be 'belong to' set.

Example: $A = \{a, b, c, d\}$ is a Set and a, b, c, d are element of Set A

Here a, b, c, d belongs to A or $a, b, c, d \in A$

- Representation of Sets:

(a) Roster or Tabular form

e.g.: Set of all Natural Numbers less than 5 = $\{1, 2, 3, 4\}$

(b) Set-builder form

e.g.: Set of all Natural Numbers less than 5 = $\{x : x \in \mathbb{N}, x < 5\}$

- **Types of sets:**

(a) Empty /Null/Void Set: Set which does not contain any element. It is denoted by ϕ or $\{ \}$

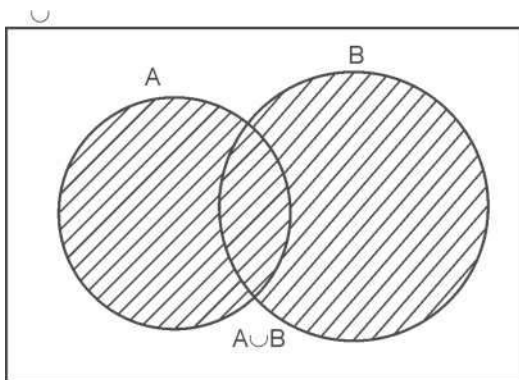
(b) Finite set : Set having finite number of elements

(c) Infinite set: Set having infinite number of elements

(d) Singleton set : Set having only one element

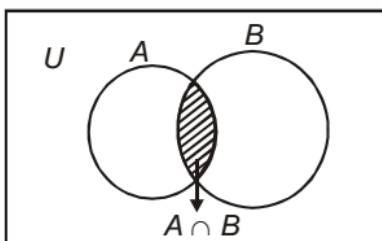
- Cardinal number of finite set: Number of distinct elements of set. It is denoted by $n(A)$.
- Equivalent sets: Two or more finite sets having same number of elements or same cardinal number.

- **Subset:** A set A is said to be subset of a set B iff $a \in A \Rightarrow a \in B$.
 $\forall a \in A$
 We write it as $A \subseteq B$.
 Note: ϕ and A itself are always subsets of set A.
 - **Super set:** If $A \subseteq B$ then B is superset of A.
 - **Proper subset :** If $A \subseteq B$, but $A \neq B$ then A is proper subset of B.
 We write it as $A \subset B$.
 - **Number of subsets of a set $A = 2^{n(A)}$**
 - **Number of proper subsets of a Set $A = 2^{n(A)} - 1$**
 - **Equal sets:** Two or more sets having exactly same elements.
 $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.
 - **Power set:** The collection of all subsets of a set A. It is denoted by $P(A)$
 $P(A) = \{X: X \subseteq A\}$
 $n[P(A)] = 2^{n(A)}$
 - **Types of Intervals**
 - (a) Open Interval $(a, b) = \{x \in \mathbb{R} : a < x < b\}$
 - (b) Closed Interval $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$
 - (c) Semi open or Semi closed Interval,
 - $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$
 - $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$
 - **Venn diagram and operations on sets**
 - (a) Union of two sets A and B :
 $A \cup B = \{x : x \in A \text{ or } x \in B\}$
-

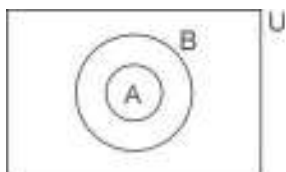


(b) Intersection of two sets A and B :

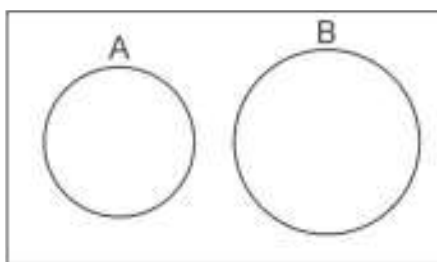
$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$



- Subset and superset: $A \subset B$

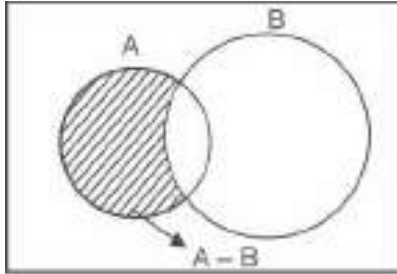


- Disjoint sets: Two sets A and B are said to be disjoint if $A \cap B = \phi$



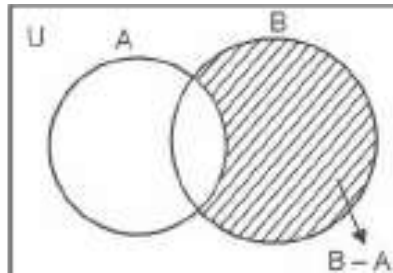
(c) Difference of sets A and B is,

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$



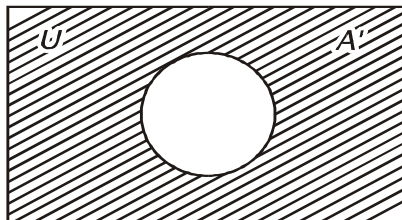
(d) Difference of sets B and A is,

$$B - A = \{x : x \in B \text{ and } x \notin A\}$$



(e) Complement of a set A, denoted by A' or A^c

$$A' = A^c = U - A = \{x : x \in U \text{ and } x \notin A\}$$



● Properties of complement of sets :

1. Complement laws

(i) $A \cup A' = U$ (ii) $A \cap A' = \phi$ (iii) $(A')' = A$

2. De Morgan's Laws

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Note : This law can be extended to any number of sets.

3. $\phi' = U$ and $U' = \phi$

4. If $A \subset B$ then $B' \subset A'$

- Laws of Algebra of sets

(i) $A \cup \phi = A$

(ii) $A \cap \phi = \phi$

- $A - B = A \cap B' = A - (A \cap B)$

- Commutative Laws :-

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

- Associative Laws :-

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

- Distributive Laws :-

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$

- $n(A \cup B) + (A \cap B) = n(A) + n(B)$

- If A and B are disjoint, then $n(A \cup B) = n(A) + n(B)$

- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

MIND MAP

Let A and B be two sets, if every element of A is an element of B, then A is called a subset of B and written as $A \subset B$ or $B \supset A$ (read as 'A' is contained in 'B' or 'B contains A'). is called superset of A.

Note:

1. Every set is a subset and superset of itself.
 2. If A is not a subset of B, we write $A \not\subset B$.
 3. The empty set is the subset of every set.
 4. If A is a set with $n(A) = m$, then no. of element A are 2^m and the number of proper subsets of A are $2^m - 1$
- Eg. Let $A = \{3, 4\}$, then subsets of A are $\phi, \{3\}, \{4\}, \{3, 4\}$. Here, $n(A) = 2$ and number of subsets of $A = 2^2 = 4$.

A set is collection of well-defined distinguished objects. The sets are usually denoted by capital letters A, B, C etc., and the members or elements of the set are denoted by lower case letters a, b, c etc., if x is a member of the set A, we write $x \in A$ (read as 'x belongs to A) and if x is not a member of set A, we write $x \notin A$ (read as 'x doesn't belong to A). if x and y both belong to A, we write $x, y \in A$. Some examples of sets are: A: odd number less than 10
N: the set of all rational numbers
B: the vowels in the English alphabates
Q: the set of all rational numbers.

In this form, we write a variable (say x) representing any member of the set followed by property satisfied by each member of the set.
e.g.: The set A of all prime number less than 10 in set builder form is written as
 $A = \{x \mid x \text{ is a prime number less than } 10\}$
the symbol " \mid " stands for the word "such that", sometimes, we use symbol " \in " in place of symbol " \mid "

In this form, we first list all the members of the set within braces (curly brackets) and separate these by commas.
E.g. The set of all natural number less than 10 in this form is written as: $A = \{1, 3, 5, 7, 9\}$
In roster form, every element of sets is listed only once. The order in which the elements are listed is immaterial.
E.g. Each of the following sets denotes the same set $\{1, 2, 3\}, \{3, 2, 1\}, \{1, 3, 2\}$

A set having one element is called singleton set.
e.g.: $\{0\}, \{9\}$ is a singleton set, whose only member is 0.
(ii) $A = \{x \mid 1 < x < 3, x \text{ is a natural number}\}$ is a singleton set which has only one member which is 2.

Two finite sets A and B are said to be equivalent, if =
 $n(A) = n(B)$. Clearly, equal set are equivalent but equivalent set need not to be equal.
e.x. The sets $A = \{4, 5, 3, 2\}$ and $B = \{1, 6, 8, 9\}$ are equivalent, but are not equal.

SETS

Subset

Representation of sets

Cardinal Number
Induction
Set Builder Form Or Rule Method

Roster Or Tambular Form

Types Of Sets

Empty Set Or Null Set

Finite And Infinite Set

Equal Set

Singleton Set

Equivalent Set

A set which has no element is called null set. It is denoted by symbol ϕ or $\{\}$.

E.g: Set of all real number whose square is -1.

In set-builder form: $\{x : x \text{ is a real number whose square is } -1\}$ in roster form: $\{\}$ or ϕ

A set which has finite number of element is called a finite set. Otherwise, it is called an infinite set.

E.g.: The set of all days in a week is a finite set where as the set of all integers, denoted by

$\{\dots, -2, -1, 0, 1, 2, \dots\}$ or $\{x \mid x \text{ is an integer}\}$ is an infinite set.

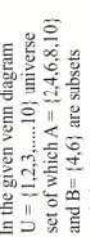
An empty set ϕ which has no element is a finite set A is called empty or void or null set.

Two sets A and B are set to be equal, written as $A = B$, if every element of A is in B and every element of B is in A.

e.g.: (i) $A = \{1, 2, 3, 4\}$ and $B = \{3, 1, 4, 2\}$ then $A = B$

(ii) $A = \{x : x - 5 = 0\}$ and $B = \{x : x \text{ is an integral positive root}\}$ Then $A = B$.

A Venn diagram is an illustration of the relationships between and among sets, groups of objects that share something common. These diagrams consist of rectangle and closed curves usually circles.

Eg: 


In the given venn diagram
 $U = \{1, 2, 3, \dots, 10\}$ universe
 set of which $A = \{2, 4, 6, 8, 10\}$
 and $B = \{4, 6\}$ are subsets
 and also $B \subset A$

- For any set A, we have
 (a) $A \cup A = A$, (b) $A \cap A = A$, (c) $A \cup \phi = A$, (d) $A \cap \phi = \phi$, (e) $A \cup U = U$,
 (f) $A \cap U = A$, (g) $A - \phi = A$, (h) $A - A = \phi$.
- For any two sets A and B we have
 (a) $A \cup B = B \cup A$, (b) $A \cap B = B \cap A$, (c) $A - B \subseteq A$, (d) $B - A \subseteq B = U$
- For any two sets A and B we have
 (a) $A \cup (B \cap C) = (A \cup B) \cap C$, (b) $A \cap (B \cup C) = (A \cap B) \cup C$
 (c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, (d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 (e) $A - (B \cup C) = (A - B) \cap (A - C)$, (f) $A - (B \cap C) = (A - B) \cup (A - C)$

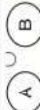
The union of two sets A and B, written as $A \cup B$ (read as A union B) is the set of all elements which are either in A or in B in both. Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$
 clearly, $x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$ and $x \in A \cup B \Rightarrow x \in A$ and $x \in B$
 $A \cup B$ eg: If $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$
 then $A \cup B = \{a, b, c, d, e, f\}$



The intersection of two sets A and B, written as $A \cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B.
 Thus, $A \cap B \Rightarrow \{x : x \in A \text{ and } x \in B\}$
 Clearly, $x \in A \cap B \Rightarrow \{x \in A \text{ and } x \in B\}$ and $x \in A \cap B \Rightarrow \{x \in A \text{ or } x \in B\}$.
 Eg: If $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$
 Then $A \cap B = \{c, d\}$



Two sets A and B are said to be disjoint, if $A \cap B = \phi$ i.e. A and B have no common element. eg: if $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$
 Then, $A \cap B = \phi$, so A and B are disjoint.



The set containing all objects of element and of which all other sets are subsets is known as universal sets and denoted by U.
 Eg: For the set of all integers, the universal set can be the set of rational numbers or the set R of real numbers

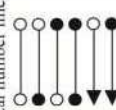
The symmetric difference of two sets A and B, denoted by $A \Delta B$, is defined as $(A \Delta B) = (A - B) \cup (B - A)$
 Eg. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$ then
 $(A \Delta B) = (A - B) \cup (B - A) = \{2, 4\} \cup \{7, 9\} = \{2, 4, 7, 9\}$

The set of natural numbers
 $N = \{1, 2, 3, 4, 5, \dots\}$
 The set of integers
 $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
 The set of irrational numbers,
 $T = \{x : x \in R \text{ and } r \in Q\}$
 The set of rational number
 $Q = \{x : x = p/q, p \in Z \text{ and } q \neq 0\}$
 Relation among these subsets are
 $N \subset Z \subset Q, Q \subset R, T \subset R, N \subset Z, T \subset R, N \subset Z, T \subset R, N \subset Z, T \subset R$

Let a and b be real numbers with $a < b$


Set of Real Numbers
 $\{x | a < x < b\}$
 $\{x | a \leq x < b\}$
 $\{x | a < x \leq b\}$
 $\{x | a \leq x \leq b\}$
 $\{x | x < a\}$
 $\{x | x \leq a\}$
 $\{x | x > a\}$
 $\{x | x \geq a\}$
 $-\infty, \infty$

Interval Notation
 Region on the real number line



The set of all subset of a given set A is called power set of A and denoted by $P(A)$.
 Eg: If $A = \{1, 2, 3\}$, then $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$
 Clearly, if A has n elements, then its power set $P(A)$ contains exactly 2^n elements.

If A and B are two sets, then their difference $A - B$ is defined as:
 $A - B = \{x : x \in A \text{ and } x \notin B\}$
 Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$
 Eg. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$ then
 $A - B = \{2, 4\}$ and $B - A = \{7, 9\}$



If U is a universal set and A is a subset of U, then complement of A is the set which contains those elements of U, which are not present in A and is denoted by A' or A^c . Thus,
 $A = \{x : x \in U \text{ and } x \notin A\}$
 e.g.: If $U = \{1, 2, 3, 4, \dots\}$ and $A = \{2, 4, 6, 8, \dots\}$ then $A^c = \{1, 3, 5, 7, \dots\}$

Properties of complement
 Complement law:
 (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$,
 De Morgan's Law:
 (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
 Double Complement law:
 $(A')' = A$
 Law of empty set and universal set
 $\phi' = U$ and $U' = \phi$

VERY SHORT ANSWER TYPE QUESTIONS

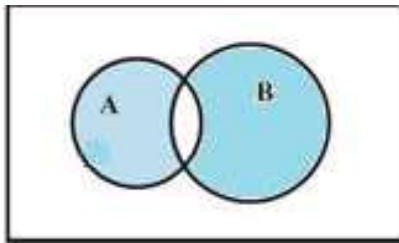
- Write set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{19}{20}\right\}$ in set builder form.
 - Write the set $\{x : x \in \mathbb{Z}^+, x^2 < 4\}$ in Roster form.
Let $A = \{1, 3, 5, 7, 9\}$. Insert the appropriate symbol \in or \notin in blank spaces: – (Question- 3,4)
 - (i) 2 _____ A (ii) $\{3\}$ _____ A (iii) $\{3, 5\}$ _____ A
 - Write the set $A = \{x : x \text{ is an integer, } -1 \leq x < 4\}$ in roster form
 - Write the set $B = \{3, 9, 27, 81\}$ in set-builder form.
Which of the following are empty sets? Justify. (Question- 6,7)
 - $A = \{x : x \in \mathbb{N} \text{ and } 3 < x < 4\}$
 - $B = \{x : x \in \mathbb{N} \text{ and } x^2 = x\}$
Which of the following sets are finite or infinite? Justify. (Question-8, 9)
 - The set of all the points on the circumference of a circle.
 - $B = \{x : x \in \mathbb{N} \text{ and } x \text{ is an even prime number}\}$
 - Are sets $A = \{-2, 2\}$, $B = \{x : x \in \mathbb{Z}, x^2 - 4 = 0\}$ equal? Why?
 - Write $(-5, 9]$ in set-builder form
 - Write $\{x : x \in \mathbb{R}, -3 \leq x < 7\}$ as interval.
 - If $A = \{1, 3, 5\}$, how many elements has $P(A)$?
 - Write all the possible subsets of $A = \{5, 6\}$.
If $A =$ Set of letters of the word 'DELHI' and $B =$ the set of letters the words 'DOLL' find (Question- 15, 16, 17)
 - $A \cup B$
-

16. $A \cap B$
17. $A - B$
18. Describe the following sets in Roster form
- (i) The set of all letters in the word 'ARITHMETIC'.
- (ii) The set of all vowels in the word 'EQUATION'.
19. Write the set $A = \{x : x \in \mathbb{Z}, x^2 < 25\}$ in roster form.
20. Write the set $B = \{x : x \text{ is a two digit number, such that the sum of its digits is } 7\}$

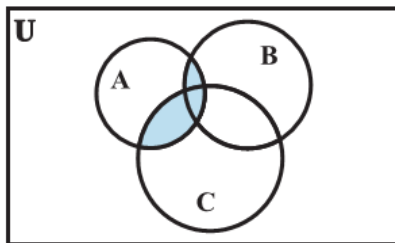
SHORT ANSWER TYPE QUESTIONS

21. Are sets $A = \{1,2,3,4\}$, $B = \{x : x \in \mathbb{N} \text{ and } 5 \leq x \leq 7\}$ disjoint? Justify?
What is represented by the shaded regions in each of the following Venn-diagrams? (Question 22, 23)

22.



23.



24. If $A = \{1, 3, 5, 7, 11, 13, 15, 17\}$

$$B = \{2, 4, 6, 8 \dots 18\}, U = \{1, 2, 3, \dots, 20\}$$

Where U is universal set then find $A' \cup [(A \cup B) \cap B']$

25. Two sets A and B are such that

$$n(A \cup B) = 21, n(A) = 10, n(B) = 15, \text{ find } n(A \cap B) \text{ and } n(A - B)$$

26. Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$ Verify the following identity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

27. If $U = \{x : x \in \mathbb{N} \text{ and } x \leq 10\}$

$$A = \{x : x \text{ is prime and } x \leq 10\}$$

$$B = \{x : x \text{ is a factor of } 24\}$$

Verify the following result

$$(i) A - B = A \cap B' \quad (ii) (A \cup B)' = A' \cap B' \quad (iii) (A \cap B)' = A' \cup B'$$

28. For any sets A and B show that

$$(i) (A \cap B) \cup (A - B) = A \quad (ii) A \cup (B - A) = A \cup B$$

29. On the Real axis, if $A = [0, 3]$ and $B = [2, 6]$, then find the following

$$(i) A' \quad (ii) A \cup B \quad (iii) A \cap B \quad (iv) A - B$$

30. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

31. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

32. Two sets A and B are such that $n(A \cup B) = 21$, $n(A' \cap B') = 9$, $n(A \cap B) = 7$ find $n(A \cap B)'$.

LONG ANSWER TYPE QUESTIONS

33. In a group of 84 persons, each plays at least one game out of three viz. tennis, badminton and cricket. 28 of them play cricket, 40 play tennis and 48 play badminton. If 6 play both cricket and badminton and 4 play tennis and badminton and no one plays all the three games, find the number of persons who play cricket but not tennis.

34. Using properties of sets and their complements prove that

(i) $(A \cup B) \cap (A \cup B)' = A$

(ii) $A - (A \cap B) = A - B$

(iii) $(A \cup B) - C = (A - C) \cup (B - C)$

(iv) $A - (B \cup C) = (A - B) \cap (A - C)$

(v) $A \cap (B - C) = (A \cap B) - (A \cap C)$.

35. Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the value of m and n .

36. A survey shows that 63% people watch news channel A whereas 76% people watch news channel B. If $x\%$ of people watch both news channels, then prove that $39 \leq x \leq 63$.

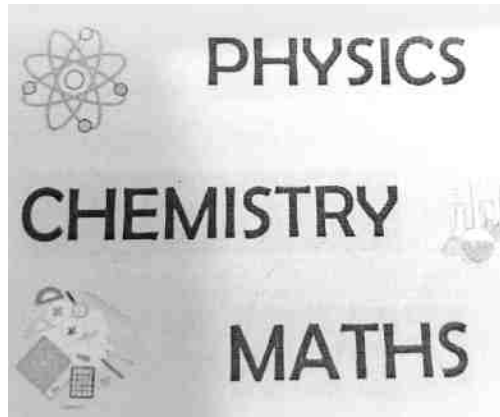
37. From 50 students taking examination in Mathematics, Physics and chemistry, each of the students has passed in at least one of the subject, 37 passes Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, almost 29 Mathematics and chemistry and at most 20 Physics and chemistry. What is the largest possible number that could have passes in all the three subjects?

CASE STUDY TYPE QUESTIONS

38. In a survey of 600 students of class XI, 150 are using YouTube videos and 225 are consulting books (other than text book) as a learning resource. 100 were using both YouTube videos and books as a learning resource.



- i. How many students are using either books or YouTube videos as the learning resource?
 - ii. How many students are neither using YouTube videos nor books as the learning resource?
 - iii. How many students are using YouTube videos only as the learning resource?
 - iv. How many students are using books only as the learning resource?
 - v. What can be the maximum number of students who will use YouTube video or books as learning resources?
39. In a class 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics. Of these 13 took both Chemistry and Mathematics, 12 took both physics and chemistry and 11 took both Physics and mathematics. If 6 students were offered all the three subjects, find:



- i. The total number of students are
(a) 47 (b) 37 (c) 35 (d) 49
- ii. How many took Mathematics but not Chemistry?
(a) 11 (b) 1 (c) 6 (d) 12
- iii. How many took exactly one of the three subjects?
(a) 12 (b) 11 (c) 13 (d) 1
- iv. How many took exactly two of these subjects?
(a) 11 (b) 13 (c) 12 (d) 18
- v. Number of students who took Physics or Mathematics but not Chemistry:
(a) 12 (b) 13 (c) 11 (d) 18
40. In a town of 10,000 families, it was found that 40% families go to shop A for their home needs groceries, 20% families go to the shop B and 10% families go to shop C. 5% families go to shops A and B, 3% go to B and C and 4% families go to A and C. 2% families go to all the three shops A, B and C. Find:



- i. The number of families which go to shop A only;
 - (a) 4000 (b) 3300 (c) 3700 (d) 4200
- ii. The number of families which don't visit/purchase from any of A, B and C.
 - (a) 4000 (b) 7000 (c) 3300 (d) 6000
- iii. The number of families which don't visit/purchase from any of A, B and C.
 - (a) 300 (b) 200 (c) 100 (d) 600
- iv. The number of families that purchase from exactly one shop.
 - (a) 4700 (b) 4000 (c) 5200 (d) 3800
- v. The number of families that buy from at least one of the shops A, B or C.
 - (a) 4000 (b) 6000 (c) 7000 (d) 1000

Multiple Choice Questions

41. In set builder method the null set is represented by
- (a) { }
 - (b) ϕ
 - (c) $\{x : x \neq x\}$
 - (d) $\{x : x = x\}$.

42. If A and B are two given sets, then $A \cap (A \cap B)'$ is equal to
 (a) A (b) B' (c) ϕ (d) $A - B$.
43. If A and B are two sets such that $A \subset B$ then $A \cap B'$ is
 (a) A (b) B' (c) ϕ (d) $A \cap B$.
44. If $n(A \cup B) = 18$, $n(A - B) = 5$, $n(B - A) = 3$ then $n(A \cap B)$ is
 (a) 18 (b) 10 (c) 15 (d) 12
45. For any two sets A and B, $A \cap (A \cup B)'$ is equal to
 (a) A (b) B (c) ϕ (d) $A \cap B$
46. If $n(A) = 5$ and $n(B) = 7$, then maximum number of elements in $A \cup B$ is
 (a) 7 (b) 5 (c) 12 (d) None of these
47. $n[P\{P(\phi)\}] =$
 (a) 2 (b) 4 (c) 8 (d) 0
48. If $A = \{1, 2, 3, 4, 5\}$, then the number of proper subsets of A is
 (a) 120 (b) 30 (c) 31 (d) 32
49. For any two sets A and B, $(A - B) \cup (B - A) =$
 (a) $(A - B) \cup A$ (b) $(B - A) \cup B$
 (c) $(A \cup B) - (A \cap B)$ (d) $(A \cup B) \cap (A \cap B)$
50. If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49n - 49 : n \in N\}$, then
 (a) $X \subset Y$ (b) $Y \subset X$
 (d) $X = Y$ (d) $X \cap Y = \phi$
51. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then $n(A^c \cap B^c) =$
 (a) 400 (b) 600

- (d) 300 (d) 200
52. If a set A has n elements, then the total number of subsets of A is
 (a) n (b) n^2
 (c) 2^n (d) 2n
53. The number of non-empty subsets of the set {1, 2, 3, 4} is
 (a) 15 (b) 14
 (c) 16 (d) 17
54. If A and B are two sets, then $A \cup B = A \cap B$ iff
 (a) $A \subseteq B$ (b) $B \subseteq A$
 (c) $A = B$ (d) None of these
55. Let A and B be two sets. Then
 (a) $A \cup B \subseteq A \cap B$ (b) $A \cap B \subseteq A \cup B$
 (c) $A \cap B = A \cup B$ (d) None of these
56. If $Q = \left\{ x : x = \frac{1}{y}, \text{ where } y \in N \right\}$, then
 (a) $0 \in Q$ (b) $1 \in Q$
 (c) $2 \in Q$ (d) $\frac{2}{3} \in Q$

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes. (a), (b), (c) and (d) given below:

- (a) Assertion is correct, reason is correct: reason is a correct explanation for assertion.
- (b) Assertion is correct, reason is correct: reason is not a correct explanation for assertion.
- (c) Assertion is correct, reason is incorrect.
- (d) Assertion is incorrect, reason is correct.
57. **Assertion:** The number of non-empty subsets of the set $\{a, b, c, d\}$ are 15.
- Reason:** Number of non-empty subsets of a set having n elements are $2^n - 1$.
58. Suppose A, B and C are three arbitrary sets and U is a universal set.
- Assertion:** If $B = U - A$, then $n(B) = n(U) - n(A)$.
- Reason:** If $C = A - B$, then $n(C) = n(A) - n(B)$.
59. **Assertion:** Let $A = \{1, \{2, 3\}\}$, then
- $P(A) = \{\{1\}, \{2, 3\}, \phi, \{1, \{2, 3\}\}\}$
- Reason:** Power set is set of all subsets of A .
60. **Assertion:** The subsets of the set $\{1, \{2\}\}$ are $\{\}, \{1\}, \{\{2\}\}$ and $\{1, \{2\}\}$.
- Reason:** The total number of proper subsets of a set containing n elements is $2^n - 1$.

ANSWERS

1. $\left\{x : x = \frac{n}{n+1}, n \in N, n \leq 19\right\}$
2. $\{1\}$

3. (i) \notin (ii) \notin (iii) \notin
4. $A = \{-1, 0, 1, 2, 3\}$
5. $B = \{x : x = 3^n, n \in \mathbb{N} \text{ and } 1 \leq n \leq 4\}$
6. Empty set because no natural number is lying between 3 and 4
7. Non-empty set because $B = \{1\}$
8. Infinite set because circle is a collection of infinite points whose distances from the centre is constant called radius.
9. Finite set because $B = \{2\}$
10. Yes, because $x^2 - 4 = 0 \Rightarrow x = 2 \text{ or } -2$ both are integers
11. $\{x : x \in \mathbb{R}, -5 < x \leq 9\}$
12. $[-3, 7)$
13. $2^3 = 8$
14. $\phi, \{5\}, \{6\}, \{5, 6\}$
15. $A \cup B = \{D, E, L, H, I, O\}$
16. $A \cap B = \{D, L\}$
17. $A - B = \{E, H, I\}$
18. (i) $\{A, R, I, T, H, M, E, C\}$ (ii) $\{E, U, A, I, O\}$
19. $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$
20. $\{16, 25, 34, 43, 52, 61, 70\}$
21. Yes, because $A \cap B = \phi$
22. $(A - B) \cup (B - A)$ or $A \Delta B$
23. $A \cap (B \cup C)$
24. $\cup = \{1, 2, 3, \dots, 20\}$

25. $n(A \cap B) = 4$, $n(A - B) = 6$

29. (i) $(-\infty, 0) \cup (3, \infty)$ (ii) $[0, 6]$ (iii) $[2, 3]$ (iv) $[0, 2)$

30. **Hint:** U = set of people surveyed
 A = set of people who play cricket
 B = set of people who play tennis

Number of people who play neither cricket nor tennis

$$\begin{aligned} &= n[(A \cup B)'] = n(U) - n(A \cup B) \\ &= 450 - 200 = 250 \end{aligned}$$

31. There are 280 students in the group.

32. 23 [Hint: Find $n(U)$]

33. 6

35. $n = 3$ $m = 6$

36. [Hint: Take $n(\cup) = 100$]

37. 14

38. i. 275 ii. 325 iii. 50 iv. 125

39. i. (c) ii. (a) iii. (b) iv. (d) v. (a)

40. i. (b) ii. (a) iii. (d) iv. (c) v. (b)

41.(c) 42.(b) 43.(c)

44.(b) 45.(c) 46.(c)

47.(a) 48.(c) 49.(c)

50.(a) 51.(c) 52.(c)

53.(a) 54.(c) 55.(b)

56.(b) 57.(a) 58.(c)

59.(d) 60.(b)