CHAPTER-13

PROBABILITY

Probability is the branch of mathematics that deals with assigning a numerical quantity $(0 \leq p \leq 1)$ to the happening/non happening of any event.

Real Life Applications

- Weather Forecasting
- Sports betting
- Investing in stock market
- Politics
- How likely a natural disaster like earthquake, hurricane etc. will strike the country in a given year.

A sports betting company may look at the current record of two teams A and B and determine which team has higher probability of winning and do the sports betting accordingly.

TOPICS TO BE COVERED AS PER CBSE LATEST CURRICULUM (2024-25)

- Conditional probability
- Multiplication theorem on probability
- Independent events
- Total probability and Baye's theorem
- Random variable and its probability distribution
- Mean of random variable

KEY POINTS

Conditional Probability : If A and B are two events associated with the same sample space of a random experiment, then the conditional probability of the event A under the condition that the event B has already occurred, written as $P(A|B)$, is given by

$$P(A|\mathrm{B}) = \frac{(P \cap B)}{P(B)}, \quad P(B) \neq 0.$$

Properties :

(1) $P(S|F) = P(F|F) = 1$ where S denotes sample space

(2) $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$

(3) $P(E'|F) = 1 - P(E|F)$

Multiplication Rule : Let E and F be two events associated with a sample place of an experiment. Then

$$P(E \cap F) = P(E)\,P(F|E) \text{ provided } P(E) \neq 0$$
$$= P(F)\,P(E|F) \text{ provided } P(F) \neq 0.$$

If E, F, G are three events associated with a sample space, then

$$P(E \cap F \cap G) = P(E)\,P(F|E)\,P(G|(E \cap F))$$

Independent Events : Let E and F be two events, then if probability of one of them is not affected by the occurrence of the other, then E and F are said to be independent, i.e.,

(*a*) $P(F|E) = P(F)$, $P(E) \neq 0$

or (*b*) $P(E|F) = P(E)$, $P(F) \neq 0$

or (*c*) $P(E \cap F) = P(E)\,P(F)$

Three events A, B, C are mutually independent if

$$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$$
$$P(A \cap B) = P(A)\,P(B)$$
$$P(B \cap C) = P(B)\,P(C)$$

and $P(A \cap C) = P(A)\,P(C)$

Partition of a Sample Space : A set of events $E_1, E_2, ..., E_n$ is said to represent a partition of a sample space S if

(*a*) $E_i \cap E_j = \phi$; $i \neq j$; $i, j = 1, 2, 3, ..., n$

(*b*) $E_1 \cup E_2 \cup E_3 ... \cup E_n = S$ and

(*c*) Each $E_i \neq \phi$ i.e. $P(E_i) > 0 \ \forall \ i = 1, 2, ..., n$

Theorem of Total Probability : Let $\{E_1, E_2, ..., E_n\}$ be a partition of the sample space S. Let A be the any event associated with S, then

$$P(A) = \sum_{j=1}^{n} P(E_j)\,P(A|E_j)$$

Baye's Theorem : If E_1, E_2, ..., E_n are mutually exclusive and exhaustive events associated with a sample space S, and A is any event associated with E_i's having non-zero probability, then

$$P(E_i|A) = \frac{P(A\,|\,E_i)P(E_i)}{\sum_{i=1}^{n} P(A\,|\,E_i)P(E_i)}$$

Random Variable : A (r.v.) is a real variable which is associated with the outcome of a random experiment.

Probability Distribution of a r.v. X is the system of numbers given by

X :	x_1	x_2	....	x_n
$P(X = x)$:	p_1	p_2	....	p_n

where $\quad p_i > 0, \quad i = 1, 2, ..., n, \quad \sum_{i=1}^{n} p_i = 1.$

Mean of a r.v. *X* :

$$\mu = E(X) = \sum_{i=1}^{n} p_i\, x_i$$

Illustration:

Evaluate P(A $\cup$ B) if 2P(A) = P(B) = $\frac{5}{13}$ and P(A|B) = $\frac{2}{5}$

Solution: 2P(A) = P(B) = $\frac{5}{13}$

$\Rightarrow$ P(A) = $\frac{5}{26}$, P(B) = $\frac{5}{13}$

As $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\Rightarrow \dfrac{2}{5} = \dfrac{P(A \cap B)}{(5/13)} \quad \Rightarrow \quad \dfrac{2}{\cancel{5}} \times \dfrac{\cancel{5}}{13} = P(A \cap B)$

$\Rightarrow \dfrac{2}{13} = P(A \cap B)$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{16}$$

Illustration:

Prove that if E and F are independent events, then the events E and F′ are also independent.

Solution: $P(E \cap F) = P(E)\,P(F)$ (given)

Consider, $\quad P(E \cap F') = P(E) - P(E \cap F)$

$$= P(E) - P(E)\,P(F)$$

$$= P(E)\,(1 - P(F))$$

$$P(E \cap F') = P(E) - P(F')$$

So, E and F′ are also independent.

Illustration:

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. What is the probability that they both are diamonds.

Solution: Let E_1 = lost card is diamond

E_2 = lost card is non-diamond

A = 2 diamonds cards are drawn from the remaining cards

Using Theorem of total probability

$$P(A) = P(A|E_1)\,P(E_1) + P(A|E_2)\,P(E_2)$$

$$= \frac{12}{51} \times \frac{11}{50} \times \frac{\cancel{13}}{\cancel{52}_4} + \frac{12}{50} \times \frac{13}{51} \times \frac{\cancel{39}^3}{\cancel{52}_4}$$

$$= \frac{132}{10200} + \frac{468}{10200} = \frac{\cancel{600}}{\cancel{10200}_{17}} = \frac{1}{17}$$

Illustration:

Three cards are drawn at random (without replacement) from a well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence, find the mean of the distribution.

Solution: Let X denotes the number of red cards

$$\therefore \quad P(X = 0) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{26 \times 25 \times 24}{52 \times 51 \times 50} = \frac{2}{17} = \frac{4}{34}$$

$$P(X = 1) = \frac{{}^{26}C_1 \times {}^{26}C_2}{{}^{52}C_3} = 26 \times \frac{26 \times 25}{2} \times \frac{3 \times 2 \times 1}{52 \times 51 \times 50} = \frac{13}{34}$$

$$P(X = 2) = \frac{{}^{26}C_2 \times {}^{26}C_1}{{}^{52}C_3} = \frac{26 \times 25 \times 26 \times 3 \times 2 \times 1}{2 \times 52 \times 51 \times 50} = \frac{13}{34}$$

$$P(X = 3) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{4}{34}$$

$\therefore$ **Probability Distribution**

X	P(X = x)	X.P(x)
0	$\frac{4}{34}$	0
1	$\frac{13}{34}$	$\frac{13}{34}$
2	$\frac{13}{34}$	$\frac{26}{34}$
3	$\frac{4}{34}$	$\frac{12}{34}$
	$\sum p_i = 1$	$\bar{x} = \sum p_i x_i$

$$\therefore \quad \bar{x} = \sum p_i x_i = \frac{13}{34} + \frac{26}{34} + \frac{12}{34} = \frac{51}{34} = \frac{3}{2}$$

ONE MARK QUESTIONS

1. The events E and F are independent. If P(E) = 0.3 and P(EUF) = 0.5, then P(E/F) – P(F/E) equals:

   (a) $\frac{1}{7}$ (b) $\frac{2}{7}$

   (c) $\frac{3}{35}$ (d) $\frac{1}{70}$

2. For two events A and B, if P(A) = 0.4, P(B) = 0.8, P(B/A) = 0.6, then P(AUB) is:

   (a) 0.24 (b) 0.3

   (c) 0.48 (d) 0.96

3. If A and B are two events such that P(A/B) = 2 × P(B/A) and P(A) + P(B) = $\frac{2}{3}$, then P(B) is equal to

   (a) $\frac{2}{9}$ (b) $\frac{7}{9}$

   (c) $\frac{7}{9}$ (d) $\frac{5}{9}$

4. Two events A and B will be independent, if;

   (a) A and B are mutually exclusive

   (b) P(A) = P(B)

   (c) P(A′B′) = [1 – P(A)] [1 – P(B)]

   (d) P(A) + P(B) = 1

5. If for any two events A and B, P(A) = $\frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to

   (a) $\frac{1}{10}$ (b) $\frac{1}{8}$

   (c) $\frac{7}{8}$ (d) $\frac{17}{20}$

6. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is

   (a) $\frac{27}{32}$ (b) $\frac{5}{32}$

   (c) $\frac{31}{32}$ (d) $\frac{1}{32}$

7. Ashima can hit a target 2 out of 3 times. She tried to hit the target twice. The probability that she missed the target exactly one is

   (a) $\frac{2}{3}$ (b) $\frac{1}{3}$

   (c) $\frac{4}{9}$ (d) $\frac{1}{9}$

8. If sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4 is

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$

(c) $\frac{1}{18}$ (d) $\frac{1}{2}$

9. X & Y are independent events such that $P(x \cap \bar{y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) $\frac{1}{3}$ (d) $\frac{1}{5}$

10. If for two events A and B, P(A – B) = $\frac{1}{5}$ and P(A) = $\frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) $\frac{2}{3}$

11. If A and B are two events such that P(A) > 0 and P(B) ≠ 1, then $P(\bar{A} \mid \bar{B}) =$

(a) $1 - P(A/B)$ (b) $1 - P(A/\bar{B})$

(c) $\frac{1 - P(A \cup B)}{P(B)}$ (d) $\frac{P(\bar{A})}{P(B)}$

12. A and B are events such that P(A/B) = P(B/A) then

(a) $A \subset B$ (b) B = A

(c) $A \cap B = \phi$ (d) P(A) = P(B)

13. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the II plane is

(a) 0.2 (b) 0.7

(c) 0.06 (d) 0.14

14. P(E $\cap$ F) is equal to

(a) P(E) P(F/E) (b) P(F).P(E/F)

(c) Both (a) & (b) (d) None of these

15. Two dice are thrown. If it is known that the sum of the numbers on the dice is less than 6, the probability of getting a sum 3 is

(a) $\frac{1}{8}$ (b) $\frac{2}{5}$

(c) $\frac{1}{5}$ (d) $\frac{5}{18}$

In following questions Q16 to Q20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(c) (A) is true and (R) is false

(d) (A) is false but (R) is true

16. Assertion (A) : The mean of a random variable X is also called the expectation of x, denoted by E(x).

Reason (R) : The mean or expectation of a random variable X is the sum of the producuts of all possible values of x by their respective probabilities.

17. Assertion (A) : Let A and B be two independent events. The $P(A \cap B) = P(A) + P(B)$

Reason (R) : Three events A, B and C are said to be independent if

$P(A \cap B \cap C) = P(A).P(B).P(C)$

18. Assertion (A) : Two coins are tossed simultaneously. The probabiltiy ofgetting two heads, if it is known that atleast one head comes up is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then

$$P(F/E) = \frac{P(F \cap E)}{P(E)}$$

19. Assertion (A) : The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is 2

Reason (R): E(X) = mean of $x = \sum_{i=1}^{n} p_i x_i$

20. Assertion (A) : Bag P contains 6 Red and 4 Blue balls and Bag Q contains 5 red and 6 Blue Balls. A ball is transferred from Bag P to bag Q and then a ball is drawn from Bag Q. The probability that the ball drawn from bag Q is blue is $\frac{8}{15}$.

Reason (R) : According to the law of total probability

$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$ where E_1 and E_2 partitions the sample space S and A is any event connected with E_1 and E_2.

TWO MARKS QUESTIONS

1. *A* and *B* are two events such that $P(A) \neq 0$, then find $P(B|A)$ if (*i*) *A* is a subset of *B* (*ii*) $A \cap B = \phi$.

2. A random variable *X* has the following probability distribution, find *k*.

X	0	1	2	3	4	5
$P(X)$	$\frac{1}{15}$	K	$\frac{15K-2}{15}$	K	$\frac{15K-1}{15}$	$\frac{1}{15}$

3. Out of 30 consecutive integers two are chosen at random. Find the probability so that their sum is odd.

4. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. Find the probability that the eldest child is a girl given that the family has atleast one girl.

5. If *A* and *B* are such that $P(A \cup B) = \frac{5}{9}$ and $P(\bar{A} \cup \bar{B}) = \frac{2}{3}$, then find $P(\bar{A}) + P(\bar{B})$.

6. Prove that if *A* and *B* are independent events, then *A* and *B′* are also independent events.

7. If *A* and *B* are two independent events such that $P(A) = 0.3$, $P(A \cup B) = 0.5$, then find $P(A|B) - P(B|A)$

8. Three faces of an ordinary dice are yellow, two faces are red and one face is blue. The dice is rolled 3 times. Find the probability that yellow, red and blue face appear in the first, second and third throw respectively.

9. Find the probability that a leap year will have 53 Fridays or 53 Saturdays.

10. A person writes 4 letters and addresses on 4 envelopes. If the letters are placed in the envelopes at random, then what is the probability that all the letters are not placed in the right envelopes.

11. Find the mean of the distribution

X = x	0	1	2	3	4	5
P(X = x)	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

12. In a class XII of a school, 40% of students study Mathematics, 30% of the students study Biology and 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, then find the probability that he will be studying Mathematics or Biology.

THREE MARKS QUESTIONS

Q.1. A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem is solved ?

Q.2. If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$ then find $P(A)$ and $P(B)$.

Q.3. From a lot of 20 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random, one by one with replacement. Find the probability distribution of the number of defective bulbs. Also, find the mean of the distribution.

Q.4. Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is 1/5 and that of Nisha's selections is 1/6. What is the probability that

(i) both of them are selected?

(ii) only one of them is selected?

(iii) none of them is selected?

Q.5. In a game, a man wins a rupee for a six and looses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/looses.

Q.6. Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability that the selected person is male assuming that there are 60% males and 40% females ?

Q.7. Two dice are thrown once. Find the probability of getting an even number on the first die or a total of 8.

Q.8. Two aeroplanes X and Y bomb a target in succession. There probabilities to hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if first miss the target. Find the probability that target is hit by Y plane.

Q.9. The random variable X can take only the values 0, 1, 2. Given that $P(X = 0) = P(X = 1) = p$ and that $E(X^2) = E(X)$, find the value of p.

Q.10. An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of 3 balls. Find the mean of X.

Q11. A box contains 10 tickets, 2 of which carry a prize of Rupees 8 each, 5 of which carry a prize of Rupees 4 each and remaining 3 carry a prize of Rupees 2 each. If one ticket is drawn at random, find the mean value of the prize. Using the concept of probability distribution.

Q.12. The probability distribution of a random variable X is given below:

X	1	2	3
P(X)	K/2	K/3	K/6

(i) Find the value of K

(ii) Find $P(1 \leq X < 3)$

(iii) Find E(X), the mean of X.

Q.13. A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

Q.14. A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of x?

Q.15. There are two coins. One of them is a biased coin such that

P(Head) : P(tail) is 1 : 3 and the other is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

Q.16. Two numbers are selected from first six even natural numbers at random without replacement. If X denotes the greater of two numbers selected, find the probability distribution of X.

Q.17. A fair coin and an unbiased die are tossed. Let A be the event "Head appears on the coin" and B' be the event, "3 comes on the die". Find whether A and B are independent or not.

FIVE MARKS QUESTIONS

Q.1. By examining the chest X-ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of a healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB. A person is selected at random and is diagnosed to have TB. What is the probability that he actually has TB ?

Q.2. Three persons *A*, *B* and *C* apply for a job of Manager in a private company. Chances of their selection (*A*, *B* and *C*) are in the ratio 1 : 2 : 4. The probabilities that *A, B* and *C* can introduce charges to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change doesn't take place, find the probability that it is due to the appointment of *C*.

Q.3. A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope, just two consecutive letters TA are visible. What is the probability that the letter came from TATANAGAR.

Q.4. The probability distribution of a random variable *X* is given as under :

$$P(X = x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3 \\ 2kx & \text{for } x = 4, 5, 6 \\ 0 & \text{Otherwise} \end{cases}$$

where *k* is a constant. Calculate

(*i*) $E(X)$ (*ii*) $E(3X^2)$ (*iii*) $P(X \geq 4)$

Q.5. Three critics review a book. Odds in favour of the book are 5 : 2, 4 : 3 and 3 : 4 respectively for the three critics. Find the probability that the majority are in favour of the book.

Q.6. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6, 7. Let *X* denotes the larger of the two numbers obtained. Find the mean of the probability distribution of *X*.

Q.7. An urn contains five balls. Two balls are drawn and are found to be white. What is the probability that all the balls are white?

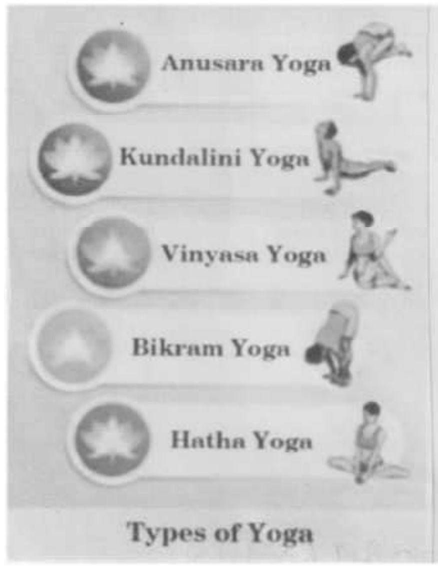
Q.8. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained.

Q.9. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the possibility of the lost card being of club.

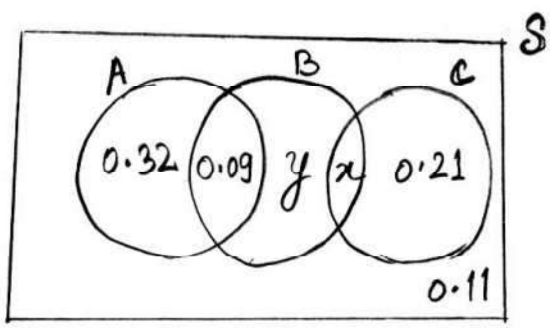
Q.10. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II at random. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

CASE STUDY QUESTIONS

Q.1. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:



The Venn Diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further it is given that probability of a member performing type C Yoga is 0.44.



On the basis of the above information, answer the following questions:

(i) Find the value of x.

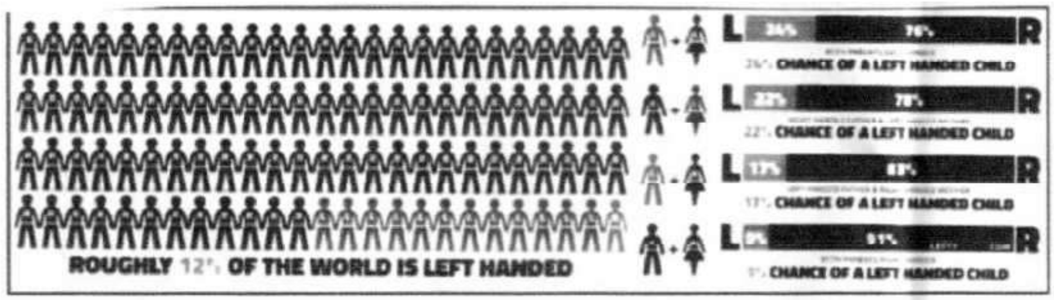
(ii) Find the value of y.

(iii) (a) Find $P\left(\frac{C}{B}\right)$

OR

(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

Q.2. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of havig a left handed child are as follows:

A : When both father and mother are left handed

Chances of left handed child is 24%

B : When father is right handed and mother is left handed:

Chances of left handed child is 22%

C : When father is left handed and mother is right handed:

Chances of left handed child is 17%

D : When both father and mother are right handed:

Chances of left handed child is 9%

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions:

(i) Find $P(L/C)$

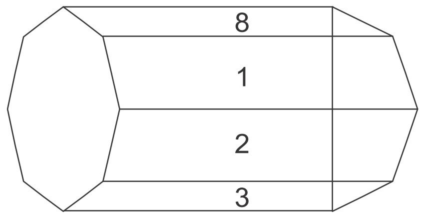
(ii) Find $P(\bar{L}/A)$

(iii) (a) Find $P(A/L)$

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

Q.3 An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtianed on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X):	P	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$

Based on the above information, answer the following questions:

(i) Find the value o f p

(ii) FInd P(X > 6)

(iii) (a) Find P(x = 3m), where m is a natural number

OR

(iii) (b) Find the mean E(X)

Q.4. In a birthday party, a magician was being invited by a parent and he had 3 bags that contain number of red and white balls as follows:

Bag 1 contains : 3 red balls, Bag 2 contains : 2 white balls and 1 Red ball

Bag 3 contains : 3 white balls

The probability that the bag *i* will be chosen by the magician and a ball is selected from it is $\frac{i}{6}, i = 1, 2, 3$.

Based on the above information, answer the following questions.

(a) What is the probability that a red ball is selected by the magician?

(b) What is the probability that a white ball is selected by the magician?

(c) Given that the magician selects the white ball, what is the probability that the ball was from Bag 2.

Q.5. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information answer the following :

(*i*) Find the conditional probability that an error is committed in processing given that Sonia processed the form?

(*ii*) What is probability that Sonia processed the form and committed an error?

(*iii*) What is total probability of committing an error in processing the form?

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. If A and B are independent events such that P(A) = 0.4, P(B) = x and P(A $\cup$ B) = 0.5, then x = ?

(a) $\frac{4}{5}$ (b) 0.1

(c) $\frac{1}{6}$ (d) None of these

2. If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4, then P(A/B) is

(a) 0.32 (b) 0.64

(c) 0.16 (d) 0.25

3. A couple has two children. What is the probability that both are boys if it is known that one of them is a boy?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

4. The random variable X has a probability distribution P(X) of the following form, where '*k*' is some number.

$$P(X = x) = \begin{cases} k, & \text{if} \quad x = 0 \\ 2k, & \text{if} \quad x = 1 \\ 3k, & \text{if} \quad x = 2 \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of *k*.

5. If two events are independent, then

(a) they must be mutually exclusive

(b) the sum of their probabilities must be equal to 1

(c) (a) and (b) both are correct

(d) none of the above is correct

SELF ASSESSMENT-2

EAH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability that both cards are queens is

(a) $\frac{1}{13} \times \frac{1}{13}$ (b) $\frac{1}{13} + \frac{1}{13}$

(c) $\frac{1}{13} \times \frac{1}{17}$ (d) $\frac{1}{13} \times \frac{4}{51}$

2. The probability distribution of a discrete random variable X is given below:

X	2	3	4	5
P(X = x)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The values of k is

(a) 8 (b) 16

(c) 32 (d) 48

3. Three persons A, B and C fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

(a) 0.024 (b) 0.188

(c) 0.336 (d) 0.452

4. If $4P(A) = 6P(B) = 10P(A \cap B) = 1$, then $P(B/A) = ?$

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{7}{10}$ (d) $\frac{19}{60}$

5. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is

(a) $\frac{5}{17}$ (b) $\frac{12}{17}$

(c) $\frac{17}{30}$ (d) $\frac{3}{5}$

ANSWER

One Mark Questions

1. (d) $\frac{1}{70}$	2. (d) 0.96	3. (a) $\frac{2}{9}$	4. (c)
5. (c) $\frac{7}{8}$	6. (c) $\frac{31}{32}$	7. (c) $\frac{4}{9}$	8. (d) $\frac{1}{2}$
9. (c) $\frac{1}{3}$	10. (d) $\frac{2}{3}$	11. (b) $1-P(A/\overline{B})$	12. (d) P(A) = P(B)
13. (d) 0.14	14. (c) Both (a) & (b)	15. (c) $\frac{1}{5}$	16. (a)
17. (d)	18. (a)	19. (a)	20. (a)

Two Marks Questions

1. (*i*) 1 (*ii*) 0
2. $\frac{4}{15}$
3. $\frac{15}{29}$
4. $\frac{4}{7}$
5. $\frac{10}{9}$
7. $\frac{1}{70}$
8. $\frac{1}{36}$
9. $\frac{3}{7}$
10. $\frac{23}{24}$
11. $\frac{35}{18}$
12. 0.6

Three Marks Questions

1. $\frac{3}{4}$
2. $P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{6}$ or $P(A) = \frac{5}{6}$ and $P(B) = \frac{4}{5}$
3. $\frac{1}{2}$

x	0	1	2
P(x)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

4. (i) $\frac{1}{30}$ (ii) $\frac{3}{10}$ (iii) $\frac{2}{3}$
5. $-\frac{91}{54}$
6. $\frac{3}{4}$
7. $\frac{5}{9}$
8. $\frac{7}{50}$
9. $\frac{1}{2}$
10. $\frac{9}{7}$
11. 4.2 ₹
12. (i) k = 1 (ii) $\frac{5}{6}$ (iii) $\frac{5}{3}$
13. $\frac{1}{3}$ and $\frac{1}{4}$, $\frac{3}{4}$ and $\frac{2}{3}$
14.

X	0	1	2	3	4	5
P(X)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

15. $\frac{1}{3}$
16.

X	4	6	8	10	12
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

17. Yes, A and B are independent.

Five Marks Questions

1. $\frac{110}{221}$
2. $\frac{7}{10}$
3. $\frac{7}{11}$
4. (*i*) 4.31, (*ii*) 61.9, (*iii*) $\frac{15}{22}$

5. $\frac{209}{343}$ 6. $\overline{x} = \frac{17}{3}$ 7. $\frac{1}{2}$ 8. $\overline{x} = \frac{6}{13}, \sigma^2 = \frac{60}{169}$

9. $\frac{11}{50}$ 10. $\frac{16}{31}$

CASE STUDY QUESTIONS

1. (i) x = 0.23 (ii) y = 0.04 (iii) (a) $\frac{23}{36}$ or (b) 0.46

2. (i) P(L/C) = 0.17, (ii) $P(\overline{L}/A) = 0.76$ (iii) (a) $P(A/L) = \frac{1}{3}$ or (*b*) 0.39

3. (i) $P = \frac{1}{10}$ (ii) $P(x > 6) = \frac{19}{100}$

   (iii) (a) $\frac{21}{100}$ or (b) $E(x) = 4.06$

4. (*a*) $\frac{5}{18}$ (*b*) $\frac{13}{18}$ (*c*) $\frac{4}{13}$

5. (*i*) 0.04 (*ii*) 0.008 (*iii*) 0.047

SELF ASSESSMENT-1

1. (c) 2. (b) 3. (a) 4. $k = \frac{1}{6}$ 5. (d)

SELF ASSESSMENT-2

1. (a) 2. (c) 3. (b) 4. (a) 5. (b)