# CHAPTER-12

# LINEAR PROGRAMMING

Linear programming is used to obtain optimal solutions for operations research. Using LPP, researchers find the **best**, most economical **solution** to a problem within all of its **limitations**, or constraints.

Few examples of applications of LPP

(i) **Food and Agriculture:** In nutrition, Linear programming provides a powerful tool to aid in planning for dietary needs. Here, we determine the different kinds of foods which should be included in a diet so as to **minimize** the cost of the desired diet such that it contains the minimum amount of each nutrient.

(ii) **Transportation:** Systems rely upon linear programming for cost and time efficiency.

**Airlines** use linear programming to optimize their profits according to different seat prices and customer demand. Because of this only, efficiency of airlines increases and expenses are decreased.

| TOPICS TO BE COVERED AS PER CBSE LATEST CURRICULUM 2024-25 |
|---|
| • Introduction, constraints, objective function, optimization. |
| • Graphical method of solution for problems in two variables. |
| • Feasible and infeasioble region (bounded or unbounded) |
| • Feasible and infeasible solutions. |
| • Optimal feasible solutions (upto three non-trival constraints) |

**KEY POINTS :**

- **OPTIMISATION PROBLEM :** is a problem which seeks to maximize or minimize a function. An optimisation problem may involve maximization of profit, minimization of transportation cost etc, from available resources.
- **A LINEAR PROGRAMMING PROBLEM (LPP) :** LPP deals with the optimisation (maximisation/minimisation) of a linear function of two variables (say x and y) known as objective function subject to the conditions that the variables are non negative and satisfy a set of linear inequalities (called linear constraints). A LPP is a special type of optimisation problem.
- **OBJECTIVE FUNCTION :** Linear function $z = ax + by$ where a and b are constants which has to be maximised or minimised is called a linear objective function.
- **DECISION VARIABLES :** In the objective function $z = ax + by$, $x$ and $y$ are called decision variables.
- **CONSTRAINTS :** The linear inequalities or restrictions on the variables of an LPP are called constraints.

  The conditions $x \geq 0, y \geq 0$ are called non-negative constraints.
- **FEASIBLE REGION :** The common region determined by all the constraints including non-negative constraints $x \geq 0, y \geq 0$ of a LPP is called the feasible region for the problem.
- **FEASIBLE SOLUTION :** Points within and on the boundary of the feasible region for a LPP represent feasible solutions.
- **INFEASIBLE SOLUTIONS :** Any point outside the feasible region is called an infeasible solution.
- **OPTIMAL (FEASIBLE) SOLUTION :** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
- **THEOREM 1 :** Let R be the feasible region (convex polygon) for a LPP and let $z = ax + by$ be the objective function. When $z$ has an optimal value (maximum or minimum), where x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.
- **THEOREM 2 :** Let R be the feasible region for a LPP. & let $z = ax + by$ be the objective function. If R is bounded, then the objective function $z$ has both a maximum and a minimum value on R and each of these occur at a corner point of R.

  If the feasible region R is unbounded, then a maximum or minimum value of the objective function may or not exist. However, if it exists it must occur at a corner point of R.

- **MULTIPLE OPTIMAL POINTS :** If two corner points of the feasible region are optimal solutions of the same type i.e both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type.

**Illustration:**

A company produces two types of belts A and B. Profits on these belts are Rs. 2 and Rs. 1.50 per belt respectively. A belt of type A requires twice as much time as belt of type B. The company can produce atmost 1000 belts of type B per day. Material for 800 belts per day is available. Atmost 400 buckles for belts of type A and 700 for type B are available per day. How much belts of each type should the company produce so as to maximize the profit?

**Solution:** Let the company produces $x$ no. of belts of type A and $y$ no. of belts of type B to maximize the profit.

$\therefore$ **Objective function** Max $z = 2x + 1.5y$

As, maximum 1000 belts of type B : 1 day

$\therefore$ 1 belt of type B : $\left(\frac{1}{1000}\right)^{th}$ of a day

ATQ, 1 belt of type A : $\left(\frac{2}{1000}\right)^{th}$ of a day

$\therefore \frac{2x}{1000} + \frac{y}{1000} \le 1$

$\Rightarrow 2x + y \le 1000$

L.P.P becomes

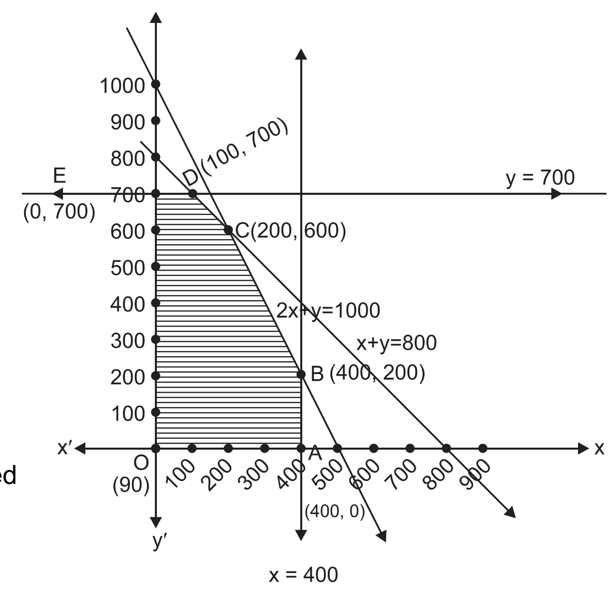
Max $z = 2x + 1.5y$

s.t. $2x + y \le 1000$

$x + y \le 800$

$x \le 400$, $y \le 700$, $x \ge 0$, $\ge 0$

Here, the feasible negion is bounded given be region OABCDE.



Using Corner point method.

| Corner Points | Obj. fn. z = 2x + 1.5y |
|---|---|
| O (0, 0) | 0 |
| A (400, 0) | 800 |
| B (400, 200) | 1100 |
| C (200, 600) | 1300 ← max z. |
| D (100, 700) | 1250 |
| E (0, 700) | 1050 |

∴ Optimal solution is given by C(200, 600)

i.e. company should produce 200 belts of type A and 600 belts of type B so as to maximize the profit of Rs. 1300.

## ONE MARK QUESTIONS

1. The solution set of the inequation $3x + 4y < 7$ is:

   (a) Whole xy plane except the points lying on the line $3x + 5y = 7$

   (b) Whole xy plane alogn with the points lying on the line $3x + 5y = 7$

   (c) Open half plane containging the origin except the point of line $3x + 5y = 7$

   (d) Open half plane not containging the origin except the point of line $3x + 5y = 7$

2. Which of the following points solisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?

   (a) (–2, 4) (b) (3, 2)

   (c) (–5, 6) (d) (4, 2)

3. The objective function $Z = ax + by$ of LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true?

   (a) $a = 9, b = 1$ (b) $a = 5, b = 2$

   (c) $a = 3, b = 5$ (d) $a = 5, b = 3$

4. The corner points of the feasible region of a LPP are (0, 4), (7, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If

   $z = 30x + 24y$ is the objective functions, then (maximum value of z-minimum value of z) is equal to

   (a) 40 (b) 96

   (c) 120 (d) 136

5. The minimum value of $z = 3x + 8y$ subject to the constraints $x \leq 20$, $y \geq 10$ and $x \geq 0$, $y \geq 0$ is

   (a) 80 (b) 140

   (c) 0 (d) 60

6. The number of corner points of the feasible region determined by the constraints

   $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is

   (a) 2 (b) 3

   (c) 4 (d) 5

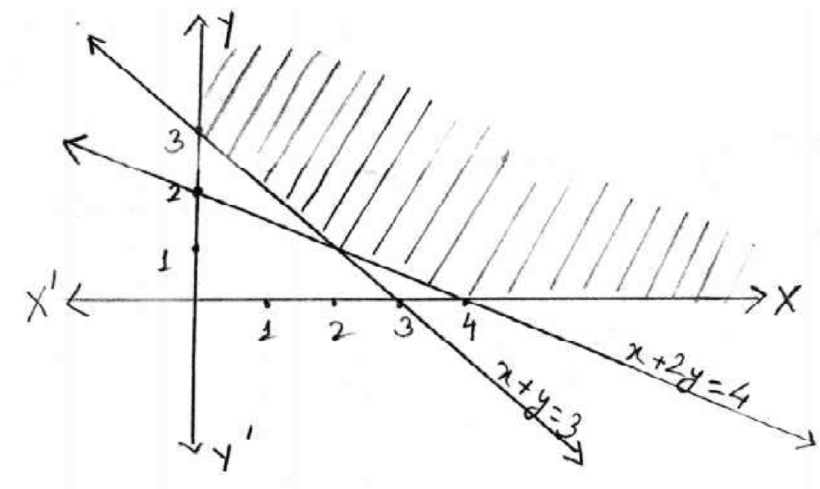
7. The no. of feasible solutions of the L.P.P. given as maximise $z = 15x + 30y$ subject the constraints:

   $3x + y \leq 12$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$ is

   (a) 1 (b) 2

   (c) 3 (d) infinite

8. The feasible region of a linear programming problem is shown in the figure below:




Which of the following are the possible constraints?

(a) $x + 2y \geq 4, x + y \leq 3, x \geq 0, y \geq 0$

(b) $x + 2y \leq 4, x + y \leq 3, x \geq 0, y \geq 0$

(c) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$

(d) $x + 2y \geq 4, x + y \leq 3, x \leq 0, y \leq 0$

9. L.P.P. is a process of finding

(a) Maximum value of the objective function

(b) Minimum value of the objective function

(c) Optimum value of the objective function

(d) None of these

10. Which of the following statements is correct?

(a) Every L.P.P. admits an optimal solution

(b) A L.P.P. admits a unique optimal solution

(c) If a L.P.P. admits two optimal solutions, it has an infinite number of optimal solutions

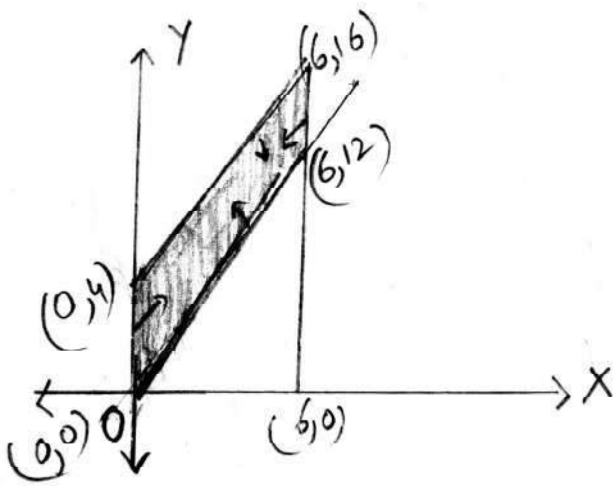
(d) The set of all feasible solution of a L.P.P. is not a convex set

11. Region represented by $x \geq 0$, $y \geq 0$ is

(a) First quadrant (b) Second quadrant

(c) Third quadrant (d) Fourth quadrant

12. The feasible region for L.P.P. is shown shaded in the figure. Let $f = 3x - 4y$ be the objective function, then maximum value of $f$ is




(a) 12 (b) 8

(c) 0 (d) –18

13. The area of the feasible region fo the following constraints

$3y + x \geq 3$, $x \geq 0$, $y \geq 0$ will be

(a) Bounded (b) Unbounded

(c) Convex (d) Concave

14. The line $5x + 4y \geq 20, x \leq 6, y \leq 4$ form,

(a) A square (b) A rhombus

(c) A triangle (d) A qudrilateral

15. The graph of inequations $x \leq y$ and $y \leq x + 3$ is located in

(a) II quadrant (b) I, II quadrant

(c) I, II and III quadrant (d) II, III, IV quadrant

## ASSERTION-REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion (A) and Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(c) (A) is true and (R) is false

(d) (A) is false but (R) is ture

16. Assertion (A) : If a L.P.P. admits two optional solution then it has infinitely many optimal solution.

    Reason (R) : If the value of the objective function of a L.P.P. is same at two corners then it is same at every point on the line segment joining the two corner pionts.

17. Assertion (A): The solution region satisfied by the inequalities $x + y \le 5$, $x \le 4$, $y \le 4$, $x \ge 0$, $y \ge 0$ is bounded.

    Reason (R) : A region in x-y plane is said to be bounded if it can be enclosed within a circle.

18. Assertion (A) : Minimize $z = x^2 + 2xy + y^2$ can be considered as the objective function for the L.P.P.

    Reason (R) : Objective function of the L.P.P. is of this type $z = ax + by$; a and b are real numbers i.e. z is linear function of x and y.

19. Assertion (A) : The region represented by the inequalities $x \ge 6$, $y \ge 2$, $2x + y \ge 10$, $x \ge 0$, $y \ge 0$ is empty.

    Reason (R) : There is no (x, y) that satisfies all the constraints.

20. Assertion (A) : Corner points of the feasible region for an L.P.P. are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function. The minimum value of F occurs at (0, 2) only.

    Reason (R) : Minimum value of F occurs at all the infinite no. points that lie on the line segment joining (0, 2) and (3, 0).

## THREE MARKS QUESTIONS

1. Solve the following linear programming problem graphically:

   Maximise $z = -3x - 5y$

   subject to the constraints

   $-2x + y \le 4$

   $x + y \ge 3$

   $x - 2y \le 2$

   $x \ge 0, y \ge 0$

2. Solve the following LPP graphically:

Maximise $z = 5x + 3y$

s.t. the constraints

$3x + 5y \leq 15$

$5x + 2y \leq 10$

$x, y \geq 0$

3. Solve the following LPP graphically

Maximise $z = x + 2y$

s.t. $x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x \geq 0, y \geq 0$

4. The objective function $z = 4x + 3y$ of a LPP under some constraints is to be maximized and minimized. The corner points of the feasible region are A(0, 700), B(100, 700), C(200, 600) and D(400, 200). Find the point at which z is maximum and the point at which z is minimum. Also find the corresponding maximum and minimum values of z.

5. Solve graphically

Minimise : $z = -3x + 4y$

s.t. $3x + 2y \leq 12$

$x, y \geq 0$

6. Solve the following LPP graphically

Minimise: $Z = 60x + 80y$

s.t. $3x + 4y \geq 8$

$5x + 2y \geq 11$

$x, y \geq 0$

7. Solve graphically

Maximise : $z = 600x + 400y$

s.t. $x + 2y \leq 12$

$2x + y \leq 12$

$x + 1.25y \geq 5$

$x, y \geq 0$

8. Solve graphically

Maximise : $P = 100x + 5y$

s.t. $x + y \leq 300$

$3x + y \leq 600$

$y \leq x + 200$

9. Solve the LPP graphically

Minimize $z = 5x + 10y$

s.t. $x + 2y \leq 120,\ x + y \geq 60,\ x - 2y \geq 0$

$x \geq 0,\ y \geq 0$

10. Determine graphically the minimum value of the following objective function:

$z = 500x + 400y$

s.t. $x + y \leq 200$

$x \geq 20$

$y \geq 4x$

$y \geq 0$

## FIVE MARKS QUESTIONS

Q. 1 Solve the following LPP graphically.

Maximize $z = 3x + y$ subject to the constraints

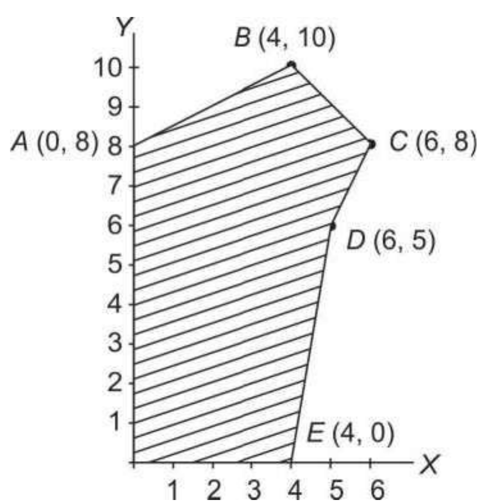
$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

Q.2 The corner points of the feasible region determined by the system of linear constraints are as shown below.



Answer each of the following :

(i) Let $z = 3x - 4y$ be the objective function. Find the maximum and minimum value of $z$ and also the corresponding points at which the maximum and minimum value occurs.

(ii) Let $z = px + qy$ where $p, q > 0$ be the objective function. FInd the condition on $p$ and $q$ so that the maximum value of $z$ occurs at B (4, 10) and C (5, 8). Also mention the number of optimal solutions in this case.

Q. 3 There are two types of fertilisers A and B. A consists of 10% nitrogen and 6% phosphoric acid and B consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If A costs Rs. 6 per kg and B costs Rs. 5 per kg. determine how much of each type of fertiliser should be used so that nutrient requirements are met at minimum cost. What is the minimum cost?

Q. 4 A man has Rs. 1500 to purchase two types of of shares of two different companies S1 and S2. Market price of one share of S1 is Rs. 180 and S2 is Rs 120. He wishes to purchase a maximum of ten shares only. If one share of type S1 gives a yield of Rs 11, and of type S2 yields Rs 8 then how much shares of each type must be purchased to get maximum profit? and what will be the maximum profit?

Q. 5 A company manufactures two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requiires 2 hours of the cutter's time and 1 hour of the finisher's time. Lamp B required 1 hr of cutter's, 2 hrs of finisher's time. The cutter has 100 hours and finisher has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp B is Rs 13.00. Assuming that he can sell all that he produces how many of each type of lamps should be manufactured to obtain maximum profit and what will be the maximum profit?

Q.6 A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for atmost 20 items. A fan and sewing machine cost Rs 360 and Rs. 240 respectively. He can sell a fan at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest money to maximise his profit?

Q. 7 A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of X, 2 units of capital and 1 unit of labour is required. To produce one unit of Y, 3 units of labour and 1 unit of capital is required. If X and Y are priced at Rs. 80 and Rs100 per unit respectively, how should the producer use his resources to maximize revenue?

Q. 8 A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows :

| Machine | Area Occupied | Labour Force | Daily Output (in units) |
|---|---|---|---|
| A | 1000 $m^2$ | 12 men | 50 |
| B | 1200 $m^2$ | 8 men | 40 |

He has maximum area of 7600 $m^2$ available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output?

Q.9 A manufacturer makes two types of cups A and B. Three machines are required to manufacture the cups and the time in minutes required by each in as given below :

| Types of Cup | Machines | | |
|---|---|---|---|
| | I | II | III |
| A | 12 | 18 | 6 |
| B | 6 | 0 | 9 |

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paisa and on B is 50 paisa, find how many cups of each type should be manufactured to maximise the profit per day.

Q. 10 An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and as profit of Rs. 300 is made on each second class ticket. The airline reserves atleast 20 seats for first class. However at least four times as many passengers prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximize profit for the airline.

Q. 11 A diet for a sick person must contains at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories whereas one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food A and B should be used to have least cost but it must satisfy the requirements of the sick person.

Q.12 Anil wants to invest at most Rs. 12000 in bonds A and B. According to the rules, he has to invest at least Rs. 2000 in Bond A and at least Rs. 4000 in bond B. If the rate of interest on bond A and B are 8% and 10% per annum respectively, how should he invest this money for maximum interest? Formute the problem as LPP and solve graphically.

## CASE STUDY QUESTIONS

Q. 1 A man rides his motorcycle at the speed of 50 km/hr. He has to spend Rs 2/km on petrol. But if he rides it at a faster speed of 80 km/hr, the petrol cost increases to Rs 3/km. He has atmost Rs 120 to spend on petrol and one hr's time. he wishes to find the maximum distance that he can travel.

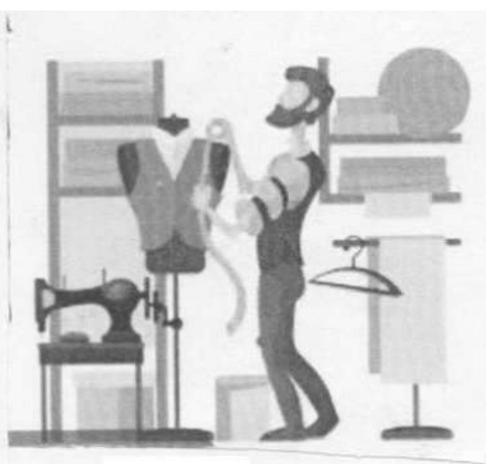
Based on the above information answer the following questions.

(1) If he travels x km with the speed of 50 km/hr and y km with the speed of 80 km/hr, then write the objective function

(2) Find the Maximum distance man can travel?

Q.2 Two tailors A and B earn Rs 150 and Rs 200 per day respectively. A can sticth 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. it is desired to produce atleast 60 shirts and 32 pants at a minimum labour cost.

**Tailor A**



**Tailor B**

Based on the above information answer the following.

(1) If x and y are the number of days A and B work respectively then find the objective function for this LPP

(2) Find the optimal solution for this LPP and the minimum labour cost?

## SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. Objective functin of a L.P.P. is
   (a) A constraint
   (b) A function of be optimised
   (c) A relation between the variables
   (d) None of these
2. The solution set of the inequation $2x + y > 5$ is
   (a) Open half plans that contains the origion
   (b) Open half plane not containing the origin
   (c) Whole xy-plane except the points lying on the line $2x + y = 5$
   (d) None of these
3. Which of the following statements is correct?
   (a) Every L.P.P admits an optimal solution
   (b) A L.P.P. admits unique optimal solution
   (c) If a LPP admits two optimal solutions, it has an infinite number of optimal solutions
   (d) None of these
4. Solution set of inequation $x \geq 0$ is
   (a) Half plane on the left of y-axis
   (b) Half plane on the right of y-axis excluding the points on y-axis
   (c) Half plane on the right of y-axis including the points on y-axis
   (d) None of these
5. In a LPP, the constraints on the decision variables $x$ and $y$ are
   $x - 3y \geq 0$, $y \geq 0$, $0 \leq x \leq 3$.
   The feasible region
   (a) is not in the first quadrant
   (b) is bounded in the first quadrant
   (c) is unbounded in the first quadrant
   (d) doesn't exist

**SELF ASSESSMENT-2**

EAH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. Solution set of the inequation $y \leq 0$ is
   (a) Half plane below the x-axis excluding the points on x-axis
   (b) Half plane below the x-axis including the points on x-axis
   (c) Half plane above the x-axis
   (d) None of these
2. Regions represented by inequations $x \geq 0$, $y \geq 0$ is
   (a) first quadrant
   (b) second quadrant
   (c) third quadrant
   (d) fourth quadrant
3. The feasible region for an LPP is always
   (a) concavo convex polygen
   (b) concave poloygon
   (c) convex polygon
   (d) None of these
4. If the constraints in a linear programming problem are changed then
   (a) the problem is to be reevaluated
   (b) solution not defined
   (c) the objective function has to be modified
   (d) the change in constraints is ignored
5. L.P.P. is as follows:

   Minimize $Z = 30x + 50y$

   Subject to the constraints,

   $3x + 5y \geq 15$

   $2x + 3y \leq 18$

   $x \geq 0$, $y \geq 0$

   In the feasible region, the minimum value of Z occurs at
   (a) a unique point
   (b) no point
   (c) infinitely many points
   (d) two points only

## ANSWER

### One Marks Questions

| | | |
|---|---|---|
| 1. (c) | 2. (d) (4, 2) | 3. (c) *a* = 3, *b* = 5 |
| 4. (d) 136 | 5. (a) 80 | 6. (a) 2 |
| 7. (d) infinite | 8. (c) | 9. (c) |
| 10. (c) | 11. (a) | 12. (c) 0 |
| 13. (b) unbounded | 14. (c) A triangle | 15. (c) |
| 16. (a) | 17. (a) | 18. (d) |
| 19. (a) | 20. (d) | |

### Three Marks Questions

1. Optimal solution $\left(\frac{8}{3}, \frac{1}{3}\right)$, maximize = $\frac{-29}{3}$ feasible region unbounded.
2. Optimal solution $\left(\frac{20}{19}, \frac{45}{19}\right)$, maximize = $\frac{235}{19} = 12.3$
3. Optimal solution (0, 200), maximize = 400
4. Maximize z = 2600 at C(200, 600) and minimize z is 2100 at A(0, 700)
5. Minimize z = – 12 at (4, 0)
6. Unbounded, minimize z = 160. It occurs at all the points onthe line segment joining $\left(2, \frac{1}{2}\right)$ and $\left(\frac{8}{3}, 0\right)$. So, infinite optimal solutions.
7. Maximize z = 4000 at (4, 4)
8. Maximize z = 20,000 at (200,0)
9. Mixímize z = 300 at (60, 0)
10. Miximize z =42000 at (20, 80)

### Five Marks Questions

1. Max z = 250 at x = 50, y = 100
2. (i) Max z = 12 at (4,0) and min z = –32 at (0,8)

   (ii) P = 2q, infinite solutions lying on the line segment joining the points B and C
3. 100 kg of fertilizer A and 80 kg of fertilizer B, minimum cost Rs 1000
4. Maximum profit = Rs 95 with 5 shares of each type.
5. Lamps of type A = 40, Lamps of type B = 20 Max profit = Rs 540
6. Fans : 8, sewing machines : 12, max profit : Rs 392
7. X : 2 units, Y : 6 units, max revenue is Rs 760.

8. Type A : 4, Type B : 3
9. Cup A : 15, cup B : 30
10. No of first class ticket = 40, No of second class tickets = 160
11. Food A :5 units, food B: 30 units
12. Maximum interest is Rs 1160 at (2000, 10000)

**CASE STUDIES QUESTIONS**

1. (i) maximize z = x + y (ii) $54\frac{2}{7}$ km
2. (i) minimize z = 150x + 200y (ii) (5, 3) and Rs. 1350

**SELF ASSESSMENT-1**

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (b) | 3. (c) | 4. (c) | 5. (b) |

**SELF ASSESSMENT-2**

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (a) | 3. (c) | 4. (a) | 5. (c) |