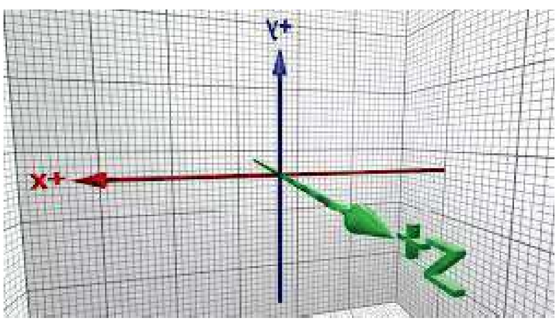
CHAPTER 11

THREE-DIMENSIONAL GEOMETRY

In the real world, everything you see is in a three-dimensional shape, it has length, breadth, and height. Just simply look around and observe. Even a thin sheet of paper has some thickness.



Applications of geometry in the real world include the computer-aided design (CAD) for construction blueprints, the design of assembly systems in manufacturing such as automobiles, nanotechnology, computer graphics, visual graphs, video game programming, and virtual reality creation.

The next time you play a mobile game, thank three-dimension geometry for the realistic look to the landscape and the characters that exhibit the game's virtual world.

THREE DIMENSIONAL GEOMETRY

Topics to be covered as per C.B.S.E. revised syllabus (2024-25)

- Direction cosines and direction ratios of a line joining two points.
- Cartesian equation and vector equation of a line.
- Skew lines
- Shortest distance between two lines.
- Angle between two lines.

POINTS TO REMEMBER

- **Distance Formula:** Distance (d) between two points(x_1 , y_1 , z_1)and(x_2, y_2, z_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- **Section Formula:** line segment AB is divided by P (x, y, z) in ratio m:n

(a) Internally	**(b) Externally**
$\left(\frac{m\,x_2 + n\,x_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$	$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$

- **Direction ratio** of a line through $(x_1, y_1, z_1)$and (x_2, y_2, z_2) are$x_2 - x_1, y_2 - y_1, z_2 - z_1$

- **Direction cosines** of a line having direction ratios as a, b, c are:

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

- **Equation of line in space:**

Vector form	**Cartesian form**
(i) Passing through point $\vec{a}$ and parallel to vector $\vec{b}$; $\vec{r} = \vec{a} + \lambda\vec{b}$	(i) Passing through point (x_1, y_1, z_1) and having direction ratios a, b, c;

	$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
(ii)Passing through two points $\vec{a}$ and $\vec{b}$; $\vec{r} = \vec{a} + \lambda\,(\vec{b} - \vec{a})$	(ii) Passing through two points $(x_1, y_1 z_1)$ and $(x_2, y_2 z_2)$; $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

- **Angle between two lines:**

Vector form	Cartesian form
(i) For lines $\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\,\overrightarrow{b_2}$, $\cos\theta = \frac{\lvert\vec{b}_1 \cdot \vec{b}_2\rvert}{\lvert\vec{b}_1\rvert\lvert\vec{b}_2\rvert}$ where 'θ' is the angle between two lines.	(ii) For lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ $\cos\theta = \frac{\lvert a_1a_2 + b_1b_2 + c_1c_2\rvert}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$
(iii) Lines are perpendicular if $\vec{b}_1 . \vec{b}_2 = 0$	(ii) Lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$
(iv) Lines are parallel if $\vec{b}_1 = k\vec{b}_2$; $k \neq 0$	(i) Lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- **Shortest distance between two skew lines**

The shortest distance between two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $$d = \left\|\frac{(\vec{a}_2 - \vec{a}_1)\,.\,(\vec{b}_1 \times \vec{b}_2)}{\|\vec{b}_1 \times \vec{b}_2\|}\right\|$$ If d = 0, lines are intersecting	The shortest distance between $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is $$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{D}}$$ Where $$D = \{(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2\}$$

- **Shortest distance between two parallel lines**

Let $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ are parallel lines then shortest distance between those lines

$$d = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{(\vec{b})}\right| \text{ units}$$

If d = 0, then lines coincident.

Illustration 1:
Are the following lines interesting?

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

If yes, find point of intersection.

Solution:

We can write the equations in cartesian form

$$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = l \qquad ...(i)$$

and $$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6} = m \qquad ...(ii)$$

Any point on line (i) P(λ + 3, 2λ +2, 2λ – 4)

Any point on line (ii) Q (3μ + 5, 2μ – 2, 6μ)

Comparing x, y and z coordinate respectively

$\lambda + 3 = 3\mu + 5$, $2\lambda + 2 = 2\mu - 2$, $2\lambda - 4 = 6\mu$

or $\lambda - 3\mu = 2$, $2\lambda - 2\mu = -4$, $2\lambda - 6\mu = 4$

or $\lambda - 3\mu = 2$, $\lambda - \mu = -2$, $\lambda - 3\mu = 2$

Solving first two, we get $\lambda = -4$, $\mu = -2$

$\because$ $\lambda = -4$, $\mu = -2$, Satisfies $\lambda - 3\mu = 2$

$\therefore$ lines are intersecting

and point of interesting (–1, –6, –12)

Or

Using distance formula

If

th$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 0$

Illustration 2:

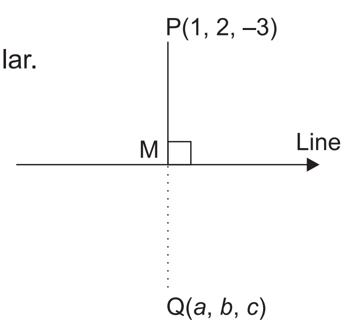
Find the foot of perpendicular from the point P(1, 2, –3) to the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$.

Also find the length of the perpendicular and image of P in the given lines.

Solution: We have

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = l \text{ (say)}$$

$\therefore$ $x = 2\lambda - 1$, $y = -2\lambda + 3$, $z = -\lambda$



Let M($2\lambda - 1$, $-2\lambda + 3$, $-\lambda$) be the foot of perpendicular.

DR's of PM are $\langle 2\lambda - 1 - 1, -2\lambda + 3 - 2, -\lambda + 3 \rangle$

or $\langle 2\lambda - 2, -2\lambda + 1, -\lambda + 3 \rangle$

$\because$ PM is perpendicular to the line

$\therefore$ $2(2\lambda - 2) - 2(-2\lambda + 1) - 1(-\lambda + 3) = 0$

$4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$

$9\lambda - 9 = 0$

$\Rightarrow \lambda = 1$

$\therefore$ Foot of the perpendicular M = (1, 1 – 1)

and PM = $\sqrt{(1-1)^2+(2-1)^2+(-3+1)^2} = \sqrt{0+1+4} = \sqrt{5}$

Let Q(a, b, c) be the image of P

As M be the mid point of PQ. (As line is plane mirror)

$\therefore \quad \frac{a+1}{2} = 1 \quad Þ \quad a = 1$

$\frac{b+2}{2} = 1 \quad Þ \quad b = 0$

$\frac{c-3}{2} = -1 \quad Þ \quad c = 1$

$\therefore$ image of P is (1, 0, 1)

ONE MARK QUESTIONS

Multiple Choice Questions (1 Mark Each)

Select the correct option out of the four given options:

1. Distance of the point (a, b, c) from x-axis is

   (a) $\sqrt{b^2+c^2}$ (b) $\sqrt{c^2+a^2}$

   (c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2+b^2+c^2}$

2. Angle between the lines 2x = 3y = – z and 6x = – y = – 4z is

   (a) 45° (b) 60°

   (c) 90° (d) 30°

3. Equation of the line passing through (2, –3, 5) and parallel to

   $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+1}{-1}$ is

   (a) $\frac{x+2}{3} = \frac{y-3}{4} = \frac{z+5}{-1}$ (b) $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{1}$

   (c) $\frac{x-2}{3} = \frac{y+3}{4} = \frac{5-z}{1}$ (d) $\frac{x-2}{-3} = \frac{y+3}{-4} = \frac{z-5}{2}$

4. If the lines $\frac{x-1}{2} = \frac{z-3}{5} = \frac{z-1}{\lambda}$ and $\frac{z-2}{3} = \frac{y+1}{-2} = \frac{z}{2}$ are perpendicular, then the value of 'λ' is

   (a) $\lambda = -2$ (b) $\lambda = 2$

   (c) $\lambda = 1$ (d) $\lambda = -1$

5. Cartesian form of line $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{j} - \hat{k})$ is

(a) $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z}{-1}$ (b) $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z}{-1}$

(c) $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{0}$ (d) $\frac{x+1}{2} = \frac{y+1}{1} = \frac{z}{0}$

6. The coordinates of the foot of the parpendicular drwan from the point (–2, 8 7) on the xz plane is

(a) (0, 8, 0) (b) (–2, 0, 7)

(c) (2, 8, –7) (d) (–2, –8, 7)

7. The length of perpendicular from the point (4, –7, 3) on the y-axis is

(a) 3 units (b) 4 units

(c) 5 units (d) 7 units

8. If cosα, cosβ and cosγ are direction cosimes of a line, then the value of cos 2α + cos2β + cos2γ is

(a) 1 (b) –1

(c) 2 (d) –2

9. If two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b$, $z = c'y + d$ are perpendicular, then

(a) $aa' + cc' = 1$ (b) $aa' + cc' + 1 = 0$

(c) $\frac{a}{a'} + \frac{c}{c'} = 1$ (d) $\frac{a}{a'} + \frac{c}{c'} + 1 = 0$

10. A point P lines on the line segment joining the points (–1, 3, 2) and (5, 0, 6), if x-coordinate of P is 2, then its z coordinate is

(a) 8 (b) 4

(c) 3 (d) –1

ASSERTION-REASON BASED QUESTIONS

In the following questions a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(c) (A) is true and (R) is false

(d) (A) is false, but (R) is true

11. Assertion (A) : The vector equation of a line passing through the points (3, 1, 2) and (4, 2, 5) is $\vec{r} = 3\hat{i} + \hat{j} + 2\hat{k} + \lambda(\hat{i} + \hat{j} + 3\hat{k})$)

Reason (R) : The vector equation of a line passing through the points with position vector $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

12. Assertion (A) : If a line joining the points (1, 0, 4) and (3, λ, 7) is perpendicular to the line joining the points (1, 2, –1) and (2, 3, 0), then λ = – 5

Reason (R) : Two lines with direction ratios (a_1, b_1, c_1) and $<a_2, b_2, c_2>$ are parallel if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

13. Assertion (A) : The coordinates of the point where the line

$\vec{r} = (3\hat{i} + \hat{j} - \hat{k}) + \lambda(-\hat{i} + 2\hat{j} + 3\hat{k})$ cuts xy-plane and $\left(\frac{8}{3}, \frac{-5}{3}, 0\right)$

Reason (R) : The z-coordinate of any point on xy-plane is 0.

14. Assertion (A) : Lines $\frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-3}{3}$ and $\frac{2-x}{-3} = \frac{y-1}{4} = \frac{z+2}{-1}$ intersect at a point.

Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are intersecting if $(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1) \neq 0$.

TWO MARKS QUESTIONS

1. Find the equation of a line passing though (2, 0, 5) and which is parallel to line 6x – 2 = 3y + 1 = 2z –2

2. The equation of a line are 5x – 3 = 15y + 7 = 3 – 10 z. Write the direction cosines of the line

3. If a line makes angle α, β, γ with Co-ordinate axis then what is the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$

4. Find the equation of a line passing through the point (2, 0, 1) and parallel to the line whose equation is $\vec{r} = (2\lambda + 3)\hat{i} + (7\lambda - 1)\hat{j} + (-3\lambda + 2)\hat{k}$

5. Find the condition that the lines x = ay + b, z = cy + d and x = a′y + b′, z = c′y + d′ may be perpendicular to each other.

6. Show that the lines x = –y = 2z and x + 2 = 2y – 1 = –z + 1 are perpendicular to each other.

7. Find the equation of the line through (2, 1, 3) and parallel to the line $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$ in cartesian and vector form.

8. Find the cartesian and vector equation of the line through the points (2, –3, 1) and (3, –4, –5)

9. For what value of λ and μ the line joining the points (7, λ, 2), (μ, –2, 5) is parallel to the line joining the points (2, –3, 5), (–6, –15, 11)?

10. If the points (–1, 3, 2), (–4, 2, –2) and (5, 5, λ) are Collinear, find the value of λ.

THREE/FIVE MARKS QUESTIONS

1. Find vector and Cartesian equation of a line passing through a point with position vector $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and which is parallel to the line joining the points with position vectors$-\hat{\imath} + 4\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

2. Find image (reflection) of the point (7, 4, –3) in the line$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

3. Show that the lines line $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and line $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$intersect each other. Find the point of intersection.

4. Find the shortest distance between the lines:

$$\bar{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) \text{ and}$$

$$\bar{r} = (2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}).$$

5. Find shortest distance between the lines:

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{5-y}{2} = \frac{z-7}{1}$

6. Find the shortest distance between the lines:

$\overline{r} = (1-\lambda)\hat{\imath} + (\lambda-2)\hat{\jmath} + (3-2\lambda)\hat{k}$

$$\overline{r} = (\mu+1)\hat{\imath} + (2\mu-1)\hat{\jmath} - (2\mu+1)\hat{k}$$

7. Find the foot of perpendicular from the point $2\hat{\imath} - \hat{\jmath} + 5\hat{k}$ on the line $\overline{r} = \left(11\hat{\imath} - 2\hat{\jmath} - 8\hat{k}\right) + \lambda\left(10\hat{\imath} - 4\hat{\jmath} - 11\hat{k}\right)$. Also find the length of the perpendicular.

8. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonal of a cube. Prove that $cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta = \frac{4}{3}$

9. Find the length and the equations of the line of shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

10. Show that $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{2}, z = 2.$ do not intersect each other.

11. If the line $\frac{x+2}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k.

12. Find the equation of the line which intersects the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and passes through the point (1, 1, 1).

13. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angle of $\pi/3$.

14. Find the foot of perpendicular drawn from the point (2, –1, 5) to the line

$$\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$$

Also find the length of the perpendicular. Hence find the image of the point (2, –1, 5) in the given line.

15. Find the image of the point P(2, –1, 11) in the line

$$\vec{r} = (2\hat{i} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

16. Find the point(s) on the line through the point P(3, 5, 9) and Q(1, 2, 3) at a distance 14 units from the mid-point of segment PQ.

17. Find the shortest distance between the following pair of lines

$$\frac{x-1}{2} = \frac{y+1}{3} = z \text{ and } \frac{x+1}{5} = \frac{y-2}{1};\ z = 2$$

Hence write whether the lines are intersecting or not.

18. Find the foot of perpendicular from the point (1, 2, 3) to the line

$$\vec{r} = (6\hat{i} + 7\hat{j} + 7\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$$

Also find the equation of the perpendicular and length of perpendicular.

19. Find the equation of the line passing through (–1, 3, –2) and perpendicular to the lines $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z+5}{3}$ and $\frac{x-2}{-3} = \frac{y}{2} = \frac{z+1}{5}$

20. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{3-z}{-2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3)

21. The points P(4, 5, 10), Q(2, 3, 4) and R(1, 2, –1) are three vertices of a parallelogram PQRS. Find the vector equations of the sides PQ and QR and also find the coordinates of point R.

22. Find the equation of perpendicular from the point (3, –1, 11) to the line

$$\frac{x}{2} = \frac{2y-4}{6} = \frac{3-z}{-4}.$$

Also find the foot of the perpendicular and the length of the perpendicular.

23. Show that the lines $\frac{1-x}{-2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{1+y}{2} = z$ are intersecting. Also find the point of intersection.

24. For what value of 'λ', the following are Skew lines?

$$\frac{x-4}{5}=\frac{1+y}{2}=z,\ \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-\lambda}{4}$$

25. Find the vector equation of the line passing through (2, 1, –1) and parallel to the line $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$. Also find the distance between these two lines.

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. The foot of perpendicular drawn from the point (2, –1, 5) to the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$ is

(a) (2, 1, 3) (b) (3, 1, 2)

(c) (1, 2, 3) (d) (3, 2, 1)

2. The shortest distance between the lines $\vec{r}=(6\hat{i}+2\hat{j}+2\hat{k})+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and $\vec{r}=(-4\hat{i}-4\hat{k})+\mu(3\hat{i}-2\hat{j}-2\hat{k})$ is

(a) 10 units (b) 9 units

(c) 12 units (d) 9/2 units

3. If the x-coordinate of a point A on the join of B(2, 2, 1) and C(5, 1, –2) is 4 then its z-coordinate is

(a) –2 (b) –1

(c) 1 (d) 2

4. The distance of the point M(a, b, c) from the x-axis is

(a) $\sqrt{b^2+c^2}$ (b) $\sqrt{c^2+a^2}$

© $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2+c^2}$

5. The straight line $\frac{x-3}{3}=\frac{y-2}{1}=\frac{z-1}{0}$ is

(a) parallel to x-axis (b) parallel to y-axis

(c) parallel to z-axis (d) perpendicular to z-axis

SELF ASSESSMENT-2

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. The shortest distance between the line $\frac{x-3}{3}=\frac{y}{0}=\frac{z}{-4}$ and y-axis is

   (a) $\frac{12}{5}$ units (b) $\frac{1}{5}$ units

   (c) 0 units (d) 3 units

2. The point of intersection of the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ is

   (a) $\left(\frac{1}{3},\frac{-1}{3},-\frac{2}{3}\right)$ (b) $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

   (c) $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\frac{-1}{2},\frac{-3}{2}\right)$

3. If a line makes the same angle α, with each of the x and z axes and the angle β with y-axis such that $3\sin^2\alpha = \sin^2\beta$, then the value of $\cos^2\alpha$ is

   (a) $\frac{1}{5}$ (b) $\frac{2}{5}$

   (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

4. If the lines $\frac{x+3}{k-5}=\frac{y-1}{1}=\frac{5-z}{-2k-1}$ and $\frac{x+2}{-1}=\frac{2-y}{-k}=\frac{z}{5}$ are perpendicular, then the value of k is

   (a) 1 (b) –1

   (c) 2 (d) –2

5. The image of the point P(-1, 8, 4) to the line $\frac{x}{2}=\frac{y+1}{-2}=\frac{z-3}{-4}$ is

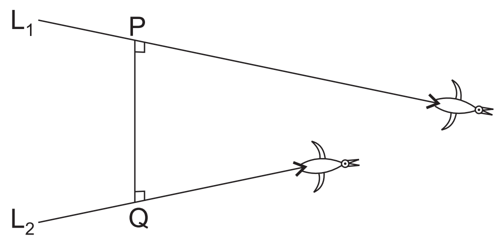
   (a) (5, 4, 4) (b) (5, 0, 4)

   (c) (-3, -6, 10) (d) (1, 8, 4)

Case Study Based Questions

1. Two birds are flying in the space along straight path L_1 and L_2

(Neither parallel nor intersecting) where,



$$L_1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

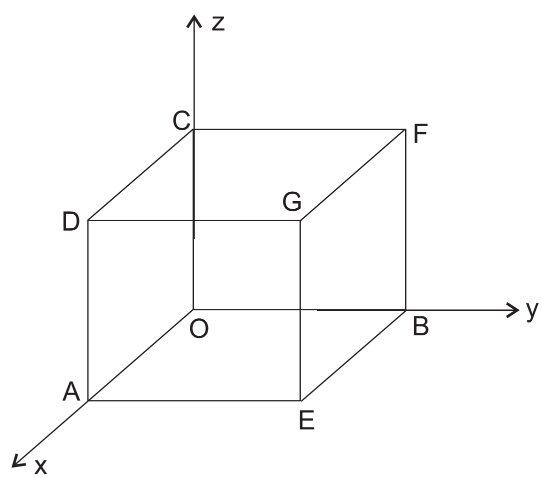
$$L_2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

P and Q are the points on the path L1 and L2 respectively such that PQ is perpendicular on both paths L1 and L2. On the basis of above information, answer the following questions

(i) Find the length PQ

(ii) Find the equation of PQ

2. A carpenter designed a Cuba of side a units and put it in 3 dimensional system such that one vertex at origin and adjacent sides on three coordinate axes as shown in figure



Based on the above information, answer the following questions:

(i) Write the coordinates of the vertices D, E, F and G.

(ii) Find the direction ratios of the diagonal OG.

(iii) Find the direction cosines of the diagonals CE and DB

OR

(iii) Find the angle between CE and DB.

ANSWERS

ONE MARK QUESTIONS

1. (a) $\sqrt{b^2 + c^2}$
2. (c) 90°
3. (c) $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-5}{-1}$
4. (b) $\lambda = 2$
5. (a) $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z}{-1}$
6. (b) (–2, 0, 7)
7. (c) 5 units
8. (b) –1
9. (b) $aa' + cc' + 1 = 0$
10. (b) 4
11. (a)
12. (b)
13. (d)
14. (c)

TWO MARK QUESTIONS

1. $\frac{x-2}{1} = \frac{y}{2} = \frac{z-5}{3}$
2. $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$
3. 2
4. $\vec{r} = (2\hat{i} + \hat{k}) + \lambda(2\hat{i} + 7\hat{j} - 3\hat{k})$
5. $aa' + cc' + 1 = 0$
6.
7. $\frac{x-2}{1} = \frac{y-1}{-7} = \frac{z-3}{2}$,
   $\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 7\hat{j} + 2\hat{k})$
8. $\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6}$,
   $\vec{r} = (2\hat{i} - 3\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} - 6\hat{k})$
9. $\lambda = 4$
   $\mu = 3$
10. $\lambda = 10$

THREE/FIVE MARK QUESTIONS

1. $\bar{r} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$

2. $\left(-\frac{51}{7}, -\frac{18}{7}, \frac{43}{7}\right)$

3. $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

4. $\frac{1}{\sqrt{6}}$

5. $2\sqrt{29}$ units

6. $\frac{8}{\sqrt{29}}$

7. $(1, 2, 3), \sqrt{14}$

9. $SD = 14\, units,\ \frac{x-5}{2} = \frac{y-7}{3} = \frac{z-3}{6}$

11. K = 12

12. $\frac{x-1}{3} = \frac{y-1}{10} = \frac{z-1}{17}$

13. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$

14. (1, 2, 3), $\sqrt{14}$, (0, 5, 1)

15. (6, 7, 3)

16. $\left(6, \frac{19}{2}, 18\right), \left(-2, \frac{-5}{2}, -6\right)$

17. $\frac{9}{\sqrt{195}}$, Not intersecting

18. (3, 5, 9), $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6}$, 7 units

19. $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$

20. $(-2, -1, 3), \left(\frac{56}{17}, \frac{43}{17}, \frac{11}{17}\right)$

21. $PQ : \vec{r} = (4\hat{i} + 5\hat{j} + 10\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + 6\hat{k})$
$QR : \vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu(\hat{i} + \hat{j} + 5\hat{k})$, Point R(3,4,5)

22. $\frac{x-3}{1} = \frac{y+1}{-6} = \frac{z-11}{4}$, (2,5,7), $\sqrt{13}$ units

23. (–1, –1, –1)

24. $\lambda \neq 69/11$

25. $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} - \hat{j} + \hat{k})$, $\sqrt{\frac{11}{6}}$ units

SELF ASSESSMENT TEST-1

1. (C) 2. (B) 3. (B) 4. (A) 5. (D)

SELF ASSESSMENT TEST-2

1. (A) 2. (D) 3. (C) 4. (B) 5. (C)

CASE STUDY BASED QUESTIONS

1. (i) $3\sqrt{30}$ units (ii) $\vec{r} = (3\hat{i} + 8\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 5\hat{j} - \hat{k})$

2. (i) D(a, 0, a), E(a, a, 0), F(0, a, a) & G(a, a, a)

(ii) <1, 1, 1>

(iii) Direction cosines of CE are $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right\rangle$

DB are $\left\langle \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right\rangle$

OR

Angle between CE and DB = $\cos^{-1}\left(\frac{1}{3}\right)$