CHAPTER 10

VECTORS

Vectors are probably the most important tool to learn in all of physics and engineering. Vectors are using in daily life following are few of the example.

- Navigating by air and by boat is generally done using vectors.
- Planes are given a vector to travel, and they use their speed to determine how far they need to go before turning or landing. Flight plans are made using a series of vectors.
- Sports instructions are based on using vectors.

VECTORS

Topics to be covered as per C.B.S.E. revised syllabus (2024-25)

- Vectors and scalars
- Magnitude and direction of a vector
- Direction consines and direction ratios of a vector.
- Types of vectors (equal, unit, zero, parallel and collinear vectors)
- Position vector of a point
- Negative of a vector
- Components of a vector
- Addition of vectors
- Multiplication of a vector by a scalar
- Position vector of a point dividing a line segment in a given ratio
- Definition, Geometrical interpretation, properties and application of scalar (dot) product of vectors
- Vector (cross) product of vectors.

POINTS TO REMEMBER

- A quantity that has magnitude as well as direction is called a vector. It is denoted by a directed line segment.

- Two or more vectors which are parallel to same line are called collinear vectors.

- Position vector of a point P(a, b, c) w.r.t. origin (0, 0, 0) is denoted by $\overrightarrow{OP}$ where $\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$ and $|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$.

- If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be any two points in space, then

  $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ and

  $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- Any vector $\vec{a}$ is called unit vector if $|\vec{a}| = 1$ It is denoted by $\hat{a}$

- If two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and direction by the two sides of a triangle in order, then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by third side of a triangle taken in opposite order. This is called triangle law of addition of vectors.

- If $\vec{a}$ is any vector and λ is a scalar, then $\lambda\,\vec{a}$ is vector collinear with $\vec{a}$ and $|\lambda\,\vec{a}| = |\lambda||\vec{a}|$.

- If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then $\vec{a} = \lambda\,\vec{b}$ where λ is some non-zero scalar.

- Any vector $\vec{a}$ can be written as $\vec{a} = |\vec{a}|\hat{a}$ where $\hat{a}$ is a unit vector in the direction of $\vec{a}$.

- If $\vec{a}$ and $\vec{b}$ be the position vectors of points A and B, and C is any point which divides $\overrightarrow{AB}$ in ratio m:n internally then position vector $\vec{c}$ of point C is given as $\vec{c} = \frac{m\vec{b}+n\vec{a}}{m+n}$. If C divides $\overrightarrow{AB}$ in ratio m:n externally, then $\vec{c} = \frac{m\vec{b}-n\vec{a}}{m-n}$. If C is mid point then $\vec{c} = \frac{\vec{a}+\vec{b}}{2}$

- The angles α, β *and* γ made by $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ with positive direction of x, y and z-axis are called direction angles and cosines of these angles are called direction cosines of $\vec{r}$ usually denoted as $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

  Also $l = \frac{a}{|\vec{r}|}, m = \frac{b}{|\vec{r}|}, n = \frac{c}{|\vec{r}|}$ and $l^2 + m^2 + n^2 = 1$

  or $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

- The numbers a, b, c proportional to l, m, n are called direction ratios.
- Scalar product or dot product of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a}\,.\,\vec{b}$ and is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$, θ is the angle between $\vec{a}$ *and* $\vec{b}$.$(0 \le \theta \le \pi)$.

- Dot product of two vectors is commutative i.e. $\vec{a}.\vec{b} = \vec{b}\,.\,\vec{a}$

- $\vec{a}.\vec{b} = 0 \iff \vec{a} = \vec{o}$ or $\vec{b} = \vec{o}$ or $\vec{a} \perp \vec{b}$.

- $\vec{a}\,.\,\vec{a} = |\vec{a}|^2$, so $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

- Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

  Projection vector of $\vec{a}$ along $\vec{b} = \left(\frac{(\vec{a}.\vec{b})}{|\vec{b}|}\right)\hat{b}$.

- Cross product or vector product of two vectors $\vec{a}$ and $\vec{b}$ is denoted as $\vec{a} \times \vec{b}$ and is defined as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$. where θ is the angle between $\vec{a}$ and $\vec{b}$.($0 \leq \theta \leq \pi$). And $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$. $\vec{b}$ and $\hat{n}$ form a right handed system.
- Cross product of two vectors is not commutative i.e., $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$, but $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.
- $\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a} = \vec{0}, \vec{b} = \vec{0}$ or $\vec{a} \parallel \vec{b}$.
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$.
- $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$
- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\pm\left(\frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}\right)$.
- $|\vec{a} \times \vec{b}|$ is the area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$
- $\frac{1}{2}|\vec{a} \times \vec{b}|$ is the area of parallelogram where diagonals are $\vec{a}$ and $\vec{b}$.
- If $\vec{a}, \vec{b}$ and $\vec{c}$ form a triangle, then area of the triangle
- $= \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}|\vec{b} \times \vec{c}| = \frac{1}{2}|\vec{c} \times \vec{a}|$.

Illustration:

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 27$

Solution:

$\because$ $\vec{d}$ is perpendicular to $\vec{a}$ and $\vec{b}$ *both*

$$\text{Let } \vec{d} = \lambda (\vec{a} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$$\vec{d} = \lambda (32\hat{i} - \hat{j} - 14\hat{k})$$

But $\quad \vec{c} \cdot \vec{d} = 27$

$\therefore \; (2\hat{i} - \hat{j} + 4\hat{k}) . \lambda (32\hat{i} - \hat{j} - 14\hat{k}) = 27$

$\Rightarrow \; \lambda (64 + 1 - 56) = 27$

$\Rightarrow \qquad \lambda = 3$

and $\vec{d} = 3(32\hat{i} - \hat{j} - 14\hat{k}) = 96\hat{i} - 3\hat{j} + 42\hat{k}$

Illustration:

Vectors $\vec{a}$, $\vec{b}$ and $\vec{b}$ are such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 5, |\vec{b}| = 7$ and $|\vec{c}| = 3$. Find the angle between $\vec{a}$ and $\vec{c}$

Solution:

Given $\quad \vec{a} + \vec{b} + \vec{c} = \vec{0}$

$$\vec{a} + \vec{c} = -\vec{b}$$

$$(\vec{a} + \vec{c}) \cdot (\vec{a} + \vec{c}) = (-\vec{b}) \cdot -(\vec{b})$$

$\Rightarrow (\vec{a})^2 + \vec{a} \cdot \vec{c} + \vec{c} \cdot \vec{a} + (\vec{c})^2 = |\vec{b}|^2 \qquad (\because \vec{a} \cdot \vec{a} = |\vec{a}|^2)$

$\Rightarrow \qquad 2\vec{a} \cdot \vec{c} = |\vec{b}|^2 - |\vec{a}|^2 - |\vec{c}|^2$

$\Rightarrow \qquad 2|\vec{a}||\vec{c}| \cos \theta = |\vec{b}|^2 - |\vec{a}|^2 - |\vec{c}|^2$

Where 'θ' be the angle between $\vec{a}$ and $\vec{c}$

$\Rightarrow 2 \times 5 \times 3 \cos \theta = 49 - 25 - 9$

$\Rightarrow \cos \theta = \dfrac{15}{30}$

$\Rightarrow \cos \theta = \dfrac{1}{2} \quad \Rightarrow \quad \theta = \dfrac{\pi}{3}$

Illustration:

Let $\vec{a}$ and $\vec{b}$ are two unit vectors and 'θ' is the angle between them, then find 'θ' if $\vec{a}+\vec{b}$ is unit vector.

Solution:

Here $|\vec{a}| = |\vec{b}| = 1$ and $|\vec{a}+\vec{b}| = 1$

$\because |\vec{a}+\vec{b}|^2 = 1$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = 1$ $\qquad (\because \vec{a}\cdot\vec{a} = |\vec{a}|^2)$

$\Rightarrow (\vec{a})^2 + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + (\vec{b})^2 = 1$

$\Rightarrow 2|\vec{a}||\vec{b}|\cos\theta = -1$

$\Rightarrow \cos\theta = -\dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{2\pi}{3}.$

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS (1 Mark Each)

Select the correct option out of the four given options:

1. If $\overrightarrow{AB} = 3\hat{i}+2\hat{j}-\hat{k}$ and the coordinate of A are (4, 1, 1), then the coordinate of B are.

   (a) (1, – 1, 2) (b) (–7, –3, 0)

   (c) (7, 3, 0) (d) (–1, 1, –2)

2. Let $\vec{a} = -2\hat{i}+\hat{j}, \vec{b} = \hat{i}+2\hat{j}$ and $\vec{c} = 4\hat{i}+3\hat{j}$, then the values of x and y such that $\vec{c} = x\vec{a}+y\vec{b}$, are:

   (a) x = 1, y = 2 (b) x = –1, y = 2

   (c) x = –1, y = –2 (d) x = 1, y = –1

3. A unit vector in the direction of the resultant of the vector $\hat{i}-\hat{j}+3\hat{k}, 2\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}+2\hat{j}-2\hat{k}$ is

   (a) $\dfrac{1}{\sqrt{21}}(4\hat{i}-2\hat{j}-\hat{k})$ (b) $\dfrac{1}{\sqrt{21}}(4\hat{i}-2\hat{j}+\hat{k})$

   (c) $4\hat{i}-2\hat{j}-\hat{k}$ (d) $\dfrac{1\,(4\hat{i}+2\hat{j}-\hat{k})}{\sqrt{21}}$

4. If $2\hat{i}+3\hat{j}+\hat{k}$ and $\hat{i}-2\hat{j}-\hat{k}$ are two vectors, then a vector of magnitude 5 units parallel to the sum of given vectors

(a) $\sqrt{\frac{5}{2}}(3\hat{i}+\hat{j})$ (b) $\frac{1}{\sqrt{30}}(\hat{i}+5\hat{j}+2\hat{k})$

(c) $\frac{1}{\sqrt{10}}(3\hat{i}+\hat{j})$ (d) $5(3\hat{i}+\hat{j})$

5. If $\vec{a}=\lambda\hat{i}+2\hat{j}+\hat{k}$ and $\vec{b}=5\hat{i}-9\hat{j}+2\hat{k}$ are perpendicular, then the value of 'λ' is:

(a) $\lambda=\frac{16}{5}$ (b) $\lambda=-\frac{16}{5}$

(c) $\lambda=4$ (d) $\lambda=\frac{10}{9}$

6. The value of p for which $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}+p\hat{j}+3\hat{k}$ are parallel vector is

(a) $p=-\frac{30}{2}$ (b) p = 15

(c) $p=\frac{2}{3}$ (d) $p=\frac{3}{2}$

7. If $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+3\hat{j}+p\hat{k})=\vec{0}$, then the value of 'p' is

(a) $p=-\frac{20}{27}$ (b) $p=\frac{27}{2}$

(c) p = 0 (d) $p=-\frac{27}{2}$

8. Value of $\hat{i}.(\hat{j}\times\hat{k})+(\hat{i}\times\hat{k}).\hat{j}$ is

(a) 2 (b) 1

(c) 0 (d) –2

9. If $\vec{a}=5\hat{i}-4\hat{j}+\hat{k}, \vec{b}=-4\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}-2\hat{k}$ than the value of $\vec{c}.(\vec{a}\times\vec{b})$ is

(a) –5 (b) 5

(c) 35 (d) 30

10. If vector $\lambda\hat{i} + 3\hat{j}$ and $4\hat{i} + \lambda\hat{j}$ are collinear, then the value of 'λ' is

(a) $\lambda = 0$ (b) $\lambda = 4$

(c) $\lambda = 3$ (d) $\lambda = \pm 2\sqrt{3}$

11. A unit vector perpendicular to $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$ is

(a) $\frac{1}{3}(-\hat{i} + 2\hat{j} + 2\hat{k})$ (b) $\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$

(c) $\frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$ (d) $\frac{1}{3}(\hat{i} + 2\hat{j} - 3\hat{k})$

12. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} \times \vec{b}| = \vec{a}.\vec{b}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) 30° (b) 45°

(c) 60° (d) 90°

13. If $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$ are the diagonals of a parallelagram, then the area of the parallelogram is

(a) 8 sq. units (b) $\sqrt{91}$ sq. units

(c) $5\sqrt{3}$ sq. units (d) $10\sqrt{3}$ sq. units

14. If scalar projection of $\lambda\hat{i} + \hat{j} + 4\hat{k}$ on $2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then the value of λ is

(a) $\lambda = 5$ (b) $\lambda = -5$

(c) $\lambda = -9$ (d) $\lambda = 9$

15. If $\vec{a} . \vec{b} = 3$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) 30° (b) 60°

(c) 120° (d) 45°

16. If $|\vec{a}| = 4$ and $-3 \leq k \leq 2$, than the range of $|k\vec{a}|$ is

(a) [0, 12] (b) [8, 12]

(c) [0, 8] (d) [–12, 8]

17. If $|\vec{a}| = 4, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 10$, than the value of $|\vec{a} . \vec{b}|^2$ is

(a) 22 (b) 44

(c) 88 (d) None of these

18. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = \sqrt{37}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

19. If $(\vec{a}+\vec{b}) \perp \vec{b}$ and $(\vec{a}+2\vec{b}) \perp \vec{a}$, then

(a) $(\vec{a}) = 2|\vec{b}|$ (b) $2|\vec{a}| = \vec{b}$

(c) $(\vec{a}) = (\vec{b})$ (d) $|\vec{a}| = \sqrt{2}|\vec{b}|$

20. If $|\vec{a}| = |\vec{b}| = |\vec{a}+\vec{b}| = 1$, then the value of $|\vec{a}-\vec{b}|$ is

(a) 0 (b) 1

(c) $\sqrt{3}$ (d) 2

Assertion-Reason Based Questions

In the following questions a statement of Assertion (A) is followed by a statement of Reason (R) Choose the correct answer out of the following couces:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A)
(c) (A) is true and (R) is false
(d) (A) is false, but (R) is true

21. Assertion (A) : If $|\vec{a}| = 3, |\vec{b}| = 5$ and $\vec{a}\,.\,\vec{b} = 10$,

$$|\vec{a}\times\vec{b}|^2 = 125$$

Reason (R) : $|\vec{a}\times\vec{b}|^2 - (\vec{a}\,.\,\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

22. Assertion (A) : If $\vec{a}$ and $\vec{b}$ are unit vector such that $|\vec{a}+\vec{b}| = \sqrt{3}$, then the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$

Reason (R) : Angle between vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$

23. Assertion (A) : If $|\vec{a}| = 4, |\vec{b}| = 5$ and $|\vec{a}\times\vec{b}| = 20$, then $\vec{a} \perp \vec{b}$

Reason (R) : Two non zero vector $\vec{a}$ and $\vec{b}$ are perpandicular if $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|$

24. Assertion (A) : If $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 12$ and $\vec{a} = 2|\vec{b}|$, then $|\vec{a}| = 4$ and $|\vec{b}| = 2$

Reason (R) : If $\vec{a}$ and $\vec{b}$ are two vectors, then $(\vec{a}+\vec{b})\,.\,(\vec{a}-\vec{b}) = |\vec{a}|^2 - |b|^2$

25. Assertion (A) : If $|2\vec{a}+\vec{b}| = |2\vec{a}-\vec{b}|$, than $\vec{a}$ parellel to $\vec{b}$

Reason (B) : Two non zero vector $\vec{a}$ and $\vec{b}$ are perpendicular if $\vec{a}\,.\,\vec{b} = 0$.

TWO MARK QUESTIONS

1. A vector $\vec{r}$ is inclined to x – axis at 45° and y-axis at 60° if | $\vec{r}$ | = 8 units. find $\vec{r}$.

2. if $|\vec{a}+\vec{b}|$ = 60, $|\vec{a}-\vec{b}|$ = 40 and $|\vec{b}|$= 46 find | $\vec{a}$ |

3. Write the projection of $\vec{b}+\vec{c}$ on $\vec{a}$ where

   $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

4. If the points (–1, –1, 2), (2, m, 5) and (3, 11, 6) are collinear, find the value of *m*.

5. For any three vectors $\vec{a},\vec{b}$ and $\vec{c}$ write value of the following.
   $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})$

6. If $(\vec{a}\times\vec{b})^2+(\vec{a}.\vec{b})^2$ = 144 and | $\vec{a}$ | = 4. Find the value of | $\vec{b}$ |.

7. If for any two vectors $\vec{a}$ and $\vec{b}$,

   $(\vec{a}+\vec{b})^2+(\vec{a}-\vec{b})^2=\lambda\left[(\vec{a})^2+(\vec{b})^2\right]$ then write the value of λ .

8. if $\vec{a},\vec{b}$ are two vectors such that $|(\vec{a}+\vec{b})|$ = | $\vec{a}$ | then prove that 2 $\vec{a}$ + $\vec{b}$ is perpendicular to $\vec{b}$.

9. Show that vectors $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$

   $\vec{b}=\hat{i}-3\hat{j}+5\hat{k}, \vec{c}=2\hat{i}+\hat{j}-4\hat{k}$ form a right angle triangle.

10. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}$ = 0 and | $\vec{a}$ | = 5, | $\vec{b}$ | = 12, | $\vec{c}$ | = 13, then find $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

11. The two vectors $\hat{i}+\hat{j}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC respectively of Δ ABC, find the length of median through A.

12. If position vectors of the points A, B and C are $\vec{a}$, $\vec{b}$ and $4\vec{a} - 3\vec{b}$ respectively, then find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

13. If position vectors of three points A, B and C are $-2\vec{a} + 3\vec{b} + 5\vec{c}$, $\vec{a} + 2\vec{b} + 3\vec{c}$ and $7\vec{a} - \vec{c}$ respectively. Then prove that A, B and C are collinear.

14. If the vector $\hat{i} + p\hat{j} + 3\hat{k}$ is rotated through an angle θ and is doubled in magnitude, then it becomes $4\hat{i} + (4p - 2)\hat{j} + 2\hat{k}$. Find the value of p.

15. If $\overrightarrow{AB} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} + 4\hat{k}$ are sides of the triangle ABC. Find the length of median through A.

16. Find scalar projection of the vector $7\hat{i} + \hat{j} + 4\hat{k}$ on the vector $2\hat{i} + 6\hat{j} + 3\hat{k}$. Also find vector porojection

17. Let $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular and $|\vec{a}| = |\vec{b}|$. Find x and y.

18. If $\vec{a}$ and $\vec{b}$ are unit vectors, find the angle between $\vec{a}$ and $\vec{b}$ so that $\vec{a} - \sqrt{2}\, b$ is a unit vector.

19. If $\vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$. Find the angle between $\vec{a}$ and $\vec{a} \times \vec{b}$.

20. Using vectors, prove that angle in a semi circle is 90°.

THREE MARKS QUESTIONS

1. The points A,B and C with position vectors $3\hat{\imath} - y\hat{\jmath} + 2\hat{k}$, $5\hat{\imath} - \hat{\jmath} + \hat{k}$ *and* $3x\hat{\imath} + 3\hat{\jmath} - \hat{k}$ are collinear. Find the values of x and y and also the ratio in which the point B divides AC.

2. If sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

3. Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ *and* $\vec{b}$ and satisfying $\vec{d}.\vec{c} = 21$

4. If $\hat{a}$ *and* $\hat{b}$ are unit vectors inclined at an angle θ then proved that

   (i) $\cos\frac{\theta}{2} = \frac{1}{2}\left|\hat{a} + \hat{b}\right|$

   (ii) $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$

   (iii) $\tan\frac{\theta}{2} = \left|\frac{\hat{a} - \hat{b}}{\hat{a} - \hat{b}}\right|$

5. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude. Prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with vectors $\vec{a}, \vec{b}$ and $\vec{c}$. Also find angle.

6. For any vector $\vec{a}$ prove that $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{\jmath}|^2 + \left|\vec{a} \times \hat{k}\right|^2 = 2|\vec{a}|^2$

7. Show that $\left(\vec{a} \times \vec{b}\right)^2 = |\vec{a}|^2\left|\vec{b}\right|^2 - \left(\vec{a}.\vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$

8. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are the position vectors of vertices A,B,C of a Δ ABC, show that the area of triangle ABC is $\frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$. Deduce the condition for points $\vec{a}, \vec{b}$ *and* $\vec{c}$ to be collinear.

9. Let $\vec{a}, \vec{b}$ *and* $\vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between b and c is $\pi/6$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

10. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

11. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k},\ \vec{c} = \hat{\jmath} - \hat{k}$ are given vectors, then find a vector $\vec{b}$ satisfying the equations $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a}.\vec{b} = 3$.

12. Find the altitude of a parallelepiped determined by the vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$ if the base is taken as parallelogram determined by $\vec{a}$ *and* $\vec{b}$ and if $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k},\ \vec{b} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{c} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$.

13. If $|\vec{a}| = 3,\ |\vec{b}| = 4$ *and* $|\vec{c}| = 5$ such that each is perpendicular to sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$

14. Decompose the vector $6\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ in two vectors which are parallel and perpendicular to the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively.

15. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c},\ \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $a \neq 0$, then show that $\vec{b} = \vec{c}$.

16. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are three non zero vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$. Prove that $\vec{a}, \vec{b}$ *and* $\vec{c}$ are mutually at right angles and $|\vec{b}| = 1$ and $|\vec{c}| = |\vec{a}|$

17. Simplify $(\vec{a} - \vec{b}) \cdot \{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\}$

18. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find the value of $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy$

19. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.

20. The magnitude of the vector product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vector $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

21. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, prove that $(\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

22. Find a vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$.

23. Prove that the angle betwen two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

24. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} + 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

25. Find a unit vector perpendicular to plane ABC, when position vectors of A,B,C are $3\hat{\imath} - \hat{\jmath} + 2\hat{k}$, $\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $4\hat{\imath} - 3\hat{\jmath} + \hat{k}$ respectively.

26. Find a unit vector in XY plane which makes an angle 45° with the vector $\hat{\imath} + \hat{\jmath}$ and angle of 60° with the vector $3\hat{\imath} - 4\hat{\jmath}$.

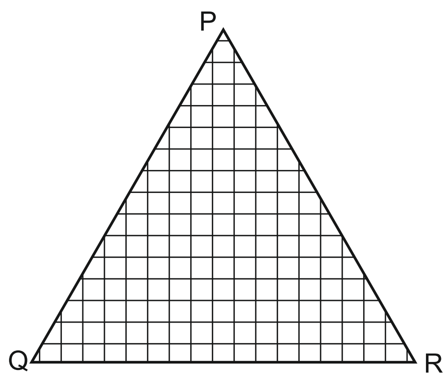
27. Suppose $\vec{a} = \lambda\hat{\imath} - 7\hat{\jmath} + 3\hat{k}$, $\vec{b} = \lambda\hat{\imath} + \hat{\jmath} + 2\lambda\hat{k}$. If the angle between $\vec{a}$ and $\vec{b}$ is greater than 90°, then prove that λ satisfies the inequality $-7 < \lambda < 1$.

28. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$ then find the value of $(2\vec{a} - 5\vec{b}).(3\vec{a} + \vec{b})$.

29. Let $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. Find a vector $\vec{d}$ such that $\vec{a}.\vec{d} = 0$, $\vec{b}.\vec{d} = 2$ and $\vec{c}.\vec{d} = 4$.

Case Study Questions (4 Marks Each)

1. A farmer moves along the boundary of a triangular field PQR. Three vertices of the triangular field are P(2, 1, –2), Q(–1, 2, 1) and R(1, –4, –2) respectively.



On the basis of above information, answer the following questions:

(i) Find the length of PQ.

(ii) Find the $\angle PQR$

(iii) Find the area of the ΔPQR

OR

(iii) Find projection of QP on QR.

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECTION CHOOSE THE CORRECT OPTION.

1. A unit vector perpendicular to both $\hat{i} + \hat{j}$ *and* $\hat{j} + \hat{k}$ is

   (A) $\hat{i} + \hat{j} + \hat{k}$ (B) $\hat{i} - \hat{j} + \hat{k}$

   (C) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (D) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

2. If $|\vec{a} \cdot \vec{b}| = 2$, $|\vec{a} \times \vec{b}| = 4$, then the value of $|\vec{a}|^2 |\vec{b}|^2$ is

   (A) 2 (B) 6

   (C) 8 (D) 20

3. The projection of vector $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ on vector $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$ is

   (A) $\frac{9}{19}$ (B) $\frac{9}{\sqrt{19}}$

   (C) $\frac{9}{\sqrt{6}}$ (D) $\frac{19}{9}$

4. If $\vec{a}$ is any vector, then the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is

   (A) $|\vec{a}|^2$ (B) $2|\vec{a}|^2$

   (C) $3|\vec{a}|^2$ (D) $4|\vec{a}|^2$

5. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is

   (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

   (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{3}$

SELF ASSESSMENT-2

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECTION CHOOSE THE CORRECT OPTION.

1. If $\vec{a}$, $\vec{b}$ and $\vec{a}+\vec{b}$ are unit vectors. Then the value of $|\vec{a}-\vec{b}|$ is

   (A) 0 (B) 1 (C) $\sqrt{2}$ (D) $\sqrt{3}$

2. If $\vec{a}$ and $\vec{b}$ are two vectors *such that* $|\vec{a}|=2, |\vec{b}|=1$ and $\vec{a}\cdot\vec{b}=1$, then the value of $(3\vec{a}-5\vec{b})\cdot(2\vec{a}+7\vec{b})$ is

   (A) 0 (B) 41 (C) 29 (D) 7

3. If $\vec{c}\cdot(\hat{i}+\hat{j})=2, \vec{c}\cdot(\hat{i}-\hat{j})=3$ and $\vec{c}\cdot\hat{k}=0$, then the vector $\vec{c}$ is

   (A) $\frac{1}{2}(5\hat{i}+\hat{j})$ (B) $\frac{1}{2}(5\hat{i}-\hat{j})$

   (C) $\frac{1}{2}(\hat{i}-5\hat{j})$ (D) $\frac{1}{2}(\hat{i}+5\hat{j})$

4. If the projection of $3\hat{i}+\lambda\hat{j}+\hat{k}$ *on* $\hat{i}+\hat{j}$ is $\sqrt{2}$ units, then the value λ is

   (A) 1 (B) –1 (C) 0 (D) 2

5. If $|\vec{a}|=2, |\vec{b}|=7$ *and* $\vec{a}\times\vec{b}=3\hat{i}+2\hat{j}-6\hat{k}$, then the angle between $\vec{a}$ and $\vec{b}$ is

   (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Answers

ONE MARK QUESTIONS

MCQ (1 Mark Each)

1. (c) (7, 3, 0)
2. (b) x = – **1**, y = 2
3. (d) $\frac{1}{\sqrt{21}}(4\hat{i}+2\hat{j}-\hat{k})$
4. (a) $\sqrt{\frac{5}{2}}(3\hat{i}+\hat{j})$
5. (a) $\lambda = \frac{16}{5}$
6. (c) 2/3
7. (b) $p = \frac{27}{2}$
8. (c) 0
9. (a) –5
10. (d) $\lambda = \pm 2\sqrt{3}$
11. (a) $\frac{1}{3}(-\hat{i}+2\hat{j}+2\hat{k}$
12. (b) 45°
13. (c) $5\sqrt{3}$ sq. units
14. (a) λ = 5
15. (b) 60°
16. (a) [0, 12]
17. (b) **44**
18. (c) $\frac{\pi}{3}$
19. (d) $|\vec{a}| = \sqrt{2}\,|\vec{b}|$
20. (c) $\sqrt{3}$
21. (c)
22. (b)
23. (a)
24. (a)
25. (d)

TWO MARK QUESTIONS

1. $4(\sqrt{2}\hat{i}+\hat{j}+\hat{k})$
2. 22
3. 2
4. $m = 8$
5. 0
6. 3
7. $\lambda = 2$

10. –169
11. $2\sqrt{2}$
12. $\overrightarrow{AC} = 3(\vec{a}-\vec{b}), \overrightarrow{BC} = 4(\vec{a}-\vec{b})$
14. $p = -\frac{2}{3}, 2$
15. $\sqrt{33}$
16. $\frac{32}{7}, \frac{32}{49}(2\hat{i}+6\hat{j}+3\hat{k})$
17. $x = -\frac{31}{12} \quad y = \frac{41}{12}$
18. $\frac{\pi}{4}$
19. $\frac{\pi}{2}$

THREE MARKS QUESTIONS

1. $x = 3,\ y = 3,\ 1:2$

3. $\vec{d} = 7\hat{\imath} - 7\hat{\jmath} - 7\hat{k}$

5. $\cos^{-1}\frac{1}{\sqrt{3}}$

8. $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

11. $\vec{b} = \frac{5}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$

12. $\frac{4}{\sqrt{38}}$ units

13. $5\sqrt{2}$

14. $\left(-\hat{\imath} - \hat{\jmath} - \hat{k}\right) + \left(7\hat{\imath} - 2\hat{\jmath} - 5\hat{k}\right)$

17. 0

18. 0

19. 60°

20. $\lambda = 1$

22. $\hat{i} - 11\hat{j} - 7\hat{k}$

24. $\vec{\beta} = \left(\frac{3}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}\right) + \left(\frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} + 3\hat{k}\right)$

25. $\frac{-1}{\sqrt{165}}\left(10\hat{\imath} + 7\hat{\jmath} - 4\hat{k}\right)$

26. $\frac{13}{\sqrt{170}}\hat{i} + \frac{1}{\sqrt{170}}\hat{j}$

28. $-\frac{11}{2}$

29. $\vec{d} = 2\hat{i} - \hat{j} + \hat{k}$

Case Study Questions

(i) $\sqrt{19}$ units

(ii) $\cos^{-1}\left(\frac{3}{\sqrt{19}}\right)$

(iii) $\frac{7}{2}\sqrt{10}$ square units

OR

(iii) 3 units

SELF ASSESSMENT-1

1. (C) 2. (D)
3. (D) 4. (B)
5. (B)

SELF ASSESSMENT-2

1. (D) 2. (A)
3. (B) 4. (B)
5. (A)