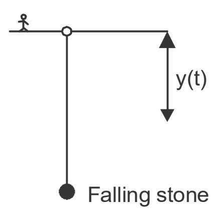
CHAPTER-9

DIFFERENTIAL EQUATIONS

Sky diving is a method of transiting from a high point in the atmosphere to the surface of the Earth with the aid of graity. This involves the control of speed during the descent using a parachute. Once the sky diver jumps from an airplane, the net force experienced by the diver can be calculated using

DIFFERENTIAL EQUATIONS.

Another eg.



D.E. is

$$my'' = mg$$

$$\Rightarrow \quad y'' = g = \text{constant}$$

where y = distance travelled by the stone at any time t.

and g = acceleration due to gravity.

TOPICS TO BE COVERED AS PER CBSE LATEST CURRICULUM 2023-24

- Definition, order and degree
- General and particular solutions of a D.E.
- Solutions of D.E. using method of separation of variables.
- Solutions of homogeneous differential equations of first order and first degree.
- Solutions of linear differential equations of the type.

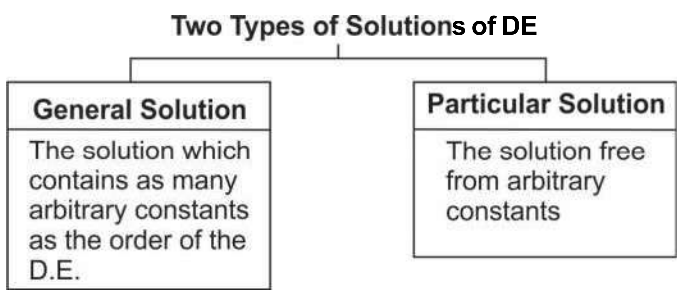
$\frac{dy}{dx} + py = q,$ where p and q are functions of x or constants.

$\frac{dx}{dy} + px = q,$ where p and q are functions of y or constants.

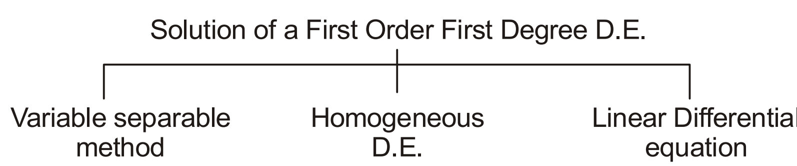
KEY POINTS :

- **DIFFERENTIAL EQUATION :** is an equation involving derivatives of the dependent variable w.r.t independent variables and the variables themselves.
- **ORDINARY DIFFERENTIAL EQUATION (ODE) :** A.D.E. involving derivatives of the dependent variable w.r.t only one independent variable is an ordinary D.E.

  In class XII ODE is referred to as D.E.
- **PARTIAL DIFFERENTIAL EQUATION (PDE) :** A.D.E involving derivatives w.r.t more than one independent variables is called a partial D.E.
- **ORDER of a DE :** is the order of the highest order derivative occurring in the D.E.
- **DEGREE of a D.E. :** is the highest power of the highest order derivative occurring in the D.E provided D.E is a polynomial equation in its derivatives. It is always a whole no.
- **SOLUTION OF THE D.E :** A relation between involved variables, which satisfy the given D.E is called its solution.



- **FORMATION OF A DIFFERENTIAL EQUATION :** We differentiate the function successively as many times as the arbitary constants in the given function and then eliminate the arbitiary constants from these equations.
- **ORDER of A D.E :** Is equal to the number of arbitrary constants in the general solution of a D.E.



- **"VARIABLE SEPARABLE METHOD" :** is used to solve D.E. in which variables can be separated completely i.e, terms containing x should remain with dx and terms containing y should remain with dy.
- **"HOMOGENEOUS DIFFERENTIAL EQUATION :** D.E. of the form $\frac{dy}{dx} = F(x, y)$ where $F(x, y)$ is a homogeneous function of degree 0

i.e. $F(\lambda x, \lambda y) = \lambda^0 F(x, y)$

or $F(\lambda x, \lambda y) = F(x, y)$ for some non-zero constant λ.

To solve this type put $y = vx$

To Solve homogenous D.E of the trype $\frac{dx}{dy} = G(x, y),$ we make substitution $x = vy$

- **LINEAR DIFFERENTIAL EQUATION :** A.D.E of the form $\frac{dy}{dx} + Py = Q$ where P and Q are constants or functions of x only is known as first order linear differential equation.

Its solution

$y.(IF) = \int Q \times (I.F.)dx + C,$ where

I. F = Integrating factor = $e^{\int Pdx}$

Another form of Linear Differential Equation is $\frac{dx}{dy} + P_1 x = Q_1$, where P_1 and Q_1 are constants or functions of y only.

Its solution is given as

$x.(I.F) = \int Q_1 X(I.F.)\, dy + C,$ where $I.F. = e^{\int P_1 dy}$

Illustration:

Write the order and degree of the Differential Equation

$$\left[1 + (y')^2\right]^{3/2} = ky''$$

Solution: Squaring both the sides

$$\left[1 + (y')^2\right]^3 = k^2 (y'')^2$$

$\therefore$ Order of D.E. = 2

and Degree of D.E. = 2

Illustration:

Solve the differential equations

$$(1+e^{2x})dy + e^x(1+y^2)dx = 0;\ y(0) = 1$$

Solution: $\dfrac{dy}{dx} = \dfrac{-e^x(1+y^2)}{1+e^{2x}}$

Using Variables separables method,

$$\frac{dy}{1+y^2} = \frac{-e^x}{1+e^{2x}}dx$$

Integrating both sides we get

$$\int \frac{1}{1+y^2}dy = -\int \frac{e^x}{1+e^{2x}}dx$$

$\Rightarrow \tan^{-1}y = -\int \dfrac{dt}{1+t^2}$; On putting $e^x = t$

$= -\tan^{-1}t$

$\Rightarrow \tan^{-1}y = -\tan^{-1}(e^x) + C$

$\Rightarrow \tan^{-1}y + \tan^{-1}(e^x) = C$

At $x = 0$, $y = 1$ given

$\therefore \tan^{-1}(1) + \tan^{-1}(1) = C$

$\Rightarrow \dfrac{\pi}{4} \times 2 = C$

$\Rightarrow C = \dfrac{\pi}{2}$

$\therefore$ Particular solution of D.E. is given by $\tan^{-1}y + \tan^{-1}(e^x) = \dfrac{\pi}{2}$.

Illustration:

Solve $(x-y)\dfrac{dy}{dx} = x + 2y$

Solution: $\dfrac{dy}{dx} = \dfrac{x+2y}{x-y} = f(x,y)$

Now $f(\lambda x, \lambda y) = \dfrac{\lambda x + 2\lambda y}{\lambda x - \lambda y} = \dfrac{\lambda(x+2y)}{\lambda(x-y)} = \lambda^o f(x, y)$

Clearly, *f* is homogeneous function in *x* and *y*.

So, given D.E. is **homogenous D.E.**

Now, Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + \frac{x\,dv}{dx}$$

$$\therefore\ v + \frac{x\,dv}{dx} = \frac{x + 2vx}{x - vx}$$

$$\Rightarrow v + \frac{x\,dv}{dx} = \frac{1 + 2v}{1 - v}$$

$$\Rightarrow \frac{x\,dv}{dx} = \frac{1 + 2v - v + v^2}{1 - v}$$

$$\Rightarrow \frac{x\,dv}{dx} = \frac{1 + v + v^2}{1 - v}$$

$$\Rightarrow \frac{(1 - v)\,dv}{1 + v + v^2} = \frac{dx}{x}$$

Integrating both sides we get

$$\Rightarrow -\frac{1}{2}\int \frac{2v - 2 + 1 - 1}{1 + v + v^2}\,dv = \log|x| + C$$

$$\Rightarrow -\frac{1}{2}\int \frac{2v + 1}{1 + v + v^2}\,dv + \frac{3}{2}\int \frac{1}{1 + v + v^2}\,dv = \log|x| + C$$

$$\Rightarrow -\frac{1}{2}\log|1 + v + v^2| + \frac{3}{2}\int \frac{1}{\left(v + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dv = \log|x| + C$$

$$\Rightarrow -\frac{1}{2}\log\left|1 + \frac{y}{x} + \frac{y^2}{x^2}\right| + \left(\frac{3}{2}\right).\left(\frac{2}{\sqrt{3}}\right)\tan^{-1}\left(\frac{2v + 1}{\sqrt{3}}\right) = \log|x| + C$$

$$\Rightarrow -\frac{1}{2}\log\left|x^2 + xy + y^2\right| + \sqrt{3}\tan^{-1}\left(\frac{2y + x}{\sqrt{3}x}\right) = C$$

Illustration:

Find the particular solution of the differential equation

$\frac{dx}{dy} + x \cot y = 2y + y^2 \cot y \, (y \neq 0)$ given that $x = 0$ when $y = \pi/2$.

Solution: Clearly, it is a Linear D.E.

$\frac{dx}{dy} + Px = Q$ where

$P = \cot y$, $Q = 2y + y^2 \cot y$

I.F. = $e^{\int Pdy} = e^{\int \cot y \, dy} = e^{\log(\sin y)} = \sin y$

$\therefore$ solution of D.E. is given by

x. (I.F) = $\int Q.IFdy$ + C; C is arbitrary constant

$\Rightarrow$ $x. (\sin y) = \int (2y + y^2 \cot y) \sin y \, dy + C$

$= \int 2y \sin y \, dy + \int y^2 \cos y \, dy + C$

$= \int 2y \, \cancel{\sin y} \, dy + y^2 . \sin y - \int 2y \, \cancel{\sin y} \, dy + C$

$\Rightarrow$ $x \sin y = y^2 \sin y + C$

Now, $x = 0$, when $y = \frac{\pi}{2}$

So, $0 = \frac{\pi^2}{4} + C \Rightarrow C = -\frac{\pi^2}{4}$

$\therefore$ $x \sin y = y^2 \sin y - \frac{\pi^2}{4}$

or $\boxed{x = y^2 - \frac{\pi^2}{4} \operatorname{cosec} y}$

ONE MARK QUESTIONS

1. The general solution of the D.E.

   $y\,dx - x\,dy = 0$; (Given $x, y > 0$), is of the form.

   (a) $xy = c$ (b) $x = cy^2$

   (c) $y = cx$ (d) $y = cx^2$

   (Where 'c' is an orbitary positive constant of integration)

2. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

   (a) Variable radii and fixed centre (0, 1)

   (b) Variable radii and fixed centre (0, –1)

   (c) Fixed radius 1 and variable centre on x-axis

   (d) Fixed radius 1 and variable centre on y-axis

3. The solution of the D.E. $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is

   (a) $e^x = \frac{y^3}{3} + e^y + c$ (b) $e^y = \frac{x^2}{3} + e^x + c$

   (c) $e^y = \frac{x^3}{3} + e^x + c$ (d) None f these

4. The order and degree of the D.E. $\frac{d^4y}{dx^4} + \sin(y''') = 0$ are respectively

   (a) 4 and 1 (b) 1 and 2

   (c) 4 and 4 (d) 4 and not defined

5. A homogeneous differential equation of the type $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution.

(a) y = vx (b) v = yx

(c) x = vy (d) x = v

6. Integrating factor of the D.E. $\frac{dy}{dx} + y\tan x - \sec x = 0$ is

(a) cosx (b) secx

(c) $e^{\cos x}$ (d) $e^{\sec x}$

7. The order and degree of the D.E. $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{6}} = 0,$ respectively are

(a) 2 and not defined (b) 2 and 2

(c) 2 and 3 (c) 3 and 3

8. The order of the D.E. of a family of curves resprsented by an equation containing four arbitrary constants, will be

(a) 2 (b) 4

(c) 6 (d) None of these

9. An equation which involves variable as well as dirivatives of the dependent variable w.r.t. the independent variable, is known as

(a) differential equation (b) integral equation

(c) linear equation (d) quadantic equation

10. $\tan^{-1}x + \tan^{-1}y = c$ is general solution of the D.E.

(a) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ (b) $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$

(c) $(1+x^2)dy + (1+y^2)dx = 0$ (d) $(1+x^2)dx + (1+y^2)dy = 0$

11. The particular solution of $\log\left(\frac{dy}{dx}\right) = 3x + 4y,\ y(0) = 0$ is

(a) $e^{3x} + 3e^{-4y} = 4$ (b) $4e^{4x} + 3e^{-4y} = 3$

(c) $3e^{3x} - 4e^{4y} = 7$ (d) $4e^{3x} + 3e^{-4y} = 7$

12. The solution of the equation $\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$ is

(a) $(x - y^2) + c = \log(3x - 4y + 1)$ (b) $x - y + c = \log(3x - 4y + 4)$

(c) $(x - y + c) = \log(3x - 4y - 3)$ (d) $x - y + c = \log(3x - 4y + 1)$

13. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

(a) $y\log\left(\frac{x}{y}\right) = cx$

(b) $x\log\left(\frac{y}{x}\right) = cy$

(c) $\log\left(\frac{y}{x}\right) = cx$

(d) $\log\left(\frac{x}{y}\right) = cy$

14. Solution of D.E. xdy – ydx = 0 respresents

(a) rectangular hyperbola

(b) parabola whose vertex is at orgain

(c) circle whose centre is at origin

(d) straight line passing through origin

15. Family $y = bx + c^4$ of curves will correspond to a differential equation of order

(a) 3

(b) 2

(c) 1

(d) infinite

16. The integrating factor of the differential equation $(1-y^2)\frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is :

(a) $\frac{1}{y^2-1}$

(b) $\frac{1}{\sqrt{y^2-1}}$

(c) $\frac{1}{1-y^2}$

(d) $\frac{1}{\sqrt{1-y^2}}$

17. The general solution of the differential equation $xdy - (1+x^2)dx = 0$ is

(a) $y = 2x + \frac{x^3}{3} + c$

(b) $y = 2\log x + \frac{x^3}{3} + c$

(c) $y = \frac{x^2}{2} + c$

(d) $y = \log x + \frac{x^2}{2} + c$

ASSERTION REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion (A) and Reason (R) Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explantion of (A)

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(c) (A) is true and (R) is false

(d) (A) is false but (R) is true

18. Assertion (A) : Order of the differential equation whose solution is $y = c_1 e^{x+c_2} + c_3 e^{x+c_4}$ is 4.

    Reason (R) : Order of the differential equation is equal to the number of independent arbitrary constant mentioned in the solution of differential equation.

19. Assertion (A) : The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is not defined.

    Reason (R) : If the differential equation is a polynomial in terms of its derivatives, then its degree is defined.

20. Assertion (A) : $\frac{dy}{dx} = \frac{x^3 - xy^2 + y^3}{x^2y - x^3}$ is a homogeneous differential equation.

    Reason (R) : The function $f(x, y) = \frac{x^3 - xy^2 + y^3}{x^2y - x^3}$ is homogeneous.

TWO MARKS QUESTIONS

1. Write the general solution of the following D.Eqns.

   (i) $\frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}$ (ii) $\frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2y}$

   (iii) $(e^x + e^{-x})\, dy = (e^x - e^{-x})dx$

2. Given that $\frac{dy}{dx} = e^{-2y}$ and y = 0 when x = 5.

   Find the value of x when y = 3.

3. Name the curve for which the slope of the tangent at any point is equal to the ratio of the abbcissa to the ordinate of the point.

4. Solve $\frac{xdy}{dx} + y = e^x$.

THREE MARKS QUESTIONS

1. (i) Show that $y = e^{m \sin^{-1} x}$ is a solution of $(1 - x^2)\frac{d^2y}{dx^2} - \frac{xdy}{dx} - m^2 y = 0$

   (ii)Show that $y = a \cos(\log x) + b \sin(\log x)$ is a solution of

   $$\frac{x^2 d^2y}{dx^2} + \frac{xdy}{dx} + y = 0$$

   (iii) Verify that y = log $\left(x + \sqrt{x^2 + a^2}\right)$ satisfies the D.E.

   $$\left(a^2 + x^2\right) y'' + xy' = 0$$

2. Solve the following differential equations.

   (i) $xdy - (y + 2x^2)dx = 0$

   (ii) $(1 + y^2)\tan^{-1} x\, dx + 2y(1 + x^2)dy = 0$

   (iii) $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

(iv) $\frac{dy}{dx}$ = 1 + x + y^2 + xy^2, y = 0 when x = 0

(v) $xdy - ydx = \sqrt{x^2 + y^2}dx, y = 0$ when $x = 1$

3. Solve each of the following differential equations

(i) $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0,\ y(0) = 0$

(ii) $(x+1)\frac{dy}{dx} = 2e^{-y} - 1, y(0) = 0$

(iii) $e^x \tan y dx + (2 - e^x)\sec^2 y dy = 0,\ y\ (0) = \frac{\pi}{4}$

(iv) $(x^2 - y^2)dx + 2xydy = 0$

(v) $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}, y = 0$ when $x = 1$

4. Solve the following differential equations

(i) Find the particular solution of

$2y\ e^{x/y}\ dx + (y - 2xe^{x/y})\ dy = 0, x = 0$ if $y = 1$

(ii) $x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$

(iii) $\frac{dy}{dx}$ = cos (x + y) + sin (x + y)

[**Hint :** Put x + y = z]

(iv) Show that the Differential Equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogenous and also solve it.

(v) $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}, |\ x\ | \neq 1$

FIVE MARKS QUESTIONS

Q. 1 Solve $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

Q. 2 Solve $(x\ dy - ydx)y \sin\left(\frac{y}{x}\right) = (ydx + xdy)x \cos\left(\frac{y}{x}\right)$

Q. 3 Find the particular solution of the D.E. $(x-y)\frac{dy}{dx} = x + 2y$ given that $y = 0$ when $x = 1$.

Q. 4 Solve $dy = \cos x\,(2 - y\,\text{cosec}\,x)dx,$ given that $y = 2$ when $x = \pi/2$

Q. 5 Find the particular solution of the D.E. $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx} = 0$ given that $y = 0$ when $x = 1$

CASE STUDY QUESTIONS

1. An equation involving derivatives of the dependent variable w.r.t. the independent variables is called a differential equation. A differential equation of the from $\frac{dy}{dx} = f(x, y)$ is said to be homogeneous if f(x, y) is a homogeneous function of degree zero, whereas a function f(x, y) is a homogeneous function of degree *n* if $f(\lambda x, \lambda y) = \lambda^n f(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = f(x, y) = g\left(\frac{y}{x}\right)$ we make the substitution $y = vx$ and then separate the variables.

   Based on the above, answer the following quations:

   (i) Show that $(x^2 - y^2)dx + 2xydy = 0$ is a differential equation of the type

   $$\frac{dy}{dx} = g\left(\frac{y}{x}\right)$$

   (ii) Solve the above equation to find its general solution.

Self Assessment Test-1 Differential Equations

Q. 1 The general solution of the D.E.

$$\log\left(\frac{dy}{dx}\right) = ax + by \text{ is}$$

(a) $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$

(b) $e^{ax} - e^{-by} = C$

(c) $be^{ax} + ae^{by} = C$

(d) none of these

Q. 2 The general solution of the DE

$$x^2\frac{dy}{dx} = x^2 + xy + y^2 \text{ is}$$

(a) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$

(b) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$

(c) $\tan^{-1}\left(\frac{y}{x}\right) = \log y + c$

(d) none of these

Q. 3 The solution of the D.E.

$dy = (4 + y^2)\, dx$ is

(a) $y = 2\tan(x + c)$

(b) $y = 2\tan(2x + c)$

(c) $2y = \tan(2x + c)$

(d) $2y = 2\tan(x + c)$

Q. 4 What is the degree of the D.E.

$$y = x\left(\frac{dy}{dx}\right)^3 + \left(\frac{dy}{dx}\right)^2$$

(a) 1

(b) 3

(c) –2

(d) Degree doesn't exist

Q. 5 Solution of D.E. $xdy - ydx = 0$ represents:

(a) a rectangular hyperbola

(b) a parabola whose vertex is at the origin

(c) a straight line passing through the origin

(d) a circle whose centre is at the origin.

Self Assessment Test-2

Q. 1 The solution of the D.E. $x\frac{dy}{dx} + 2y = x^2$ is

(a) $y = \frac{x^2 + c}{4x^2}$ (b) $y = \frac{x^2}{4} + c$

(c) $y = \frac{x^2 + c}{x^2}$ (d) $y = \frac{x^4 + c}{4x^2}$

Q. 2 The solution of the $\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$, is

(a) $y = e^{-x}(x-1)$ (b) $y = x e^x$

(c) $y = xe^{-x} + 1$ (d) $y = x e^{-x}$

Q. 3 If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, $y(0) = 1$, then $y(1)$ is equal to **[JEE mains 2021]**

(a) $\log_2 (2 + e)$ (b) $\log_2 (1 + e)$

(c) $\log_2 (2e)$ (d) $\log_2 (1 + e^2)$

Q. 4 If the solution curve of the D.E. $(2x - 10y^3)\, dy + y dx = 0$ passess through the points (0, 1) and (2, β), then β is a root of the equation

(a) $y^5 - 2y - 2 = 0$ (b) $2y^5 - 2y - 1 = 0$

(c) $2y^5 - y^2 - 2 = 0$ (d) $y^5 - y^2 - 1 = 0$ **[JEE mains 2021]**

Q. 5 Consider a curve $y = f(x)$ passing through the point (–2, 2) and the slope of the tangent to the curve at any point $(x, f(x))$ is given by

$f(x) + xf'(x) = x^2$,

(a) $x^3 + 2x\, f(x) - 12 = 0$ (b) $x^3 + xf(x) + 12 = 0$

(c) $x^3 - 3x\, f(x) - 4 = 0$ (d) $x^2 + 2xf(x) + 4 = 0$ **(HOTS)**

Answers

ONE MARK QUESTIONS

1. (c) y = cx	2. (c)	3. (c)	4. (d)
5. (c) x = vy	6. (b)	7. (a)	8. (b) 4
9. (a)	10. (c)	11. (d)	12. (d)

13. (c) 14. (d) 15. (b) 2 16. (d) $\frac{1}{\sqrt{1-y^2}}$

17. (d) 18. (d) 19. (a) 20. (a)

TWO MARKS QUESTIONS

1. (i) $y = \frac{x^6}{6} + \frac{x^3}{3} - 2\log|x| + C$ (ii) $2(y-x) + \sin 2y + \sin 2x = c$

   (iii) $y = \log_e |e^x + e^{-x}| + C$

2. $\frac{e^6 + 9}{2}$ 3. Rectangular hyperbola 4. $y.x = e^x + c$

THREE MARKS QUESTIONS

2. (i) $y = 2x^2 + cx$ (ii) $\frac{1}{2}(\tan^{-1}x)^2 + \log(1+y^2) = c$

   (iii) $\tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$ (iv) $y = tan\left(x + \frac{x^2}{2}\right)$

   (v) $y + \sqrt{x^2 + y^2} = x^2$

3. (i) $(1+x^2)y = \frac{4x^3}{3}$ (ii) $(2-e^y)(x+1) = 1$

   (iii) $\tan y = 2 - e^x$ (iv) $x^2 + y^2 = cx$

   (v) $(1+x^2)y = tan^{-1}x - \pi/4$

4. (i) $e^{x/y} = \frac{-1}{2}\log|y| + 1$ (ii) $sin(y/x) = log|x| + c$

   (iii) $\log\left|1 + \tan\left(\frac{x+y}{2}\right)\right| = x + c$ (iv) $\frac{y}{x} - log|y| = c$

   (v) $(x^2-1)y = \frac{1}{2}log\left|\frac{x-1}{x+1}\right| + c$

FIVE MARKS QUESTIONS

1. $y = -cos\,x + \frac{2\,sin\,x}{x} + \frac{2\,cos\,x}{x^2} + \frac{x\,log\,x}{3} - \frac{x}{9} + \frac{c}{x^2}$

2. $xy\,cos\left(\frac{y}{x}\right) = c$

3. $\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)-\frac{1}{2}\log|x^2+xy+y^2|=\frac{\sqrt{3}\pi}{6}$

4. $y\sin x=\frac{-1}{2}\cos(2x)+\frac{3}{2}$

5. $x=\frac{1}{2}e^{\tan^{-1}y}+\frac{1}{2}e^{-\tan^{-1}y}$

CASE STUDY QUESTIONS

1. (iii) $x^2 + y^2 = cx$; c is an arbitrary constant

SELE ASSESSMENT TEST-1

1. (a)	2. (a)	3. (b)
4. (b)	5. (c)	

SELE ASSESSMENT TEST-2

1. (d)	2. (d)	3. (b)
4. (d)	5. (c)	