CHAPTER 8

APPLICATIONS OF INTEGRALS

In real life, integrations are used in various fields such as engineering, where engineers use integrals to find the shape of building. In Physics, used in the centre of gravity etc. In the field of graphical representation. Where three-dimensional models are demonstrated.

The PETRONAS TOWERS in KUALA LUMPUR experience high forces due to wind. Integration was used to create this design of building.

APPLICATIONS OF INTEGRALS

Topics to be covered as per C.B.S.E. revised syllabus (2024-25)

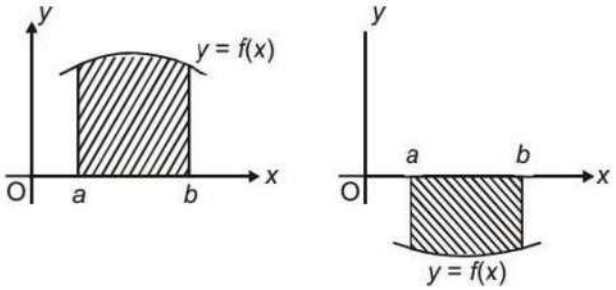
- Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipse (in standard form only)

POINTS TO REMEMBER

AREAS OF BOUNDED REGIONS

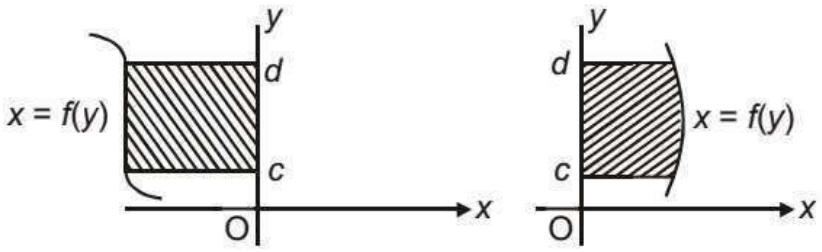
- Area bounded by the curve $y = f(x)$, the x axis and between the ordinates, $x = a$ and $x = b$ is given by

$$\left|\int_a^b f(x)dx\right|$$

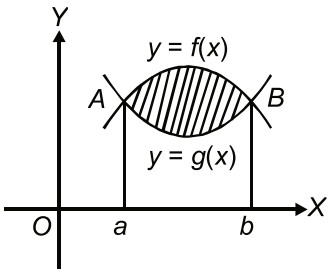


- Area bounded by the curve $x = f(y)$, the y-axis and between the abscissas, y = c and y = d is given by

$$\left|\int_c^d f(y)dy\right|$$



- Area bounded by two curves $y = f(x)$ and $y = g(x)$ such that $0 \leq g(x) \leq f(x)$ for all $x \in [a, b]$ and between the ordinates $x = a$ and $x = b$ is given by



$$\int_a^b [f(x) - g(x)]dx$$

- Area of the following shaded region = $\left|\int_a^k f(x)dx\right| + \int_k^b f(x)dx$

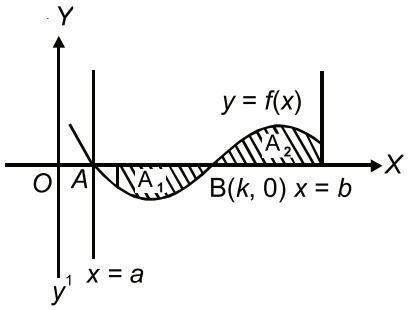


Illustration:

Using integration. Find the area of the region bounded by the line $2y + x = 8$, the x-axis and the lines $x = 2$ and $x = 4$

Solution: Required area= Areaof PQRS

= Area bounded by the line $2y + x = 8$, x-axis and ordinates $x = 2$, $x = 4$

$$= \int_2^4 y\,dx = \int_2^4 \frac{8-x}{2}\,dx$$

$$= \frac{1}{2}\left[8x - \frac{x^2}{2}\right]_2^4 = \frac{1}{2}[(32-8)-(16-2)]$$

$$= \frac{1}{2}[24-14] = \frac{1}{2}\times 10 = 5\,\text{sq.units}$$

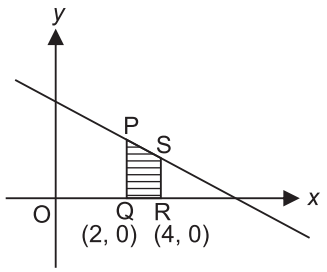


Illustration:

Draw a rough sketch of the curves $y = \sin x$ and $y = \cos x$ as x-varies from 0 to $\pi/2$. Find the area of the region enclosed by the curves and the x-axis.

Solution: Given curves $y = \sin x$

and $y = \cos x$

Area of shaded region

$$= \int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx$$

$$= -[\cos x]_0^{\pi/4} + [\sin x]_{\pi/4}^{\pi/2} = -\left[\frac{1}{\sqrt{2}} - 1\right] + \left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= \frac{-1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}} = (2 - \sqrt{2}) \text{ square units}$$

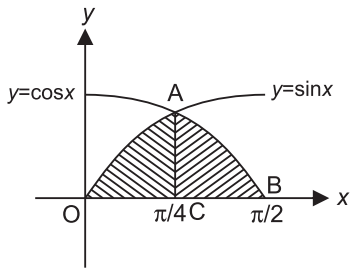


Illustration:

Using integration, find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 4$.

Solution: Given curve $y^2 = 16x$

line $x = 4$

Area of shaded region

= 2(area of AOC)

$$= 2\int_0^4 y\,dx = 2\int_0^4 4\sqrt{x}\,dx$$

$$= 8 \times \frac{2}{3}\left[x^{3/2}\right]_0^4 = \frac{16}{3}[8] = \frac{128}{3}\,\text{sq.units}$$

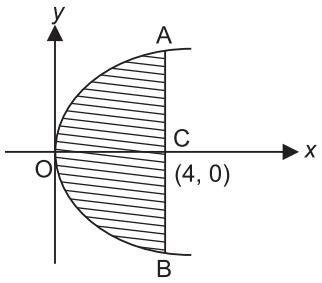


Illustration:

Using integration, find the area of the smaller portion of the circle $x^2 + y^2 = 4$ cut off by the line $x = 1$.

Solution: Circle $x^2 + y^2 = 4$

line $x = 1$

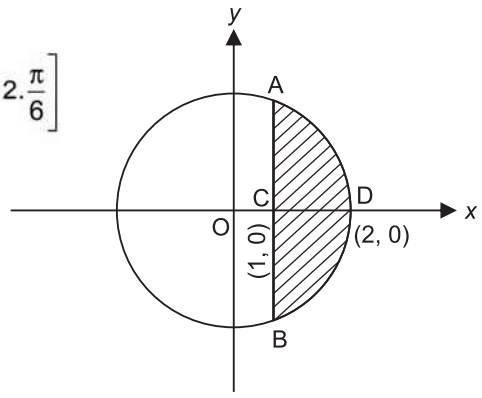
Area of shaded region

= 2(area bounded by the circle, the x-axis and ordinate $x = 1$ and $x = 2$)

$$= 2\int_1^2 y\,dx = 2\int_1^2 \sqrt{4 - x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_1^2 = 2\left[2.\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2.\frac{\pi}{6}\right]$$

$$= 2\left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right] = \frac{4\pi}{3} - \sqrt{3} \text{ sq. units}$$



ONE MARK QUESTIONS

Multiple Choice Questions (1 Mark Each)

Select the correct option out of the four given options:

1. The area of the region bounded by the curve $y = x^2$, x-axis and the lines $x = -1$, $x = 1$ is

   (a) $\frac{1}{3}$ *sq.* units (b) $\frac{2}{3}$ *sq.* units

   (c) 1 sq. unit (d) 2 sq. units

2. The area bounded by y = sin 2x, $0 \le x \le \frac{\pi}{4}$ and coordinate axes is

(a) $\frac{1}{2}$ sq. units (b) 1 sq. unit

(c) $\frac{3}{2}$ sq. units (d) 2 sq. units

3. The area bounded by the line x + 2y = 8 and the lines x = 1 and x = 3 is

(a) 16 sq. units (b) 8 sq. units

(c) 12 sq. units (d) 6 sq. units

4. The area enclosed by the parabola $y^2 = 8x$ and its latus rectum is

(a) $\frac{16}{3}$ sq. units (b) $\frac{64}{3}$ sq. units

(c) $\frac{32}{3}$ sq. units (d) $\frac{16\sqrt{2}}{3}$ sq. units

5. The area bounded by the curve y = cos x and x-axis between x = 0 and $x = \pi$ is

(a) 0 sq. units (b) 1 sq. units

(c) 2 sq. units (d) 4 sq. units

ASSERTION-REASON BASED QUESTIONS

In the following qustions a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)

(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A)

(c) (A) is true and (R) is false

(d) (A) is false, but (R) is true

6. Assertion (A) : Area enclosed by the curve $x^2 + y^2 = 4$ is given by $4\int_0^2 \sqrt{4 - x^2}\,dx$

Reason (R) : The curve $x^2 + y^2 = 4$ is symmetric about both the axes.

7. Assertion (A) : Area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum is given by $2\int_0^1 2\sqrt{x}\,dx$

Reason (R) : Length of the latus rectum of the parabola $y^2 = 4ax$ is $4a$.

TWO MARKS QUESTIONS

Using Integration:

1. Find the area of the circle $x^2 + y^2 = 16$.
2. Find the area of the parabols $y^2 = 4a\,x$ bounded by its latus rectum.
3. Find the area bounded by the curve $y^2 = x$, x – axis and the lines $x = 0$, $x = 4$.
4. Find the area bounded by the region $\{(x, y): x^2 \le y \le |x|\}$.
5. Find the area bounded by the region $y = 9x^2$, $y = 1$ and $y = 4$.
6. Find the area bounded by the curve $y = \sin x$ between $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$
7. Find the area bounded by the lines $y = 2x + 3$, $y = 0$, $x = 2$ and $x = 4$.
8. Find the area of the region bounded by $y^2 = 4x$, $x = 1$, $x = 4$ and x-axis in the first quadrant.
9. Find the area bounded by the curves $y^2 = 4ax$ and the lines $y = 2a$ and y-axis.
10. Find the area of the triangle formed by the straight lines $y = 2x$, $x = 0$ and $y = 2$

THREE/FIVE MARKS QUESTIONS

Using Integration

1. Find the area bounded by the curve $4y = 3x^2$ and the line $3x - 2y + 12 = 0$.
2. Find the area bounded by the curve $x = y^2$ and the line $x + y = 2$.
3. Find the area of the triangular region whose vertices are (1, 2), (2 – 2) and (4, 3).
4. Find the area bounded by the region $\{(x, y): x^2 + y^2 \le 1 \le x + \frac{y}{2}\}$
5. Find the area of the region bounded by the lines $x - 2y = 1$, $3x - y - 3 = 0$ and $2x + y - 12 = 0$.
6. Prove that the curve $y = x^2$and, $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$ into three equal parts.
7. Find the area of the smaller region enclosed between ellipse $b^2x^2 + a^2y^2 = a^2b^2$ and the line $bx + ay = ab$.
8. Using integration, find the area of the triangle whose sides are given by $2x + y = 4,\ 3x - 2y = 6\ and\ x - 3y + 5 = 0$.
9. Using integration, find the area of the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

10. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

11. Find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

12. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x^2 = y$ and y-axis.

13. Using integration, find the area of the region bounded by the curve $y = 1 + |x + 1|$ and lines $x = -3, x = 3, y = 0$.

14. Find the area of the region enclosed between curves $y = |x - 1|$ and $y = 3 - |x|$.

15. If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq unit then using integration find the value of m.

16. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $y = x$ and x-axis in first quadrant.

17. Find the area bounded by the parabola $y^2 = 4x$ and the straight line $x + y = 3$.

18. Find the area bounded by the parabola $y^2 = 4x$ and the line $y = 2x - 4$.

19. Find the area of region $\left\{(x,y) : \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \leq \frac{x}{3} + \frac{y}{2}\right\}$

20. Using integration, find the area of the triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

   (a) $\frac{9}{2}$ sq. units (b) $\frac{9}{3}$ sq. units

   (c) $\frac{9}{4}$ sq. units (d) $\frac{9}{5}$ sq. units

2. Area lying in first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

   (a) π sq. units (b) $\frac{\pi}{3}$ sq. units

   (c) $\frac{\pi}{2}$ sq. units (d) $\frac{\pi}{4}$ sq. units

3. The area of the region bounded by the curve y = x + 1 and the lines x = 2, x = 3 and x-axis is

(a) $\frac{13}{2}$ sq.units (b) $\frac{11}{2}$ sq.units

(c) $\frac{9}{2}$ sq.units (d) $\frac{7}{2}$ sq.units

4. The area bounded by the curve $y^2 = x$ and the line $x = 2y$ is

(a) $\frac{1}{3}$ sq.units (b) $\frac{2}{3}$ sq.units

(c) 1 sq. unit (d) $\frac{4}{3}$ sq.units

5. The area of the region bounded by the $y = \sin x$, $y = \cos x$ and y-axis, $0 \le x \le \frac{\pi}{4}$ is

(a) $(\sqrt{2}+1)$ sq.units (b) $(\sqrt{2}-1)$ sq.units

(c) $2\sqrt{2}$ sq.units (d) $(2\sqrt{2}-1)$ sq.units

ANSWERS

ONE MARKS QUESTION

1. (b) $\frac{2}{3}$ square units
2. (a) $\frac{1}{2}$ square units.
3. (d) 6 square units
4. (c) $\frac{32}{3}$ square units.
5. (c) 2 square units
6. (a)
7. (b)

TWO MARKS QUESTIONS

1. 16π square units.
2. $\frac{8}{3}a^2$ square units.
3. $\frac{16}{3}$ square units.
4. $\frac{1}{3}$ square units.
5. $\frac{28}{9}$ square units.
6. 2 square units.
7. 18 square units.
8. $\frac{28}{3}$ square units.
9. $\frac{2}{3}a^2$ square units.
10. 1 square units.

THREE/FIVE MARKS QUESTIONS

1. 27 square units.
2. $\frac{9}{2}$ square units.
3. $\frac{13}{2}$ square units.

4. $\left(\frac{\pi}{4}-\frac{2}{5}-\frac{1}{2}\sin^{/1}\frac{3}{5}\right)$ square units.

5. 10 square units.

7. $\left(\frac{\pi-2}{4}\right)ab$ square units.

8. 3.5 square units.

9. 4 square units.

10. $\left(\pi-\frac{1}{2}\right)$ square units.

11. $\frac{9}{8}$ square units.

12. $\frac{10}{3}$ square units.

13. 16 square units.

14. 4 square units.

15. m = 2.

16. 2π sq. units

17. $\frac{64}{3}$ sq. units

18. 9 sq. units

19. $\frac{3}{2}(\pi - 2)$ sq. units

20. 7 sq. units

SELF ASSESSMENT TEST-1

1. (C) 2. (A) 3. (D) 4. (D) 5. (B)