CHAPTER 7

INTEGRALS

There are many applications of integration in the field such as Physics, Engineering, Business, Economics etc. One of the important application of integration is finding the profit function of producing a certain number of cars if the marginal cost and revenue function are known. Companies can thus determine the maximum profit that can be earned and in this way plan their production, labour and other infrastructure accordingly.

INTEGRALS

Topics to be covered as per C.B.S.E. revised syllabus (2024-25)

- Integration as inverse process of differentiation
- Integration of a variety of functions by substitution, by partial fractions and by parts
- Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px + q}{ax^2 \pm bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx$$

$$\int \sqrt{ax^2 + bx + c}\, dx$$

  Fundamental Theorem of Calculus (without proof).

- Basic properties of definite integrals and evaluation of definite integrals.

POINTS TO REMEMBER

- Integration or anti derivative is the reverse process of Differentiation.
- Let $\frac{d}{dx}F(x) = f(x)$ then we write $\int f(x)\,dx = F(x) + c$.
- These integrals are called indefinite integrals and c is called constant of integration.
- From geometrical point of view, an indefinite integral is the collection of family of curves each of which is obtained by translating one of the curves parallel to itself upwards or downwards along y-axis.

STANDARD FORMULAE

1. $\int x^n\,dx = \begin{cases} \frac{x^{n+1}}{n+1} + c, & n \neq -1 \\ \log_e|x| + c, & n = -1 \end{cases}$

2. $\int (ax+b)^n\,dx = \begin{cases} \frac{(ax+b)^{n+1}}{(n+1)a} + c, & n \neq -1 \\ \frac{1}{a}\log|ax+b| + c, & n = -1 \end{cases}$

3. $\int \sin x\,dx = -\cos x + c.$

4. $\int \cos x\,dx = \sin x + c$

5. $\int \tan x.\,dx = -\log|\cos x| + c = \log|\sec x| + c.$

6. $\int \cot x\,dx = \log|\sin x| + c.$

7. $\int \sec^2 x\,dx = \tan x + c$

8. $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

9. $\int \sec x \tan x\,dx = \sec x + c$

10. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

11. $\int \sec x\, dx = \log|\sec x + \tan x| + c$

$= \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c$

12. $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + c$

$= \log\left|\tan\frac{x}{2}\right| + c$

13. $\int e^x\, dx = e^x + c$

14. $\int a^x\, dx = \frac{a^x}{\log a} + c$

15. $\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + c\,, |x| < 1$

$= -\cos^{-1} x + c$

16. $\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + c$

$= -\cot^{-1} x + c$

17. $\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + c\,, |x| > 1$

$= -\operatorname{cosec}^{-1} x + c$

18. $\int \frac{1}{a^2-x^2}\, dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

19. $\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

20. $\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

21. $\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\frac{x}{a} + c = -cos^{-1}\frac{x}{a} + c$

22. $\int \frac{1}{\sqrt{a^2+x^2}}\, dx = \log\left|x + \sqrt{a^2+x^2}\right| + c$

23. $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right| + c$

24. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

25. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right| + c$

26. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$

RULES OF INTEGRATION

1. $\int [(f_1(x) \pm f_2(x) \pm \ldots\ldots \pm f_x(x)]\,dx = \int f_1(x)dx \pm \int f_2(x)dx \pm \ldots\ldots \pm \int f_x(x)dx$

2. $\int k.f(x)dx = k\int f(x)dx.$

3. $\int e^x \{f(x) + f'(x)\}dx = e^x f(x) + c$

INTEGRATION BY SUBSTITUTION

1. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

2. $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$

3. $\int \frac{f'(x)}{[f(x)]^n}\,dx = \frac{(f(x))^{-n+1}}{-n+1} + c$

INTEGRATION BY PARTS

$$\int f(x)\, g(x)\, dx = f(x) \int g(x)\, dx - \int \left[f'(x) \int g(x)\, dx \right]$$

DEFINITE INTEGRALS

$$\int_a^b f(x)dx = F(b) - F(a), \text{ where } F(x) = \int f(x)\, dx$$

DEFINITE INTEGRAL AS A LIMIT OF SUMS.

$$\int_a^b f(x)dx = \lim_{h\to o} h\,[f(a) + f(a+h) + f(a+2h) + \cdots + f(a+n-1h)]$$

$Where\ h = \frac{b-a}{h}$ $or\ \int_a^b f(x)dx = \lim_{h\to o}[h\sum_{r=1}^{n} f(a+rh)]$

PROPERTIES OF DEFINITE INTEGRAL

1. $\int_a^b f(x) = -\int_b^a f(x)dx$

2. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt.$

3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$

4. $(i)\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx.$

   $(ii)\int_o^a f(x)\,dx = \int_o^a f(a-x)dx$

5. $\int_{-a}^{a} f(x)dx = 2\int_o^a f(x)dx, \qquad if\ f(x)\ is\ even\ function$

6. $\int_{-a}^{a} f(x)dx = 0\ if\ f(x)\ is\ an\ odd\ function$

7. $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ & \text{if } f(2a-x) = -f(x) \\ 0 & \end{cases}$

Illustration:

Evaluate $\int e^x \left(\frac{x-2}{x+4}\right)^2 dx$

Solution: $I = \int e^x \left(\frac{x-2}{x+4}\right)^2 dx = \int e^x \left(1-\frac{2}{x+4}\right)^2 dx$

$= \int e^x \left[\left(1-\frac{4}{x+4}\right)+\frac{4}{(x+4)^2}\right] dx$ It is of the form $e^x\left[f(x) + f'(x)\right] dx$

$= \int e^x [f(x)+f'(x)]dx,$ where $f(x) = 1-\frac{4}{x+4}$

$= e^x f(x)+C = e^x\left(1-\frac{4}{x+4}\right)+C = \frac{xe^x}{x+4}+C$

Illustration:

Find $\int \frac{x^2+1}{(x+1)^2} dx$

Solution: $\int \frac{x^2+1}{(x+1)^2} dx = \int \frac{(x+1)^2-2x}{(x+1)^2} dx$

$= \int \frac{(x+1)^2-2(x+1)+2}{(x+1)^2} dx$

$= \int \left[1-\frac{2}{x+1}+\frac{2}{(x+1)^2}\right] dx$

$= x-2\log|x+1|-\frac{2}{x+1}+C$

Illustration:

Evaluate $\int_0^{\pi/4} \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2} dx$

Solution: $\int_0^{\pi/4} \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2} dx = \int_0^{\pi/4} \frac{\tan^2 x\sec^2 x}{(\tan^3 x+1)^2} dx$

[dividing Num and Den by $\cos^6 x$]

Put $z = \tan^3 x + 1$,

then $dz = 3\tan^2 x \sec^2 x\, dx$

Also when $x = 0, z = 0$ and when $x = \frac{\pi}{4}, z = 2$

Now $I = \frac{1}{3}\int_2^1 \frac{dz}{z^2} = -\frac{1}{3}\left[\frac{1}{z}\right]_1^2 = -\frac{1}{3}\left[\frac{1}{2} - 1\right] = \frac{1}{6}$

Illustration:

Find $\int_{-\pi/4}^{\pi/4} \frac{x + \pi/4}{2 - \cos 2x} dx$

Solution: $\int_{-\pi/4}^{\pi/4} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx = \int_{-\pi/4}^{\pi/4} \frac{x}{2 - \cos 2x} dx + \frac{\pi}{4}\int_{-\pi/4}^{\pi/4} \frac{1}{2 - \cos 2x} dx$

$= 0 + \frac{\pi}{4}.2\int_0^{\pi/4} \frac{dx}{2 - \cos x}$ [Since first functionis an even function and second function is an odd function

$= \frac{\pi}{2}\int_0^{\pi/4} \frac{dx}{2(1 - 2\sin^2 x)}$

$= \frac{\pi}{2}\int_0^{\pi/4} \frac{dx}{2\sin^2 x + 1}$

$= \frac{\pi}{2}\int_0^{\pi/4} \frac{\sec^2 x}{3\tan^2 x + 1} dx$ [dividing num and den by $\cos^2 x$]

Put $z = \sqrt{3}\tan x$, then $dz = \sqrt{3}\sec^2 x\, dx$

Also when $x = 0, z = 0,$ and when $x = \frac{\pi}{4}, z = \sqrt{3}$

$\therefore$ From (i), $I = \frac{\pi}{2}\frac{1}{\sqrt{3}}\int_0^{\sqrt{3}} \frac{dz}{z^2 + 1} = \frac{\pi}{2\sqrt{3}}\left[\tan^{/1} z\right]_0^{\sqrt{3}}$

$= \frac{\pi}{2\sqrt{3}}\left[\tan^{-1}\sqrt{3} - \tan^{-1} 0\right]$

$= \frac{\pi}{2\sqrt{3}}\tan^{-1}\sqrt{3}$

$= \frac{\pi}{2\sqrt{3}}.\frac{\pi}{3} = \frac{\pi^2}{6\sqrt{3}}$

ONE MARK QUESTIONS

Evaluate the following integrals:

1. Integrate $\int_0^2 (x^2 + x + 1)dx$

   (a) $\frac{15}{2}$ (b) 20/5

   (c) 20/3 (d) 3/20

2. $\int_0^{\pi} \sin^2 x \, dx =$

   (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

   (c) 2π (d) 4π

3. $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ equal to:

   (a) $-\frac{1}{\sin x + \cos x} + c$ (b) $\log|\sin x + \cos x| + c$

   (c) $\frac{1}{(\sin x + \cos x)^2}$ (d) $\log|\sin x - \cos x| + c$

4. $\int \frac{(1+\log x)^2}{1+x^2} dx$ is :

   (a) $\frac{1}{3}(1+\log x)^3 + c$ (b) $\frac{1}{2}(1+\log x)^2 + c$

   (c) $\log(\log 1 + x) + c^2$ (d) None of these

5. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x . \cos^2 x} dx$ is equal to

   (a) $\tan x + \cos x + c$ (b) $\tan x + \text{cosec } x + c$

   (c) $\tan x + \cot x + c$ (d) $\tan x + \sec x + c$

6. The value of $\int_{\pi/6}^{\pi/3} \frac{dx}{\sin 2x}$ is :

(a) $\frac{1}{2}\log(-1)$ (b) $\log(-1)$

(c) $\log 3$ (d) $\log\sqrt{3}$

7. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is:

(a) 1 (b) 0

(c) -1 (d) $\frac{\pi}{4}$

8. $\int \frac{x^9}{(4x^2+1)^6}dx$ is equal to

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+c$ (b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+c$

(c) $\frac{1}{10x}\left(\frac{1+4}{x^2}\right)^{-5}+c$ (d) $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+c$

9. If $\int \frac{x^3}{\sqrt{1+x^2}}dx = 9(1+x^2)^{3/2}+b\sqrt{1+x^c}+c$

(a) $a=\frac{1}{3}, b=1$ (b) $a=-\frac{1}{3}, b=1$

(c) $a=-\frac{1}{3}, b=-1$ (d) $a=\frac{1}{3}, b=-1$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of ssertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true.

1. Assertion (A) : $\int \frac{dx}{x^2+2x+3} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + c$

   Reason (R) : $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

2. Assertion (A) : $\int e^x [\sin x - \cos x] dx = e^x \sin x + c$

   Reason (R) : $\int e^x [f(x) + f'(x)] dx = e^x (f(x) + c$

3. Assertion (A) : $\int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right) dx = 0$

   Reason (R) : $\int_0^{2a} f(x) dx = 0$ *if* $f(2a - x)$

4. Assertion (A) : $\int_{\pi/6}^{\pi/3} \frac{1}{1+(\tan x)^{11/5}} dx = \frac{\pi}{12}$

   Reason (R) : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

TWO MARKS QUESTIONS

Evaluate :

1. $\int e^{[\log(x+1)-\log x]}dx$
2. $\int \frac{1}{\sqrt{x+1}+\sqrt{x+2}}dx$
3. $\int \sin x \sin 2x dx$
4. $\int\left[\frac{x}{a}+\frac{a}{x}+x^a+a^x\right]dx$
5. $\int_0^{\pi/2}\log\left(\frac{5+3\cos x}{5+3\sin x}\right)dx$
6. $\int\frac{a^x+b^x}{c^x}dx$
7. $\int\left(\sqrt{ax}-\frac{1}{\sqrt{ax}}\right)^2 dx$
8. $\int e^x 2^x dx$
9. $\int 2^{2^{2^x}} 2^{2^x} 2^x dx$
10. $\int\frac{\sin\left(2\tan^{-1}x\right)}{1+x^2}dx$
11. $\int x\log 2x\, dx$
12. $\int_0^{\pi/4}\sqrt{1+\sin 2x}\, dx$
13. $\int_0^{\pi/2} e^x(\sin x-\cos x)dx$
14. $\int_4^9\frac{\sqrt{x}}{\left(30-x^{3/2}\right)}dx$
15. $\int_0^1\frac{dx}{e^x+e^{-x}}dx$
16. $\int\frac{\log|\sin x|}{\tan x}dx$
17. $\int\frac{\sin^4 x+\cos^4 x}{\sin^3 x+\cos^3 x}dx$
18. $\int\sqrt{\tan x}\,(1+\tan^2 x)dx$
19. $\int\frac{\sin 2x}{(a+b\cos x)^2}dx$
20. $\int\frac{x^2-x+2}{x^2+1}dx$

THREE MARKS QUESTIONS

Evaluate :

1. (i) $\int \frac{x\ cosec\left(\tan^{-1} x^2\right)}{1+x^4}\ dx$

(ii) $\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\ dx$

(iii) $\int \frac{1}{\sin(x-a)\sin(x-b)}\ dx$

(iv) $\int \frac{\cos(x+a)}{\cos(x-a)}\ dx$

(v) $\int \cos 2x\ \cos 4x\ \cos 6x\ dx$

(vi) $\int \tan 2x \tan 3x \tan 5x\, dx$

(vii) $\int \sin^2 x \cos^4 x\ dx$

(viii) $\int \cot^3 x\ \mathrm{cosec}^4 x\, dx$

(ix) $\int \frac{\sin x \cos x}{\sqrt{a^2\sin^2 x+b^2\cos^2 x}} dx$ [Hint: Put $a^2\sin^2 x + b^2\cos^2 x = t$ or t^2

(x) $\int \frac{1}{\sqrt{\cos^3 x\ \cos(x+a)}}\ dx$

(xi) $\int \frac{\sin^6 x+ \cos^6 x}{\sin^2 x \cos^2 x}\ dx$

(xii) $\int \frac{\sin x+ \cos x}{\sqrt{\sin 2x}}\ dx$

Evaluate :

2. (i) $\int \frac{x}{x^4+x^2+1} dx$

(ii) $\int \frac{1}{x[6(\log x)^2+7\log x+2]} dx$

(iii) $\int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx$

(iv) $\int \frac{x^2+1}{x^4+1} dx$

(v) $\int \frac{1}{\sqrt{(x-a)(x-b)}} dx$

(vi) $\int \frac{5x-2}{3x^2+2x+1} dx$

(vii) $\int \frac{x^2}{x^2+6x+1} dx$

(viii) $\int \frac{x+2}{\sqrt{4x-x^2}} dx$

(ix) $\int x\sqrt{1+x-x^2}\, dx$

(x) $\int \frac{\sin^4 x}{\cos^8 x} dx$

(xi) $\int \sqrt{\sec x - 1}\, dx$ [Hint: Multiply and divided by $\sqrt{\sec x + 1}$]

Evaluate :

3. (i) $\int \frac{dx}{x(x^7+1)}$

(ii) $\int \frac{3x+5}{x^3-x^2-x+1} dx$

(iii) $\int \frac{\sin\theta\cos\theta}{\cos^2\theta - \cos\theta - 2}\, d\theta$

(iv) $\int \frac{dx}{(2-x)(x^2+3)}$

(v) $\int \frac{x^2+x+2}{(x-2)(x-1)}\, dx$

(vi) $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$

(vii) $\int \frac{dx}{(2x+1)(x^2+4)}$

(viii) $\int \frac{x^2-1}{x^4+x^2+1} dx$

(ix) $\int \sqrt{\tan x}\, dx$

(x) $\int \frac{dx}{\sin x - \sin 2x}$

4. Evaluate:

(i) $\int x^5 \sin x^3\, dx$

(ii) $\int \sec^3 x\, dx$

(iii) $\int e^{ax} \cos(bx + c)\, dx$

(iv) $\int \sin^{-1}\left(\frac{6x}{1+9x^2}\right) dx$ [Hint: Put $3x=\tan\theta$]

(v) $\int \cos\sqrt{x}\, dx$

(vi) $\int x^3 \tan^{-1} x\, dx$

(vii) $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right) dx$

(viii) $\int\left[\frac{1}{\log x}-\frac{1}{(\log x)^2}\right] dx$

(ix) $\int\sqrt{2ax-x^2}\, dx$

(x) $\int e^x \frac{(x^2+1)}{(x+1)^2} dx$

(xi) $\int x^3 \sin^{-1}\left(\frac{1}{x}\right) dx$

(xii) $\int\left\{\log(\log x)+\frac{1}{(\log x)^2}\right\} dx$ [Hint: Put $\log x = t$ $x = e^t$]

(xiii) $\int(6x+5)\sqrt{6+x-x^2}\, dx$

(xiv) $\int\frac{1}{x^3+1} dx$

(xv) $\int\tan^{-1}\left(\frac{x-5}{1+5x}\right) dx$

(xvi) $\int\frac{dx}{5+4\cos x}$

5. Evaluate the following definite integrals:

(i) $\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x} dx$

(ii) $\int_0^{\pi/2}\cos 2x \log\sin x\, dx$

(iii) $\int_0^1 x\sqrt{\frac{1-x^2}{1+x^2}} dx$

(iv) $\int_0^{1/\sqrt{2}}\frac{\sin^{-1}x}{(1-x^2)^{3/2}} dx$

(v) $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx$

(vi) $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx$

(vii) $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x}\, dx$

(viii) $\int_0^1 x \log\left(1 + \frac{x}{2}\right) dx$

(ix) $\int_{-1}^{1/2} |x \cos \pi x|\, dx$

(x) $\int_{-\pi}^{\pi} (\cos a\, x - \sin b\, x)^2\, dx$

6. Evaluate:

(i) $\int_2^5 [|x-2| + |x-3| + |x-4|]\, dx$

(ii) $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$

(iii) $\int_{-1}^{1} e^{\tan^{-1} x} \left[\frac{1+x+x^2}{1+x^2}\right] dx$

(iv) $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

(v) $\int_0^2 [x^2]\, dx$

(vi) $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$

(vii) $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x}\, dx$ [*Hint: use* $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

7. Evaluate the following integrals:

(i) $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

(ii) $\int_{-\pi/2}^{\pi/2} \left(\sin|x| + \cos|x|\right) dx$

(iii) $\int_{0}^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\, dx$

(iv) $\int_{0}^{\pi} \frac{x \tan x}{\sec x + \operatorname{cosec} x}\, dx$

(v) $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$

8. Evaluate

(*i*) $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx \quad x \in [0, 1]$

(*ii*) $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$

(iii) $\int \frac{x^2 e^x}{(x+2)^2} dx$

(iv) $\int \frac{x^2}{(x \sin x + \cos x)^2}\, dx$

(v) $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx$

(vi) $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin\ 2x}}\ dx$

(vii) $\int \frac{\sin x}{\sin 4x}\ dx$

(viii) $\int_{-1}^{3/2} |x\ \sin \pi x|\ dx$

(ix) $\int \frac{\sin(x-a)}{\sin(x+a)}\ dx$

(x) $\int \frac{x^2}{(x^2+4)(x^2+9)}\ dx$

(xi) $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x}\ dx$

FIVE MARKS QUESTIONS

9. Evaluate the following integrals:

(i) $\int \frac{x^5 + 4}{x^5 - x}\ dx$

(ii) $\int \frac{2e^t}{e^{3t} - 6e^{2t} + 11e^t - 6}\ dt$

(iii) $\int \frac{2x^3}{(x+1)(x-3)^2}\ dx$

(iv) $\int \frac{1 + \sin x}{\sin x\ (1 + \cos x)}\ dx$

(v) $\int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

(vi) $\int_0^1 x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

(vii) $\int_0^{\pi/2} \frac{\cos x}{1+\cos x+\sin x}\,dx$

10. Evaluate the following integrals as limit of sums:

(i) $\int_2^4 (2x+1)\,dx$

(ii) $\int_0^2 (x^2+3)\,dx$

(iii) $\int_1^3 (3x^2-2x+4)\,dx$

(iv) $\int_0^4 (3x^2+e^{2x})\,dx$

(v) $\int_0^1 e^{2-3x}\,dx$

(vi) $\int_0^1 (3x^2+2x+1)\,dx$

11. Evaluate:

(i) $\int \frac{dx}{(\sin x-2\cos x)(2\sin x+\cos x)}$

(ii) $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$

(iii) $\int_0^{\pi/2} (2\log\sin x-\log\sin 2x)\,dx$

12. $\int_0^1 x(\tan^{-1} x)^2 \, dx$

13. $\int_0^{\pi/2} \log \sin x \, dx$

14. *Prove that* $\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right) dx = 2 \int_0^1 \tan^{-1} x \, dx$

Hence or otherwise evaluate the integral $\int \tan^{-1}(1-x+x^2)\, dx.$

15. *Evaluate* $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x} \, dx.$

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. $I = \int (x^8+1)(x^4+1)(x^2+1)(x+1)(x-1)dx =$

   (a) $x^{16}-1+c$ (b) $x^{17}-x+c$

   (c) $\frac{x^{17}}{17}-x+c$ (d) $\frac{x^{16}}{16}-x+c$

2. $\int \sin(x^2+2022).d(x^2) =$

   (a) $2x.\sin(x^2+2022)+c$ (b) $-2x.\cos(x^2+2022)+c$

   (c) $\sin(x^2+2022)+c$ (d) $-\cos(x^2+2022)+c$

3. $\int \cos^3 x.\sqrt{\sin x}\, dx = \frac{2\sin^a x}{3} - \frac{2\sin^b x}{7} + c$, then $(a+b) =$

   (a) 2 (b) 4

   (c) 5 (d) 6

4. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x.\cos^2 x} dx =$

   (a) $\tan x + \cot x + c$ (b) $-\tan x + \cot x + c$

   (c) tanx + cosecx + c (d) tanx + secx + c

5. $\int_0^{\pi/2} \sin^2 x\,dx = \frac{\pi}{k}$, then $k =$

(a) 0.25 (b) 0.5

(c) 1 (d) 4

SELF ASSESSMENT-2

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. $\int_0^{\pi/2} \log\tan x\,dx =$

(a) 0 (b) 1

(c) π (d) $\frac{\pi}{2}$

2. $\int_0^{\pi} \frac{x}{1+\sin x}dx =$

(a) 4π (b) $\frac{\pi}{2}$

(c) π (d) 2π

3. $\int \log(x^2+1)dx =$

(a) $x\log(x^2+1)-2x+2\tan^{-1}x+c$ (b) $x\log(x^2+1)-2x-2\tan^{-1}x+c$

(c) $x\log(x^2+1)+2x+2\tan^{-1}x+c$ (d) None of these

4. $\int e^x.\sin x\,dx =$

(a) $\frac{e^x(\sin x-\cos x)}{2}+c$ (b) $\frac{e^x(\sin x-\cos x)}{2}-c$

(c) $\frac{e^x(-\sin x+\cos x)}{2}+c$ (d) $\frac{-e^x(\sin x-\cos x)}{2}+c$

5. $\int \cos^2 x\,dx = ax+b\sin 2x+c$, then $(2a+4b+1) =$

(a) 0 (b) 1

(c) 3 (d) –7

Answers

ONE MARKS QUESTIONS

1. (c) $\frac{20}{3}$

2. (a) $\frac{\pi}{2}$

3. (b) $\log|\sin x+\cos x|+c$

4. (a) $\frac{1}{3}(1+\log x)^3+c$

5. (c) tanx + cot x + c

6. (c) log 3

7. (b) 0

8. $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+c$

9. $a=\frac{1}{3}, b=-1$

INTEGRAL ASSERTION REASONS

1. A is true and R is correct explanation of A
2. Option (d) is correct
3. Option (b) is correct
4. (a) A is true and R is correct explanation of A

TWO MARKS QUESTIONS

1. $x + \log x + c$

2. $\frac{2}{3}\left[(x+2)^{3/2} - (x+1)^{3/2}\right] + c$

3. $\frac{-1}{2}\left[\frac{\sin 3x}{3} - \sin x\right] + c$

4. $\frac{1}{a}\frac{x^2}{2} + a\log|x| + \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$

5. 0

6. $\frac{\left(\frac{a}{c}\right)^x}{\log\left|\frac{a}{c}\right|} + \frac{\left(\frac{b}{c}\right)^x}{\log\left|\frac{b}{c}\right|} + c$

7. $\frac{ax^2}{2} + \frac{\log|x|}{a} - 2x + c$

8. $\frac{2^x e^x}{\log(2e)} + c$

9. $\frac{2^{2^{2x}}}{(\log 2)^3} + C$

10. $\frac{-\left[\cos\left(2\tan^{-1}x\right)\right]}{2} + C$

11. $\frac{x^2}{2}\log 2x - \frac{x^2}{4} + C$

12. 1

13. 1

14. $\frac{19}{99}$

15. $\tan^{-1} e - \frac{\pi}{4}$

16. $\frac{\log|\sin x|^2}{2} - C$

17. $\log|\sec x + \tan x| + \log|\operatorname{cosec} x - \cot x| + C$

18. $\frac{2}{3}(\tan x)^{3/2} + C$

19. $-\frac{2}{b^2}\left[\log|a + b\cos x| + \frac{a}{a + b\cos x}\right] + C$

20. $x - \frac{1}{2}\log|x^2 + 1| + \tan^{-1}x + C$

THREE MARKS QUESTIONS

1. (i) $\frac{1}{2}\log\left[\text{cosec}(\tan^{-1}x^2) - \frac{1}{x^2}\right] + c$

(ii) $\frac{1}{2}\left(x^2 - x\sqrt{x^2-1}\right) + \frac{1}{2}\log\left|x + \sqrt{x^2-1}\right| + c$

(iii) $\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c$

(iv) $x\cos 2a - \sin 2a\ \log|\sec(x-a)| + c$

(v) $\frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$

(vi) $\frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$

(vii) $\frac{1}{32}\left[2x + \frac{1}{2}\sin 2x - \frac{1}{2}\sin 4x - \frac{1}{6}\sin 6x\right] + c$

(viii) $-\left(\frac{\cot^6 x}{6} + \frac{\cot^4 x}{4}\right) + c$

(ix) $\frac{1}{a^2-b^2}\sqrt{a^2\sin^2 x + b^2\cos^2 x} + c$

(x) $-2\text{cosec}\ a\sqrt{\cos a - \tan x \sin a} + c$

(xii) $\tan x - \cot x - 3x + c$

(vi) $\sin^{-1}[\sin x - \cos x] + c$

2. (i) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$

(ii) $\log\left|\frac{2\log x}{3\log x}\right| + c$

(iii) $\frac{-2}{\sqrt{\tan x}} + \frac{2}{3}\tan^{3/2}x + c$

(iv) $\frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(x-\frac{1}{x}\right)\right\}+c$

(v) $2\log\left|\sqrt{x-a}+\sqrt{x-b}\right|+c$

(vi) $\frac{5}{6}\log|3x^2+2x+1|+\frac{-11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+c$

(vii) $x-3\log|x^2+6x+12|+2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right)+c$

(viii) $-\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+c$

(ix) $-\frac{1}{3}(1+x-x^2)^{3/2}+\frac{1}{8}(2x-1)\sqrt{1+x-x^2}+\frac{5}{16}\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)+c$

(x) $\frac{\tan^5 x}{5}+\frac{\tan^7 x}{7}+c$

(xi) $-\log\left|\cos x+\frac{1}{2}+\sqrt{\cos^2 x+\cos x}\right|+c$

3. (i) $\frac{1}{7}\log\left|\frac{x^7}{x^7+1}\right|+c$

(ii) $\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{x-1}+c$

(iii) $\frac{-2}{3}\log|\cos\theta-2|-\frac{1}{3}\log|1+\cos\theta|+c$

(iv) $\frac{1}{14}\log\left|\frac{x^2+3}{(2-x)^2}\right|+\frac{2}{7\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

(v) $x+4\log\left|\frac{(x-2)^2}{x-1}\right|+c$

(vi) $x+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)-3\ \tan^{-1}\left(\frac{x}{2}\right)+c$

(vii) $\frac{2}{17}\log|2x+1| - \frac{1}{17}\log|x^2+4| + \frac{1}{34}\tan^{-1}\frac{x}{2} + c$

(viii) $\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right| + c$

(ix) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\log\left|\frac{\tan x - \sqrt{2\tan x}+1}{\tan x + \sqrt{2\tan x}+1}\right| + c$

(x) $-\frac{1}{2}\log|\cos x - 1| - \frac{1}{6}\log|\cos x + 1| + \frac{2}{3}\log|1 - 2\cos x| + c$

4. (i) $\frac{1}{3}[-x^3\cos x^3 + \sin x^3] + c$

(ii) $\frac{1}{2}[\sec x\tan x + \log|\sec x + \tan x|] + c$

(iii) $\frac{e^{ax}}{a^2+b^2}[a\cos(bx+c) + b\sin(bx+c)] + c$

(iv) $2x\ \tan^{-1}3x - \frac{1}{3}\log|1+9x^2| + c$

(v) $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$

(vi) $\left(\frac{x^4-1}{4}\right)\tan^{-1}x - \frac{x^3}{12} + \frac{x}{4} + c$

(vii) $\frac{1}{2}e^{2x}\tan x + c$

(viii) $\frac{x}{\log x} + c$

(ix) $\left(\frac{x-a}{2}\right)\sqrt{2ax - x^x} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + c$

(x) $e^x\left(\frac{x-1}{x+1}\right) + c$

(xi) $\frac{x^4}{4}\sin^{-1}\left(\frac{1}{x}\right) + \frac{x^2+2}{12}\sqrt{x^2-1} + c$

(xii) $x\log|\log x| - \frac{x}{\log x} + c$

(xiii) $-2(6+x-x^2)^{\frac{3}{2}} + 8\left[\frac{2x-1}{4}\sqrt{6+x-x^2} + \frac{25}{8}\sin^{-1}\left(\frac{2x-1}{5}\right)\right] + c$

(xiv) $\frac{1}{3}\log|x+1| - \frac{1}{6}\log|x^2 - x + 1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$

(xv) $x\tan^{-1}x - \frac{1}{2}\log|1 + x^2| - x\tan^{-1}5 + c$

(xvi) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right) + c$

5. (i) $\frac{1}{20}\log 3$

(ii) $-\pi/4$

(iii) $\frac{\pi}{4} - \frac{1}{2}$

(iv) $\frac{\pi}{4} - \frac{1}{2}\log 2$

(v) $\frac{\pi}{2}$

(vi) $\pi/4$

(vii) $\pi/2$

(viii) $\frac{3}{4} + \frac{3}{2}\log\frac{2}{3}$

(ix) $\frac{3}{2\pi} - \frac{1}{\pi^2}$

(x) $2\pi + \frac{1}{2a}\sin 2a\pi - \frac{1}{2b}\sin 2b\pi$

6. (i) $\frac{1}{2}$

(ii) π

(iii) $e^{\pi/4} + e^{-\pi/4}$

(iv) $\frac{1}{4}\pi^2$

(v) $5 - \sqrt{3} - \sqrt{2}$

(vi) $\frac{\pi^2}{16}$ (vii) $\frac{\pi^2}{2a}$

7. (i) $\frac{\pi}{12}$ (ii) 2

(iii) $\frac{\pi}{2}$ (iv) $\frac{\pi^2}{4}$

(v) $a\pi$

8. (i) $\frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x}+\frac{2\sqrt{x-x^2}}{\pi}-x+c$

(ii) $-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x-x^2}+c$

(iii) $\frac{x-2}{x+2}e^x+c$

(iv) $\frac{\sin x-x\cos x}{x\sin x+\cos x}+c$

(v) $(x+a)\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+c$

(vi) $2\sin^{-1}\frac{\sqrt{3}-1}{2}$

(vii) $\frac{1}{8}\log\left|\frac{1-\sin x}{1+\sin x}\right|-\frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|+c$

(viii) $\frac{3}{\pi}+\frac{1}{\pi^2}$

(ix) $(\cos 2a)(x+a)-(\sin 2a)\log|\sin(x+a)|+c$

(x) $-\frac{4}{5}\log|x^2+4|+\frac{9}{5}\log|x^2+9|+c$

(xi) $-\left(\frac{1}{2}\sin 2x+\sin x\right)+c$

9. (i) $x-4\log|x|+\frac{5}{4}\log|x-1|+\frac{3}{4}\log|x+1|+\log|x^2+1|$

(ii) $\frac{-1}{2}\tan^{-1}x+c$

(iii) $\log\left|\frac{(e^t-1)(e^t-3)}{(e^t-2)^2}\right|+c$

(iv) $2x-\frac{1}{8}\log|x+1|+\frac{81}{8}\log|x-3|-\frac{27}{2(x-3)}+c$

(v) $\frac{1}{4}\log\left|\frac{1-\cos x}{1+\cos}\right|+\frac{1}{2(1+\cos\ )}+\tan\frac{x}{2}+c$

(vi) $\frac{\pi}{\sqrt{2}}$ (vii) $\frac{\pi-2}{4}$

(viii) $\frac{\pi}{4}-\frac{1}{2}\log 2$

10. (i) 14 (ii) $\frac{26}{3}$

(iii) 26 (v) $\frac{1}{3}\left(e^2-\frac{1}{e}\right)$

(iv) $\frac{1}{2}(127+e^8)$ (vi) 3

11. (i) $\frac{1}{5}\log\left|\frac{\tan x-2}{2\tan x+1}\right|+c$ (ii) $\frac{\pi}{8}\log 2$

(iii) $\frac{\pi}{2}\log\frac{1}{2}$

12. $\frac{\pi^2}{16}-\frac{\pi}{4}+\frac{1}{2}\log 2$

13. $\frac{-\pi}{2}\log 2$

14. $\log 2$

15. $\frac{1}{\sqrt{2}}\log|\sqrt{2}+1|$

SELF ASSESSMENT TEST-1

1. (c) 2. (d) 3. (c) 4. (a) 5. (d)

SELF ASSESSMENT TEST-2

1. (a) 2. (c) 3. (a) 4. (b) 5. (c)