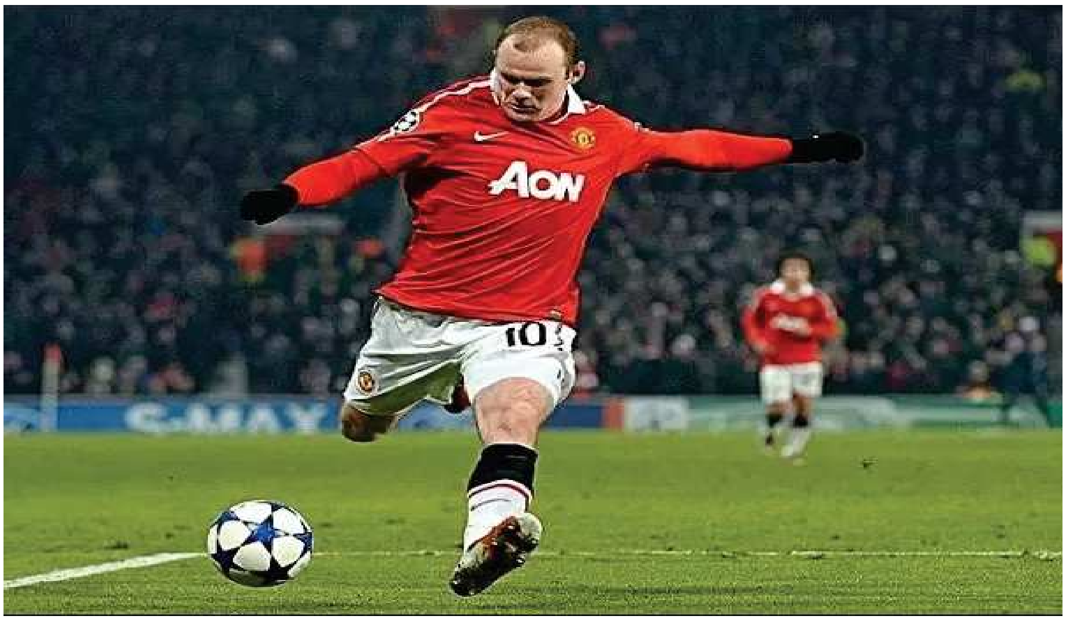
CHAPTER 5

CONTINUITY AND DIFFERENTIABILITY



Many real life events, such as trajectory traced by Football where you see player hit the soccer ball, angle and the distance covered animation on the screen is shown to the viewers using technology can be described with the help of mathematical functions. The knowledge of Continuity and differentiation is popularly used in finding speed, directions and other parameters from a given function.

CONTINUITY AND DIFFERENTIABILITY

Topics to be covered as per C.B.S.E. revised syllabus (2024-25)

- Continuity and differetiability
- Chain rule
- Derivative of inverse trigonometric functions, like $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$
- Derivative of implicit functions.
- Concept of exponential and logarithmic function
- Derivatives of logarithmic and exponential functions.
- Logarithmic differentiation, derivative of functions expressed in parametric forms.
- Second order derivatives.

POINTS TO REMEMBER

- A function $f(x)$ is said to be continuous at $x = c$ iff $\lim_{x\to c} f(x) = f(c)$

  i.e., $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$

- $f(x)$ is continuous in (a, b) iff it is continuous at $x = c$ $\forall c \in (a, b)$.

- $f(x)$ is continuous in $[a, b]$ iff

  (i) $f(x)$ is continuous in (a, b)

  (ii) $\lim_{x\to a^+} f(x) = f(a)$

  (iii) $\lim_{x\to b^-} f(x) = f(b)$

- Modulus functions is Continuous on R

- Trigonometric functions are continuous in their respective domains.

- Exponential function is continuous on R

- Every polynomial function is continuous on R.

- Greatest integer function is continuous on all non-integral real numbers

- If $f(x)$ and $g(x)$ are two continuous functions at $x = a$ and if $c \in R$ then

  (i) $f(x) \pm g(x)$ are also continuous functions at $x = a$.

  (ii) $g(x).f(x)$, $f(x) + c$, $cf(x)$, $|f(x)|$ are also continuous at $x = a$.

  (iii) $\frac{f(X)}{g(X)}$ is continuous at $x = a$, provided $g(a) \neq 0$.

- A function $f(x)$ is derivable or differentiable at $x = c$ in its domain iff

  $\lim_{x\to c^-} \frac{f(X)-f(c)}{X-c} = \lim_{x\to c^+} \frac{f(X)-f(c)}{X-c}$, and is finite

  The value of above limit is denoted by $f'(c)$ and is called the derivative of $f(x)$ at $x = c$.

  $\frac{d}{dx}(u \pm v) = \frac{du}{dx} + \frac{dv}{dx}$

- $\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v.\frac{du}{dx}$ (Product Rule)

- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$ (Quotient Rule)

- If y= f(u) and u=g(t) then $\frac{dy}{dt} = \frac{dy}{du} \times \frac{du}{dt} = f'(u)g'(t)$ (Chain Rule)

- If y =f(u), x=g(u) then,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{f'(u)}{g'(u)}$$

Illustration:

Discuss the continuity of the function $f(x)$ given by

$$f(x) = \begin{cases} 4 - x, & x < 4 \\ 4 + x, & x \geq 4 \end{cases} \text{ at } x = 4$$

Solution: We have $f(x) = \begin{cases} 4 - x, & x < 4 \\ 4 + x, & x \geq 4 \end{cases}$

$$\text{LHL} = \lim_{x \to 2^{/}} f(x) = \lim_{x \to 4^-} (4 - x) = \lim_{h \to 0^-} 4 - (4 - h) = 0$$

$$\text{RHL} = \lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (4 + x) = \lim_{h \to 0^+} 4 + (h + 4) = 8 + 0 = 8$$

Here LHL ≠ RHL

Hence $f(x)$ is not continuous at $x = 4$

Illustration:

Show that the function $f(x)$ given by

$$f(x) = \begin{cases} \frac{\tan x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0 \end{cases} \text{ is continuous at } x = 0$$

Solution: We have $f(x) = \begin{cases} \frac{\tan x}{x} + \cos x, & x \neq 4 \\ 2, & x = 4 \end{cases}$

Now $f(0) = 2$ ... (i)

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(\frac{\tan x}{x} + \cos x \right) = \lim_{h \to 0^-} \frac{\tan(0 - h)}{(o - h)} + \cos(0 - h) = \lim_{h \to 0} \left[\frac{-\tan h}{-h} + \cos h \right]$$

$$= \lim_{h \to 0} \frac{\tan h}{h} + \lim_{h \to 0} \cos h = 1 + \cos(0) = 1 + 1 = 2 \qquad ...(ii)$$

$$\text{RHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} \left(\frac{\tan x}{x} + \cos x \right) = \lim_{h \to 0^+} \frac{\tan(0+h)}{(0+h)} + \cos(0+h)$$

$$= \lim_{h \to 0} \frac{\tan h}{h} + \lim_{h \to 0} \cos h = 1 + \cos(0) = 1 + 1 = 2 \qquad ...(iii)$$

$$\text{LHL} = \text{RHL} = f(0)$$

Hence $f(x)$ is continuous at $x = 0$

ONE MARK QUESTIONS

Continuity and Differentiability

This section comprises Multiple Choice Questions (MCQ) of one mark each

1. The value of k for which the function f given by

$$f(x) = \begin{pmatrix} \frac{k \cos x}{\pi - 2x}, & if\ x \neq \frac{\pi}{2} \\ 5, & if\ x = \frac{\pi}{2} \end{pmatrix}$$ is continuous at $x = \frac{\pi}{2}$ is :

(a) 6 (b) 5

(c) $\frac{5}{2}$ (d) 10

2. The value of k for which

$$f(x) = \begin{cases} 3x + 5, & x \geq 2 \\ k\ x^2, & x < 2 \end{cases}$$ is a continuous function is :

(a) $\frac{-11}{4}$ (b) $\frac{4}{11}$

(c) 11 (d) $\frac{11}{4}$

3. For what value of k, may the function $f(x) = \begin{cases} k(3x^2 - 5x), & x \leq 0 \\ \cos x, & x > 0 \end{cases}$ becomes continuous ?

(a) 0 (b) 1

(c) $\frac{-1}{2}$ (d) No value

4. If $f(x)=\begin{cases}\dfrac{\sin \pi x}{5x}, x\neq 0\\ k, x=0\end{cases}$ is continuous at x = 0, then k is equal to :

(a) $\frac{5}{\pi}$ (b) $\frac{\pi}{5}$

(c) 1 (d) 0

5. if $f(x)=\begin{cases}\dfrac{\sqrt{x^2+5-3}}{x+2}, x\neq -2\\ k \quad , x=-2\end{cases}$ is continuous at x = –2 , then the value of k is equal to :

(a) $\frac{-2}{3}$ (b) 0

(c) $\frac{2}{3}$ (d) none of these

6. If $f(x)\begin{cases}\dfrac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}, x\neq \frac{\pi}{4}\\ k \quad , x=\frac{\pi}{4}\end{cases}$ is continuous at x = $\frac{\pi}{4}$, then the value of k is :

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) none of these

7. The number of points of discontinuity of the rational function $f(x)=\dfrac{x^2-3x+2}{4x-x^3}$ is :

(a) 1 (b) 2

(c) 3 (d) none of these

8. The function f(x) =[x], where [x] denotes the greatest integer function, is continuous at x =

(a) –2 (b) 1

(c) 4 (d) 1.5

9. The function f(x) = |x| at x = 0 is :

(a) continuous but not differentiable

(b) diffcrcntiablc but not continuous

(c) continuous and differentiable

(d) neither continuous nor differentiable

10. The function f(x) = |x| + |x-1| is :

(a) differentiable at x = 0 but not at x = l

(b) differentiable at x = l but not at x = 0

(c) neither differentiable at x = 0 nor at x = 1

(d) differentiable at x = 0 as well as at x = 1

11. The set of numbers where the function f given by $f(x)$ = |2x – 1| cos x is differentiable is:

(a) R (b) $R-\left(\frac{1}{2}\right)$

(c) $(0,\infty)$ (d) none of these

12. If $y=\log\left(\frac{1-x^2}{1+x^2}\right), |x|<1$ then $\frac{dy}{dx}=$

(a) $\frac{4x^3}{1-x^4}$ (b) $\frac{-4x}{1-x^4}$

(c) $\frac{1}{4-x^4}$ (d) $\frac{-4x^3}{1-x^4}$

13. The derivative of sec($\tan^{-1}$x) w.r.t. x is

(a) $\frac{x}{1+x^2}$ (b) $\frac{1}{\sqrt{1+x^2}}$

(c) $\frac{x}{\sqrt{1+x^2}}$ (d) $x\sqrt{1+x^2}$

14. If $y=\cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)+\operatorname{cosec}^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$ then $\frac{dy}{dx}$ is equal to :

(a) $\frac{\pi}{2}$ (b) 0

(c) 1 (d) none of these

15. Differential of log [log (log x^5)] w.r.t. x is :

(a) $\frac{5}{x\log(x^5)\log(\log x^5)}$ (b) $\frac{5}{x\log(\log x^5)}$

(c) $\frac{5x^4}{\log(x^5)\log(\log x^5)}$ (d) $\frac{5x^4}{\log(\log x^5)}$

16. If $y = \sin(m \sin^{-1} x)$ then which of the following equations is true?

(a) $(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + m^2y = 0$

(b) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

(c) $(1+x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$

(d) $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$

17. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{\cos x}{2y-1}$ (b) $\frac{\cos x}{1-2y}$

(c) $\frac{\sin x}{1-2y}$ (d) $\frac{\sin x}{2y-1}$

Q no (18-22) are Assertion Reason Based questions carrying one mark each. These type of questions consists of two statements , one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (a), (b),(c), and (d) as given below.

(a) Bolh Assertion (A) and Reason (R)arc true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R)arc true and Reason (R) is **not** the correct explanation of the Assertion (A).

(c) Assertion (A)is true and Rcason(R) is false.

(d) Assertion (A)is false and Reason(R) is true

18. Let $f(x) = \frac{1}{1-x} - \frac{3}{1-x^3}, x \neq 1$

Statement -I: The value of f(1)so that f is continuous function is 1

Statement-II : $g(x) = \frac{x+2}{x^2+x+1}$ is continuous function

Answer (d) Assertion (A)is false and Reason(R) is true

19. Consider the function f(x) = |x – 2| + |x – 5|, $x \in R$

Statement - I : f′(4)=0

Statement -II : f is continuous on [2, 5] differentiable on (2, 5) and f(2) = f(5)

Solution (b) Both Assertion (A) and Reason (R)are true and Reason (R) is **not** the correct explanation of the Assertion (A).

20. **Statement -I :** $f(x)=\begin{cases} x^2 \sin\frac{1}{x}, x \neq 0 \\ 0 , x=0 \end{cases}$

Statement-II : Both $h(x) = x^2$ and $g(x)=\begin{cases} \sin\frac{1}{x}, x \neq 0 \\ 0, x=0 \end{cases}$ continuous at $x = 0$

21. F(x) is defined as the product of two real functions $f_1(x) = x \quad \forall\, x \in R$ and

$f_2(x)=\begin{cases} x\sin\frac{1}{x}, x \neq 0 \\ 0, x=0 \end{cases}$ as follows

$F(x)=\begin{cases} f_1(x), f_2(x) \text{ if } x \neq 0 \\ 0, \text{ if } x=0 \end{cases}$

Statement -I: F(x) is continuous on R

Statement-II : $f_1(x)$ and $f_2(x)$ are continuous on R

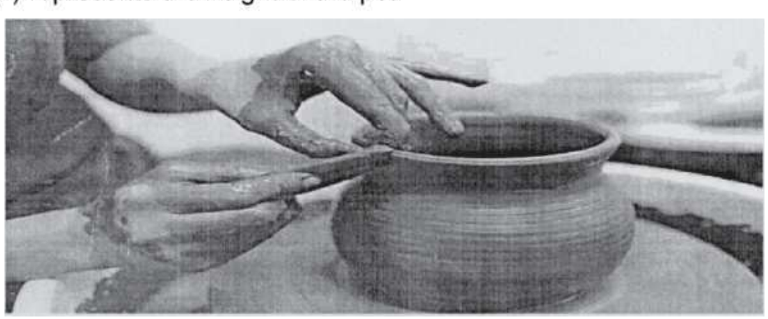
22. Let f(x) be a differentiable function such that $f(2)=4$ and $f'(2)=4$

Statement -I: $\lim\limits_{x\to 2}\frac{xf(2)-2f(x)}{x-2}=-4$

Statement -II : $f'(a)=\lim\limits_{x\to a}=\frac{f(x)-f(a)}{x-a}$

CASE BASED

23. A plotter made a mud vessel , where the shape of pot is based on f(x) = |x-3| + |x-2|, where f(x) represents the height of the pot.



Based on the information given above answer the following questions

(1) When x > 4 what will be the height in terms of x?

(2) When the value of x lies between (2, 3) then find the value of f(x).

(3) If the potter is trying to make pot using the function f(x)=[x], will he get a pot or not? why?

Q24. Let x = f(t) and y = g(t) be the parametric forms with t as parameter, then

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{g'(t)}{f'(t)}$ where $f'(t) \neq 0$

On the basis of the above information answer the following questions :

(1) What will be the derivative of f(tanx) w.r.t g(secx) at $x = \frac{\pi}{4}$ where $f'(1)$ and $g'(\sqrt{2}) = 4$?

(2) Find the derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t $\cos^{-1} x$.

(3) If y = $\frac{1}{4}u^4$ and $u = \frac{2}{3}x^3$ then find $\frac{dy}{dx}$

25. A function f(x) is said to be differentiable at x=c if

(i) Left hand derivative (L.H.D) = $f'(c) = \lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}$ exists finitely.

(ii) Right hand derivative (R.H.D) = $f'(c) = \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$ exists finitely.

(iii) R.H.D = L.H.D, i.e. if the function f(x) is differentiable at x = c, then f' (c) = $\lim_{h \to c} \frac{f(x) - f(c)}{x - c}$

Based on the above information answer the following :

(1) If f(x) is differentiable at x = 3. then find the value of $\lim_{h \to 3} \frac{x^2 f(3) - 9f(x)}{x - 3}$

(2) Find $\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{h}$ if it exists.

TWO MARKS QUESTIONS

1. Differentiate sin (x^2) w. r. t. $e^{\sin x}$

2. $y = x^y$ then find $\frac{dy}{dx}$

3. If $y = x^x + x^3 + 3^x + 3^3$, find $\frac{dy}{dx}$

4. If $y = 2\sin^{-1}(\cos x) + 5\,\text{cosec}^{-1}(\sec x)$. Find $\frac{dy}{dx}$

5. If $y = e^{[\log(x+1) - \log x]}$ find $\frac{dy}{dx}$

6. Differentiate $\text{Sin}^{-1}[x\sqrt{x}]$ w. r. t. x.

7 Find the derivative of $|x^2+2|$ w.r.t. x

8. Find the domain of the continuity of $f(x) = \sin^{-1}x - [x]$

9. Find the derivative of cos $(\sin x^2)$ w.r.t. x at $x = \sqrt{\frac{\rho}{2}}$

10. If $y=e^{3\log x + 2x}$, Prove that $\frac{dy}{dx} = x^2(2x+3)\,e^{2x}$.

11. Differentiate $\sin^2(\theta^2+1)$ w.r.t. θ^2

12. Find $\frac{dy}{dx}$ if $y = \sin^{/ 1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$

13. If $x^2 + y^2 = 1$ verify that $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$

14. Find $\frac{dy}{dx}$ when $y = 10^{x10^x}$

15. If $y = x^x$ find $\frac{d^2y}{dx^2}$

16. Find $\frac{dy}{dx}$ if $y = \cos^{-1}(\sin x)$

17. If f(x) = x + 7, and g(x) = x – 7, $x \in R$, them find $\frac{d}{dx}$ (fog) (x).

18. Differntiate log (7 logx) w.r.t x

19. If $y = f(x^2)$ and $f'(x) = \sin x^2$. Find $\frac{dy}{dx}$

20. Find $\frac{dy}{dx}$ if $y = \sqrt{\sin^{-1}\sqrt{x}}$

THREE MARKS QUESTIONS

1. Examine the continuity of the following functions at the indicated points.

(I) $f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ at $x = 0$

(II) $f(x) = \begin{cases} x - [x], & x \neq 1 \\ 0, & x = 1 \end{cases}$ at $x = 1$

(III) $f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ at $x = 0$

(IV) $f(x) = \begin{cases} \frac{x - \cos(sin^{-1}x)}{1 - \tan(sin^{-1}x)} & x \neq \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & x = \frac{1}{\sqrt{2}} \end{cases}$ at $x = \frac{1}{\sqrt{2}}$

2. For what values of constant K, the following functions are continuous at the indicated points.

(i) $f(x) = \begin{cases} \frac{\sqrt{1+Kx} - \sqrt{1-Kx}}{x} & x < 0 \\ \frac{2x+1}{x-1} & x > 0 \end{cases}$ at $x = 0$

(ii) $f(x) = \begin{cases} \frac{e^x - 1}{\log(1+2x)} & x \neq 0 \\ K & x = 0 \end{cases}$ at $x = 0$

(iii) $f(x) = \begin{cases} \frac{1 - cos4x}{x^2} & x < 0 \\ K & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}} - 4} & x > 0 \end{cases}$ at $x = 0$

3. For what values a and b

$$f(x) = \begin{cases} \dfrac{x+2}{|x+2|} + a & if\ x < -2 \\ a+b & if\ x = -2 \\ \dfrac{x+2}{|x+2|} + 2b & if x > -2 \end{cases}$$

Is continuous at $x = -2$

4. Find the values of a, b and c for which the function

$$f(x) = \begin{cases} \dfrac{\sin[(a+1)x] + sinx}{x} & x < 0 \\ C & x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & x > 0 \end{cases}$$

Is continuous at $x = 0$

5. $f(x) = \begin{cases} [x] + [-x] & x \neq 0 \\ \lambda & x = 0 \end{cases}$

Find the value of λ, *f* is continuous at $= 0$?

6. Let $f(x) = \begin{cases} \frac{1-sin^3x}{3cos^2x} ; & x < \frac{\pi}{2} \\ a ; & x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} ; & x > \frac{\pi}{2} \end{cases}$

If $f(x)$ is continuous at $x = \frac{\pi}{2}$, find *a* and *b.*

7. If $f(x) = \begin{cases} x^3 + 3x + a & x \leq 1 \\ bx + 2 & x > 1 \end{cases}$

Is everywhere differentiable, find the value of *a* and *b*.

8. Find the relationship between a and b so that the function defined by

$$f(x) = \begin{Bmatrix} ax+1 \ , & x \leq 3 \\ bx+3 \ , & x > 3 \end{Bmatrix} is\ continous\ at\ x = 3.$$

9. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ where $x \neq 0$.

10. If $y = x^{x^x}$, then find $\frac{dy}{dx}$.

11. Differentiate $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$ w.r.t. x.

12. If $(x+y)^{m+n} = x^m . y^n$ then prove that $\frac{dy}{dx} = \frac{y}{x}$

13. If $(x-y).e^{\frac{x}{x-y}} = a$, prove that y$\left(\frac{dy}{dx}\right)$ + x = 2y

14. If $x = \tan\left(\frac{1}{a}\log y\right)$ then show that

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$$

15. If $y = x\log\left(\frac{x}{a+bx}\right)$ prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

16. Differentiate $\sin^{-1}\left[\frac{2^{x+1}.3^x}{1+(36)^x}\right]$ w.r.t x.

17. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$, prove that

$\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$, Where –1 < x < 1 and –1 < y < 1 [HINT: put x^3 = sin A and y^3 = sin B]

18. If $f(x) = \sqrt{x^2+1}, g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$ find $f'[h'(g'(x))]$.

19. If x = $\sec\theta - \cos\theta$ and $y = sec^n\theta - cos^n\theta$, then prove that $\frac{dy}{dx} = n\sqrt{\frac{y^2+4}{x^2+4}}$

20. If $x^y + y^x + x^x = m^n$, then find the value of $\frac{dy}{dx}$.

21. If $x = a\,cos^3\theta, y = a\,sin^3\theta$ then find $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{6}$

22. If $y = \tan^{-1}\left[\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right]$ where $0 < x < \frac{\pi}{2}$ find $\frac{dy}{dx}$

23. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then show that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

24. If $= \left[x + \sqrt{x^2+1}\right]^m$, show that $(x^2+1)y_2 + xy_1 - m^2y = 0$.

25. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log\ )^2}$

26. If $y^{1/m} + y^{-1/m}$ = 2x then prove that $(x^2-1)y_2 + xy_1 = m^2y$.

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ONE

1. If $y = \sin^2 x - \cos^2 x$, then $\frac{dy}{dx} =$

   (a) $2\sin x$ (b) $2\cos x$

   (c) $2\sin 2x$ (d) $-2\sin 2x$

2. The value of '4k' for which the function $f(x)$ is continuous at $x = 3$.

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & \text{when } x \neq 3 \\ 2k+1, & \text{when } x = 3 \end{cases}$$

   (a) 4 (b) 6

   (c) 11 (d) 22

3. Derivative of $\sin x$ with respect to $\cos x$ is

   (a) $\tan x$ (b) $-\tan x$

   (c) $\cot x$ (d) $-\cot x$

4. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{dx} + x\frac{dy}{dx} =$

(a) n^2y (b) ny

(c) y (d) $-ny$

5. If $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ then $\frac{d^2y}{dx^2} =$

(a) $\frac{\sec^3\theta}{a}$ (b) $\frac{\sec^3\theta}{a\theta}$

(c) $\sec^3\theta$ (d) $\theta\sec^3\theta$

SELF ASSESSMENT-2

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ONE

1. A Function defined as

$$f(x) = \begin{cases} |x| - 3, & \text{when } x < 0 \\ 5 - |x|, & \text{when } x \geq 0 \end{cases}$$

is continuous on

(a) R (b) $R - \{0\}$

(c) $[0, \infty)$ (d) $(-\infty, 0]$

2. The function g(x) = $(\sin x + \cos x)$ is continuous at

(a) R (b) $R - \{0\}$

(c) $R - \left\{\frac{\rho}{2}\right\}$ (d) $R - \{\pi\}$

3. The value of the derivative of $|x - 2| + |x - 3|$ at $x = 2$ is

(a) 1 (b) 3

(c) 2 (d) 0

4. If $\sin y = x.\cos(a + y)$ then $\frac{dy}{dx}$?

(a) $\frac{\cos^2(a+y)}{\cos a}$ (b) $\frac{\cos^2(a+y)}{\sin a}$

(c) $\frac{\sin^2(a-y)}{\cos a}$ (d) $\frac{\sin^2(a+y)}{\sin a}$

5. If $y = \left(\frac{x^a}{x^b}\right)^{a-b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$, then $\frac{dy}{dx} =$

(a) 1 (b) abc

(c) $a + b + c$ (d) 0

ANSWERS

ONE MARK QUESTIONS

1. (d) 10
2. (d) $\frac{11}{4}$
3. (d) No value
4. (b) $\frac{\pi}{5}$
5. (a) $\frac{-2}{3}$
6. (c) $\frac{1}{2}$
7. (c) 3
8. (d) 1.5
9. (c) continuous and differentiable
10. (c) neither differentiable at x = 0 nor at x = 1
11. (b) $R-\left\{\frac{1}{2}\right\}$
12. (b) $\frac{-4x}{1-x^4}$
13. (c) $\frac{x}{\sqrt{1+x^2}}$
14. (b) 0
15. (a) $\frac{5}{x\log(x^5)\log(\log x^5)}$
16. (b) $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+x^2y=0$
17. (a) $\frac{\cos x}{2y-1}$
18. (a) $\frac{x}{\sqrt{1+x^2}}$

ASSERTION REASONING

18. Answer (d) Assertion (A) is false and Reason (R) is true
19. (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
20. Solution (c) Assertion (A) and Reason (R) is false
21. Ans (a) Both Assertion (A) is true and Reason (R) are true.
22. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)

CASE BASED QUESTIONS

23. (a) f(x) = 2x – 5

    (b) f(x) = 1

    (c) Since the function is not continuous he will not get a pot.

24. (a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) $\frac{16}{27}x^{11}$

25. (a) 9f' (3) + 6f(3) (b) 2f'(x)

TWO MARKS QUESTIONS

1. $\frac{2x\cos(x^2)}{\cos x\, e^{\sin x}}$
2. $\frac{y^2}{x[1-y\log x]}$
3. $x^x\,[1 + \log x] + 3x^2 + 3^x \log_e 3$
4. –7
5. $-\frac{1}{x^2}$
6. $\frac{3}{2}\sqrt{\frac{x}{1-x^3}}$
7. $\frac{2x(x^2+2)}{|x^2+2|}$
8. $(-1,0) \cup (0,1)$
9. 0
11. $\text{Sin}\,(2\theta^2 + 2),\ \theta \neq 0$
12. 0
14. $10^{x^{10^x}}\, 10^x \log 10(1 - x\log 10)$
15. $x^x[1 - \log x]$
16. -1
17. 1
18. $\frac{1}{x\log x}$
19. $2x \sin x^4$
20. $\frac{1}{4\sqrt{x}\sqrt{1-x}\sqrt{\sin^{-1}\sqrt{x}}}$, where $0 < x < 1$

THREE MARKS QUESTIONS

1. (I) Continuous (II) Discontinuous
   (III) Not Continuous at $x = 0$ (IV) Continuous
2. (I) K = -1 (II) K = $^1/_2$
   (III) K = 8
3. $a = 0,\ b = -1$
4. $a = \frac{-3}{2},\ b = R - \{0\},\ c = \frac{1}{2}$
5. $\lambda = -1$
6. $a = \frac{1}{2},\ b = 4$
7. $a = 3,\ b = 5$
8. 3a – 3b = 2
9. $-\frac{1}{2}$
10. $x^x x^{x^x}\left\{(1+\log x)\log x + \frac{1}{x}\right\}$
11. $(x\cos x)^x[1 - x\ \tan x + (\log x\ \cos x)] + (x\sin x)^{1/x}\left[\frac{1+x\ cot\,x - log(x\ sin)^x}{x^2}\right]$
16. $\left[\frac{2^{x+1}3^x}{1+(36)^x}\right]\log 6$
18. $\frac{2}{\sqrt{5}}$
20. $\frac{dy}{dx} = \frac{x^x(1+\log x)+yx^{y-1}-y^x\log y}{x^y\log x + xy^{x-1}}$
21. $\frac{32}{27a}$
22. $-\frac{1}{2}$

SELF ASSESSMENT TEST-1

1. (C) 2. (C) 3. (D) 4. (A) 5. (B)

SELF ASSESSMENT TEST-2

1. (B) 2. (A) 3. (C) 4. (A) 5. (D)