CHAPTER-4

DETERMINANTS

One of the important aplications of inverse of a non-singular square matrix is in cryptography.

Cryptography is an art of communication between two people by keeping the information not known to others. It is based upon two factors, namely encryption and decryption.

Encryption means the process of transformation of an information (plain form) into an unreadable form (coded form). On the other hand, *Decryption* means the transformation of the coded message back into original form. Encryption and decryption require a secret technique which is known only to the sender and the receiver.

This secret is called a *key*. One way of generating a key is by using a non-singular matrix to encrypt a message by the sender. The receiver decodes (decrypts) the message to retrieve the original mesage by using the inverse of the matrix. The matrix used for encryption is called encryption matrix (encoding matrix) and that used for decoding is called decryption matrix (decoding matrix).

TOPIC TO BE COVERED AS PER CBSE LATEST CURRICULUM (2024-25)

- Determinant of a square matrix (up to 3 × 3 matrice), minors, co-factors and applications of determinants in finding the area of a triangle.
- Adjoint and inverse of a square matrix.
- Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

A determinant of order 2 is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ where a, b, c, d are complex numbers (As Complex Number Include Real Number). It denotes the complex number $ad - bc$.

Even though the value of determinnants Represented by Modulus symbol but the value of a determinant may be positive, negative or zero.

In other words,

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ (Product of diagonal elements – Product of non-diagonal elements)

- Determinant of order 1 is the number itself.
- We can expand the determinants along any Row or Column, but for easier calculations we shall expand the determinant along that row or column which contains maximum number of zeroes.

MINORS AND COFATORS

Minor of an Element

If we take an element of the determinant and delete/remove the row and column containing that element, the determinant of the elements left is called the minor of that element. It is denoted by M_{ij}. For example,

Let us consider a Determinant $|A|$

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \Rightarrow$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \Rightarrow M_{11} = \begin{vmatrix} e & f \\ q & r \end{vmatrix} \text{ (Minor of } a_{11} = M_{11})$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \Rightarrow M_{11} = \begin{vmatrix} e & f \\ q & r \end{vmatrix} \text{ (Minor of } a_{11} = M_{11})$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ p & q & r \end{vmatrix} \Rightarrow M_{11} = \begin{vmatrix} e & f \\ q & r \end{vmatrix} \text{ (Minor of } a_{11} = M_{11})$$

Hence a determinant of order two will have “4 minors” and a determinant of order three will have “9 minors”.

Minor of an Element:

Cofactor of the element a_{ij} is $c_{ij} = (-1)^{i+j} M_{ij}$; where i and j denotes the row and column in which the particular element lies. (Means Magnitude of Minor and Cofactor of aij are equal).

- **Property:** If we multiply the elements of any row/column with their respective Cofactors of the same row/column, then we get the value of the determinant.

  For example,

  $|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

  $|A| = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$

- **Property:** If we multiply the elements of any row/column with their respective Cofactors of the other row/column, then we get zero as a result.

  For example,

  $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33}$

Note that the value of a determinant of order three in term sof 'Minor' and 'Cofactor' can be written as:

$|A| = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$ OR $|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

Clearly, we see that, if we apply the appropriate sign to the minor of an element, we have its Cofactor. The signs form a check-board pattern.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

PROPERTIES OF DETERMINANTS

- The value of a determinant remains unaltered, if the row and solumns are inter changed.

$$|A| = |A^T|$$

$$\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

- If any two rows (or columns) of a determinant be interchanged, the value of determinant is changed in sign only . e.g.

$$\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} = \begin{vmatrix} b & y & q \\ a & x & p \\ c & z & r \end{vmatrix}$$

- If all the elements of a row (or column) are zero, then the determinant is zero.

$$\begin{vmatrix} a & 0 & x \\ b & 0 & y \\ c & 0 & z \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ p & q & r \\ x & y & z \end{vmatrix} = 0$$

- If the all elements of a row (or column) are proportional (identical) to the elements sof some other row (or column), then the determinant is zero.

$$\begin{vmatrix} a & ka & x \\ b & kb & y \\ c & kc & z \end{vmatrix} = \begin{vmatrix} mp & mq & mr \\ p & q & r \\ x & y & z \end{vmatrix} = 0$$

- If all the elements of a determinant above or below the main diagonal consist of zeros (Triangular Matrix), then the determinant is equal to the product of diagonal elements.

$$\begin{vmatrix} a & 0 & 0 \\ x & b & 0 \\ y & z & c \end{vmatrix} = \begin{vmatrix} a & x & y \\ 0 & b & z \\ 0 & 0 & c \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc$$

- If all the elements of one row/column of a determinant are multiplied by "k" (A scalar), the value of the new determinant is k times the original determinant.

$$\begin{vmatrix} ka & p & x \\ kb & q & y \\ kc & r & z \end{vmatrix} = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$\begin{vmatrix} ka & kp & x \\ kb & kq & y \\ kc & kr & z \end{vmatrix} = k^2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$$\begin{vmatrix} ka & kp & kx \\ kb & kq & ky \\ kc & kr & kz \end{vmatrix} = k^3 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

$|kA| = k^n |A|$, where n is the order of determinant.

AREA OF A TRIANGLE

Area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ (sq. units)}$$

ADJOINT OF A MATRIX

Let $A = [a_{ij}]_{m \times n}$ be a square matrix and C_{ij} be cofactor of a_{ij} in $|A|$.

Then, $(\text{adj } A) = [C_{ij}] \Rightarrow \text{adj } A = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$

- $A.(\text{adj } A) = (\text{adj } A).A = |A|$
- $(\text{adj } AB) = (\text{adj } B).(\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$, where n is the order of a Matrix A

SINGULAR MATRIX

A Matrix A is singular if $|A| = 0$ and it is non-singular if $|A| \neq 0$

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 5 \neq 0. \text{ So } A \text{ is Non-singular Matrix.}$$

$$|A| = \begin{vmatrix} 2 & 8 \\ 1 & 4 \end{vmatrix} = 8 - 8 = 0. \text{ So } A \text{ is singular Matrix.}$$

INVERSE OF A MATRIX

A square matrix A is said to be invertible if there exists a square matrix B of the same order such that $AB = BA = I$ then we write $A^{-1} = B$, (A^{-1} exists only if $|A| \neq 0$)

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{|A|}\begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$$

- $(AB)^{-1} = B^{-1}.A^{-1}$
- $(A^{-1})^{-1} = A$
- $(A^T)^{-1} = (A^{-1})^T$
- $AA^{-1} = A^{-1}A = I$
- $|A^{-1}| = \dfrac{1}{|A|}$
- $|A.\text{adj } A| = |A|^n$ (Where n is the order of Matrix A)

Illustration:

For what value of k, the matrix $A = \begin{pmatrix} 2 & 10 \\ 5k-2 & 15 \end{pmatrix}$ is singular matrix.

Solution: As, Matrix is singular, so its determinant will be zero.

$|A| = 2(15) - 10(5k - 2) = 30 - 50k + 20$

$|A| = 50 - 50k = 0$

$\Rightarrow\ 50k = 50$

$\therefore\ \boxed{k = 1}$

Illustration:

Without expanding the determinants prove that $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

Solution: Let $A = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

We observe here $a_{ij} = -a_{ij}$ (A is skew-symmetric matrix)

$\Rightarrow$ $A^T = -A$

$\Rightarrow$ $|A^T| = |-A|$

$\Rightarrow$ $|A| = (-1)^3 \, |A|$

> Property USED: $|A^T| = |A|$, $|kA| = k^n|A|$
> Where n is the order of the determinant

$\Rightarrow$ $|A| = -|A|$

$\Rightarrow$ $2|A| = 0$

$$\Rightarrow |A| = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

Illustration:

If A is an invertible matrix of order 2 and $|A| = 4$, then write the value of $|A^{-1}|$.

Solution: As we know that,

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{4}$$

$$\Rightarrow \boxed{|A^{-1}| = \frac{1}{4}}$$

Illustration:

Find the inverse of the matrix $\begin{pmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{pmatrix}$ and hence solve the system of equations:

$3x + 4y + 5z = 18$

$5x - 2y + 7z = 20$

$2x - y + 8z = 13$

Solution: Let, $A = \begin{pmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{pmatrix}$

Cofactors are,

$C_{11} = \begin{vmatrix} -1 & 8 \\ -2 & 7 \end{vmatrix} = -7 + 16 = 9$ $\quad C_{21} = -\begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = -38$ $\quad C_{31} = \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = 37$

$C_{12} = -\begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix} = -(14 - 40) = 26$ $\quad C_{22} = \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = -4$ $\quad C_{32} = -\begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = -14$

$C_{13} = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -4 + 5 = 1$ $\quad C_{23} = -\begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = 26$ $\quad C_{33} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -11$

$$\text{Adj } A = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix} = \begin{pmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{pmatrix}$$

$|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = 3(9) + 4(26) + 5(1) = 27 + 104 + 5 = 136$

$$\text{So, } A^{-1} = \frac{1}{|A|}(\text{Adj } A) = \frac{1}{136}\begin{pmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{pmatrix}$$

Given system of equation can be written as

$$\begin{pmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 18 \\ 20 \\ 13 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 4 & 5 \\ 5 & -2 & 7 \\ 2 & -1 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 18 \\ 13 \\ 20 \end{pmatrix}$$

$\Rightarrow A.X = B \Rightarrow A^{-1} AX = A^{-1}.B$

$IX = A^{-1}.B \Rightarrow X = A^{-1}.B$

$$X = \frac{1}{136}\begin{pmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{pmatrix}\begin{pmatrix} 18 \\ 13 \\ 20 \end{pmatrix} = \frac{1}{136}\begin{pmatrix} 9\times 18 - 38\times 13 + 37\times 20 \\ 26\times 18 - 4\times 13 - 14\times 20 \\ 1\times 18 + 26\times 13 - 11\times 20 \end{pmatrix}$$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{136}\begin{pmatrix} 408 \\ 136 \\ 136 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

$\Rightarrow$ So, $x = 3$, $y = 1$, $z = 1$

ONE MARK QUESTIONS

1. If f(x)= $\begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$, then determinant of $\left(f\left(\frac{\pi}{6}\right).f\left(\frac{\pi}{3}\right)\right) =$

   (a) 0 (b) 1 (c) –1 (d) $\frac{\pi}{2}$

2. If for a square matrix $A, A^2 - A + I = 0$, then A^{-1} equal

   (a) A (b) I + A (c) A – I (d) I – A

3. If $\begin{vmatrix} x & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix}$ = 0, then value of x is

   (a) 0 (b) 1 (c) 4 (d) 2

4. The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is

   (a) xyz (b) (x + y + z) (c) 2(x + y + z) (d) 0

5. If $A = \begin{pmatrix} 2 & 2023 & 2024 \\ 0 & 1 & 2022 \\ 0 & 0 & 5 \end{pmatrix}$, then A. (adj A) equals

   (a) 2I (b) I (c) 5I (d) 10 I

6. If $A = \begin{pmatrix} 3 & 1 \\ 19 & 7 \end{pmatrix}$, then A.(adj A) equals

   (a) $\begin{pmatrix} 3 & 1 \\ 19 & 7 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 7 & -1 \\ -19 & 3 \end{pmatrix}$

7. If the area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units, then |k| =

   (a) 0 (b) 6 (c) 3 (d) 9

8. If the area of a triangle with vertices (2, –6), (5, 4) and (k, 4) is 35sq. units, then the sum of all possible values of k is

(a) 2 (b) 10 (c) 12 (d) 14

9. If $A = \begin{pmatrix} 2023 & 1 \\ 2024 & 1 \end{pmatrix}$, then A^{-1} =

(a) $\begin{pmatrix} -2023 & 1 \\ 2024 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 1 \\ 2024 & -2023 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & -1 \\ -2024 & -2023 \end{pmatrix}$

10. If $A = \begin{pmatrix} k & 16 \\ -9 & -k \end{pmatrix}$ is singular matrix, then sum of all possible values of k is

(a) 0 (b) 12 (c) 10 (d) 24

11. If $A = \begin{pmatrix} k & 12 \\ 3 & 6 \end{pmatrix}$ is non-invertible matrix, then value of k is

(a) 0 (b) 3 (c) 6 (d) 12

12. If $A.(adjA) = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$, then $|A| + |adjA|$ =

(a) 5 (b) 10 (c) 25 (d) 30

13. If $A.(adjA) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, then $\frac{|A| + |A^T|}{|A^{-1}|}$ =

(a) 2 (b) 8 (c) 4 (d) 16

ASSERTION-REASON BASED QUSTIONS (Q. 14 & Q.15)

In the following questions, a statement of assertion (A) is following by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A are R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explnantion of A.

(c) A is true but R is false

(d) A is false but R is true

14. Assertion: For Matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, value of $4C_{31} + 5C_{32} + 6C_{33}$ is 0.

Reasoning : The sum of the products of elements of any row of a matrix A with the co-factors of elements of other row is always equal to Zero.

15. Assertion : If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, then determinant of matrix A is zero.

Reasoning : The determinant of a skew-symmetric matrix of order 3 × 3 is always zero.

TWO MARKS QUESTIONS

16. Without expanding the determinants prove that $\begin{vmatrix} 0 & 2023 & -2021 \\ -2023 & 0 & -2022 \\ 2021 & 2022 & 0 \end{vmatrix} = 0$

17. Let A be a 3 × 3 matrix such that $|A| = -2$, then find the value of $|-2A^{-1}| + 2|A|$.

18. If $A = \begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix}, B = \begin{pmatrix} yr - zq & cq - br & bz - cy \\ zp - xr & ar - cp & cx - az \\ xq - yp & bp - aq & ay - bx \end{pmatrix}$. Find $|B|$ if $|A| = 4$

19. If $A = \begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix}, B = \begin{pmatrix} yr - zq & cq - br & bz - cy \\ zp - xr & ar - cp & cx - az \\ xq - yp & bp - aq & ay - bx \end{pmatrix}$. Find $|A|$ if $|B| = 25$

20. Find the Adjoint of Matrix A,

$$A = \begin{pmatrix} 2\cos\frac{\pi}{3} & -2\sin\frac{\pi}{3} \\ 2\sin\frac{\pi}{3} & -2\cos\frac{\pi}{3} \end{pmatrix}$$

THREE MARKS QUESTIONS

21. If A is a square matrix of order 3, such that $|\text{Adj } A| = 25$, then find the value of

(a) $|A|$ (b) $|-2A^T|$ (c) $|4A^{-1}|$

(d) $|5A|$ (e) $A.\text{Adj } A$ (f) $|A.\text{Adj } A|$

(g) $|A^3|$

22. If A is a square matrix of order 3, such that $|A| = 5$, then find the value of

(a) $|3A|$ (b) $|-2A^T|$ (c) $|4A^{-1}|$

(d) $|\text{Adj } A|$ (e) $A.\text{Adj } A$ (f) $|A.\text{Adj } A|$

(f) $|A^3|$

23. If $A = \begin{pmatrix} 1 & 2020 & 2021 \\ 0 & 1 & 2022 \\ 0 & 0 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 & 0 \\ 2021 & 1 & 0 \\ 2020 & 2022 & 1 \end{pmatrix}$ then find the value of

(a) $|AB|$ (b) $|(AB)^{-1}|$ (c) $|A^2.B^3|$

(d) $|3(AB)^T|$ (e) $|\text{Adj } (AB)|$

24. Find matrix 'X' such that $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 7 & -2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}$

25. Find matrix 'X' such that

(a) $X \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & -2 \\ 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} X = \begin{pmatrix} 7 & -2 \\ 1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} X \begin{pmatrix} 7 & -2 \\ 1 & 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

FOUR/FIVE MARKS QUESTIONS

26. (a) A school wants to award its students for regularity and hardwork with a total cash award of ₹ 6,000. If three times the award money for hardwork added to that given for regularity amounts of ₹ 11,000 represent the above situation algebraically and find the award money for each value, using matrix method.

(b) A shopkeeper has 3 varieties of pen A, B and C. Rohan purchased 1 pen of each variety for total of ₹ 21. Ayush purchased 4 pens of A variety, 3 pens of B variety and 2 pen of C variety for ₹ 60. While Kamal purchased 6 pens of A variety, 2 pens of B variety and 3 pen of C variety for ₹ 70. Find cost of each variety of pen by Matrix Method.

27. Find A^{-1}, where $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$. Hence use the result to solve the following system of linear equations:

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

28. Find A^{-1}, where $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$. Hence, solve the system of linear equations:

$$x + 2y + 3z = 8$$
$$2x + 3y - 3z = -3$$
$$-3x + 2y - 4z = -6$$

29. If $A = \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$ find AB. Hence using the product solve the system of eq.

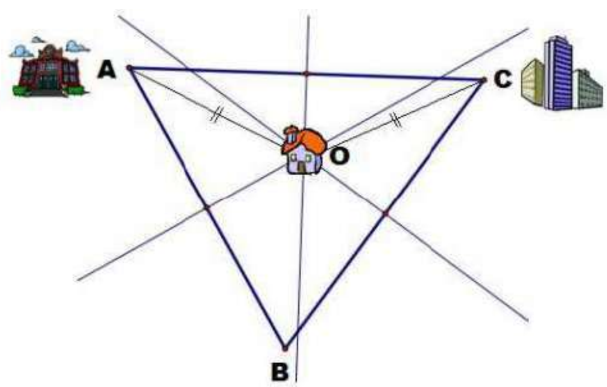
$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

30. Find the product of matrices AB, where $A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{pmatrix}$ and use the result to solve following system of equations:

$$x - 2y - 3z = 1$$
$$-2x + 4y + 5z = -1$$
$$-3x + 7y + 9z = -4$$

CASE STUDY BASED QUESTIONS

A. A family wanted to buy a home, but they wanted it to be close both to both the children's school and the parents' workplace. By looking at a map, they cold find a point that is equidistant from both the workplace and the school by finding the *circumcenter* of the triangular region.



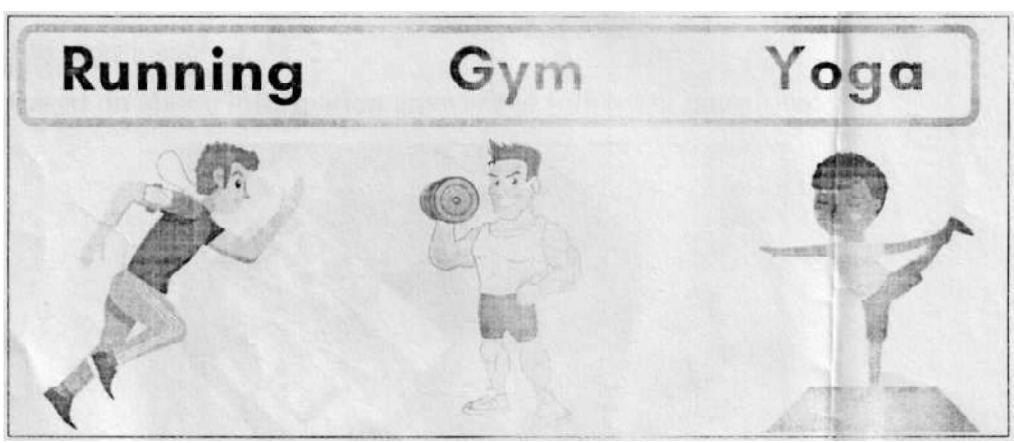
If the coordinates are $A(12, 5)$, $B(20, 5)$ and $C(16, 7)$, on the basis of this answer the following: (Figure is for reference only, Not as per scale)

(a) Using the concept of Determinants. Find the equation of AC.

(b) If any point $P(2, k)$ is collinear with point $A(12, 5)$ and $O(16, 2)$, then find the value of $(2k - 15)$.

(c) If any point P(2, k) is collinear with point A (12, 5) and 0 (16, 2), then find the value of $(2k - 15)$.

B. For keeping Fit, X people believes in morning walk, Y people believes in yoga and Z people join Gym. Total no of people are 70. Further 20%, 30% and 40% people are suffering from any disease who believe in morning walk, yoga and GYM respectively. Total no. of such people is 21. If morning walk cost ₹ 0 Yoga cost ₹ 500/month and GYM cost ₹ 400/ month and total expenditure is ₹ 23000.



On the basis of above information, answer the following:

(a) If matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 0 & 5 & 4 \end{pmatrix}$, then find A^{-1}.

(b) On solving the given situational problem using matrix method, find the total number of person who prefer GYM.

C. An amount of ₹ 600 crores is spent by the government in three schemes. Scheme *A* is for saving girl child from the cruel parents who don't want girl child and get the abortion before her birth.

Scheme *B* is for saving of newlywed girls from death due to dowry. Scheme *C* is planning for good health for senior citizen. Now twice the amount spent on Scheme *C* together with amount spent on Scheme *A* is ₹ 700 crores. And three times the amount spent on Scheme *A* together with amount spent on Scheme *B* and Scheme *C* is ₹ 1200 crores.

If we assume government invest (In crores) ₹ X, ₹ Y and ₹ Z in scheme *A*, *B* and *C* respectively. Solve the above problem using Matrices and answer the following:

C. Gautam buys 5 pens, 3 pens, 3 bags & 1 instrumental box and pays a sum of Rs. 160. From the same shop, Vikram buys 2 pens, 1 bag & 3 instrumental boxes and pays a sum of Rs. 190. Also Ankur buys 1 pen, 2 bags & 4 instrumental boxes and pays a sum of Rs. 250.

Based on above informatin answer the following questions:

(a) Convert the given situation into a matrix equation of the form AX = B.

(b) Find |A|.

(c) Find A^{-1}.

OR

Determine $P = A^2 - 5A$

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. If $A = \begin{bmatrix} 2 & 3 & 6 \\ 0 & 1 & 8 \\ 0 & 0 & 5 \end{bmatrix}$, then $|A|$ =

(a) 2 (b) 5

(c) 8 (d) 10

2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$, then $|A^T|$ =

(a) 2 (b) 5

(c) 8 (d) 10

3. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then $|A^{-1}|$ =

(a) 0 (b) 1

(c) $\cos x . \sin x$ (d) –1

4. If $A = \begin{bmatrix} 6x & 8 \\ 3 & 2 \end{bmatrix}$ is singular matrix, then the value of x is

(a) 2 (b) 3

(c) 5 (d) 7

5. The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k will be

(a) 6 (b) 9

(c) 3 (d) 0

SELF ASSESSMENT-2

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE

1. If the value of a third order determinant is 12, then the value of the determinant formed by replacing each element by its co-factor will be

(a) 0 (b) 1

(c) 12 (d) 144

2. If the points (3, –2), (x, 2), (8, 8) are collinear, then x =

(a) 2 (b) 5

(c) 4 (d) 3

3. $\begin{vmatrix} \cos 15^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 75^\circ \end{vmatrix} =$

(a) 0 (b) 1

(c) –1 (d) 2

4. The minor of 6 in the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is

(a) 9 (b) –6

(c) 6 (d) 10

5. The cofactor of 4 in the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is

(a) 9 (b) –6

(c) 6 (d) 10

ANSWER

One Mark Questions

1. (b) 1
2. (d) I – A
3. (c) 4
4. (d) 0
5. (d) $10I$
6. (c) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$
7. (c) 3
8. (b) 10
9. (b) $\begin{pmatrix} -1 & 1 \\ 2024 & -2023 \end{pmatrix}$
10. (a) 0
11. (c) 6
12. (d) 30
13. (b) 8
14. (a) Both A and R are true and R is the correct explanation of A.
    (b) Both A and R are true and R is not the correct explanation of A.

Two Marks Questions

17. 0
18. 16
19. ±5
20. $\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$

Three Marks Questions

21. (a) ±5 (b) ±40 (c) $\frac{\pm 64}{5}$ (d) ±625 (e) $\pm 5I$
    (f) ±125 (g) ±125
22. (a) 135 (b) –40 (c) $\frac{64}{5}$ (d) 25 (e) $5I$
    (f) 125 (e) 125
23. (a) 6 (b) $\frac{1}{6}$ (c) 72 (d) 162 (e) 36
24. $X = \frac{1}{9}\begin{pmatrix} 2 & 31 \\ -1 & -11 \end{pmatrix}$
25. (a) $X = \begin{pmatrix} 16 & -25 \\ 1 & -1 \end{pmatrix}$ (b) $X = \begin{pmatrix} 11 & -7 \\ -5 & 4 \end{pmatrix}$ (c) $X = \frac{1}{9}\begin{pmatrix} 5 & -17 \\ -3 & 12 \end{pmatrix}$

Five Marks Questions

26. (a) Award money given for
    Honesty = ₹ 500
    Regularity = ₹ 2000 and

Hard work = ₹ 3500

(b) Cost of pen of

Variety A = ₹ 5

Variety B = ₹ 8 and

Variety C = ₹ 8

27. $x = 3, y = -2, z = 1$ 28. $x = 0, y = 1, z = 2$ 29. $x = 3, y = -2, z = -1$

30. $x = -4, y = -1, z = -1$

CASE STUDIES QUESTIONS

A. (a) x – 2y = 2 (b) 10 sq. units (c) 10

B. (a) $A^{-1} = \frac{-1}{6}\begin{pmatrix} -8 & 1 & 1 \\ -8 & 4 & -2 \\ 10 & -5 & 1 \end{pmatrix}$ (b) 20

C. (a) $\begin{pmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 160 \\ 190 \\ 250 \end{pmatrix}$

A X = B

(b) –22

(c) $\frac{1}{-22}\begin{pmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{pmatrix}$

OR PART

$\begin{pmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{pmatrix}$

SELF ASSESSMENT-1

1. (d) 2. (d) 3. (b) 4. (a) 5. (c)

SELF ASSESSMENT-2

1. (d) 2. (b) 3. (a) 4. (b) 5. (c)