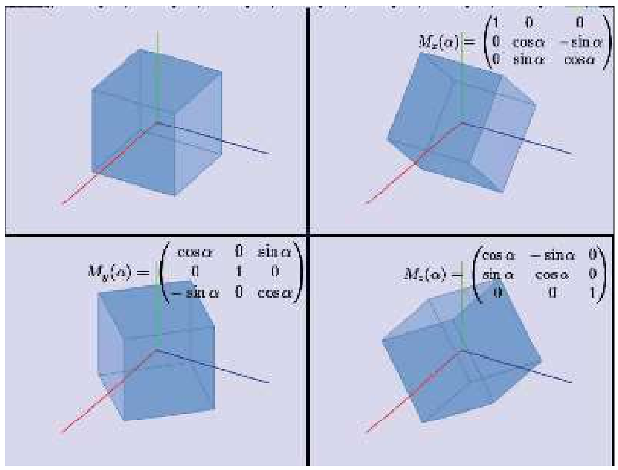
CHAPTER-3

MATRICES

Matrices find many applications is scientific field and apply to practical real life problem. Matrices can be solved physical related application and one applied in the study of electrical circuits, quantum mechanics and optics, with the help of matrices, calculation of battery power outputs, resistor conversion of electrical energy into another useful energy, these matrices play a role in calculation, with the help of matrices problem related to Kirchhoff law of voltage and current can be easily solved.



Matrices can play a vital role in the projection of three dimensional images into two dimensional screens, creating the realistic decreeing motion. Now day's matrices are used in the ranking of web pages in the Google search. It can also be used in generalization of analytical motion like experimental and derivatives to their high dimensional.

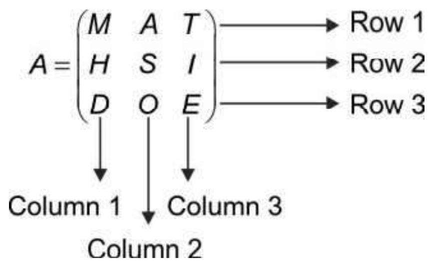
Matrices are also used in geology for seismic survey and it is also used for plotting graphs. Matrices are also used in robotics and automation in terms of base elements for the robot movements. The movements of the robots are programmed with the calculation of matrices 'row and column' controlling of matrices are done by calculation of matrices.

TOPIC TO BE COVERED AS PER CBSE LATEST CURRICULUM (2024-25)

- Concept, notaion, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices.
- Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Oncommutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2).
- Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Matrices are defined as a rectangular arrangement of numbers of functions. Since it is a rectangular arrangement, it is 2-dimensional.

A two-dimensional matrix consists of the number of rows (m) and a number of columns (n). Horizontal ones are called Rows and Vertical ones are called columns.



ORDER OF MATRIX

The order of matrix is a relationship with the number of elements present in a matrix.

The order of a matrix is denoted by $m \times n$, where m and n are the number of Rows and Columns Respectively and the number of elements in a matrix will be equal to the product of m and n.

TYPES OF MATRICES

Row Matrix

A matrix having only one row is called a row matrix.

Thus $A = [A_{ij}]_{mxn}$ is a row matrix if m = 1. So, a row matirx can be represented as $A = [A_{ij}]_{1xn}$

It is called so because it has only one row and the order of a row matrix will hence be $1 \times n$.

For example,

$A = [1 \quad 2 \quad 3 \quad 4]$ is row matrix of order 1 × 4. Another example of the row matrix is

$B = [0 \quad 9 \quad 4]$ which is of the order 1 × 3.

Column Matrix

A matrix having only one column is called a column matrix. Thus, $A = [A_{ij}]_{mxn}$ is a column matrix if $n = 1$.

Hence, the order is $m \times 1$. An example of a column matrix is:

$$A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, B = \begin{pmatrix} M \\ A \\ T \\ H \end{pmatrix}$$

In the above example, A and B are 3 × 1 and 4 × 1 order matrices respectively.

Square Matrix

If the number of rows and the number of columns in a matrix are equal, then it is called a square matrix.

Thus, $A = [A_{ij}]_{mxn}$ is a square matrix if $m = n$; For example is a square matrix of order 3 × 3.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

> For Additional Knowledge:
>
> The sum of the diagonal elements in a square matrix A is called the trace of matrix A, and which is sdenoted by tr(A);
>
> $\text{tr}(A) = a_{11} + a_{22} + ... + a_{nn}$

Zero or Null Matrix

If in a matrix all the elements are zero then it is called a zero matrix and it is generlly denoted by O. Thus, $A = [A_{ij}]_{mxn}$ is a zero-matrix if $a_{ij} = 0$ for all i and j; For example

$$A = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Here A and B are Null matrix of order 3 × 1 and 2 × 2 respectively.

Diagonal Matrix

If all the non-diagonal elements of a square matrix, are zero, then it is called a diagonal matrix.

Thus, a square matrix $A = [a_{ij}]$ is a diagonal matrix if aij = 0, when $i \neq j$;

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}, C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}, D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

A, B and C are diagonal matrix of order 3 × 3, and D is a diagonal matrix of order 2 × 2.

Diagonal matrix can also be denoted by A = diagonal [2 3 4], B = diag [2 0 4], C = [0 0 4]

> Important things to note:
>
> (i) A diagonal matrix is always a square matrix.
>
> (ii) The diagonal elements are characterized by this general form: a_{ij}, where $i = j$. This means that a matrix can have only one diagonal.

Scalar Matrix

If all the elements in the diagonal of a diagonal matrix are equal, it is called a scalar matrix.

Thus, a square matrix $A = [a_{ij}]$ is a scalar matrix if

$$A = [a_{ij}] = \begin{cases} 0; & i \neq j \\ k; & i = j \end{cases} \quad \text{Where, } k \text{ is constant.}$$

For example A and B are scalar matrix of order 3 × 3 and 2 × 2 respectively.

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}, B = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix}$$

Unit Matrix or Identity Matrix

If all the elements of a principal diagonal in a diagonal matrix are 1, then it is called a unit matrix.

A unit matrix of order n is denoted by I_n. Thus, a square matrix $A = [a_{ij}]_{m \times m}$ is an identity matrix if

$$A = [a_{ij}] = \begin{cases} 0; & i \neq j \\ 1; & i = j \end{cases}$$

For example I_3 and I_2 are identity matrix of order 3 × 3 and 2 × 2 respectively.

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- All identity matrices are scalar matrices
- All scalar matrices are diagonal matrices
- All diagonal matrices are square matrices

Triangular Matrix

A square matrix is said to be a triangular matrix if the elements above or below the principal diagonal are zero. There are two types of Triangular Matrix:

Upper Triangular Matrix

A square matrix $[a_{ij}]$ is called an upper triangular matrix, if $a_{ij} = 0$, when $i > j$.

$$A = \begin{pmatrix} D & O & E \\ 0 & D & O \\ 0 & 0 & E \end{pmatrix}$$ is an upper triangular matrix of order 3 × 3.

Lower Triangular Matrix

A square matrix is called a lower triangular matrix, if $a_{ij} = 0$, when $i > j$.

$$A = \begin{pmatrix} D & 0 & 0 \\ O & D & 0 \\ E & O & E \end{pmatrix}$$

is a lower triangular matrix of order 3 × 3.

Transpose of a Matrix

Let A be any matrix, then on interchanging rows and columns of A. The new matrix so obtained is transpose of A donated A^T or A'.

[order of $A = m \times n$, then order of $A^T = n \times m$]

Properties of transpose matrices A and B are:

(a) $(A^T)^T = A$

(b) $(kA)^T = kA^T$ (k = constant)

(c) $(A + B)^T = A^T + B^T$

(d) $(AB)^T = B^T . A^T$

Symmetric Matrix and Skew-Symmetric matrix

- A square matrix $A = [a_{ij}]$ is symmetric if $A^T = A$ i.e. a_{ij} $\forall$ i and j
- A square matrix $A = [a_{ij}]$ is skew-symmetric if $A^T = -A$ i.e. $a_{ij} = -a_{ij}$ $\forall$ i and j

  (All diagonal elements are zero in skew-symmetric matrix)

Illustration:

A is matrix of order 2022 × 2023 and B is a matrix such that AB^T and $B^T A$ are both defined, then find the order of matrix B.

Solution: Let the order of matrix be R × C, So,

$(A)_{2022\times2023} (B^T)_{C\times R}$ $\Rightarrow$ $C = 2023$ (As AB^T is defined)

$(B^T)_{C\times R} (A)_{2022\times2023}$ $\Rightarrow$ $R = 2022$ (As $B^T A$ is defined)

Thus order of matrix B is (2022 × 2023).

Illustration:

If A is a skew symmetric matrix, then show that A^2 is symmetric.

Solution: As A is skew-symmetric, $A^T = -A$

$(A^2)^T = (A.A)^T = A^T.A^T = (-A)(-A) = A^2$

As $(A^2)^T = A^2$

$\Rightarrow$ Thus, A^2 is symmetric.

Illustration:

If $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} + X = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$, where $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then find the value of $a + c - b - d$.

Solution: As, $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$,

$$\Rightarrow \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 3-1 & 4+1 \\ 5-2 & 6-3 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 3 & 3 \end{pmatrix}$$

On compairing the corresponding elements, we get,

$a = 2$, $b = 5$, $c = 3$, $d = 3$

Thus, $a + c - b - d = 5 - 5 - 3 = -3$

Illustration:

If A is a diagonal matrix of order 3 × 3 such that $A^2 = A$, then find number of possible matrices A.

Solution: As, A is a diagonal matrix of order 3 × 3

Let, $A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$

$$\Rightarrow \quad A^2 = \begin{pmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix}$$

As $A^2 = A \quad \Rightarrow \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} = \begin{pmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix}$

So, $a = 0$ or -1, similarly b and c can take 2 values (0 and –1).

Thus, total number of possible matrices are 2 × 2 × 2 = 8.

ONE MARK QUESTIONS

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. If $A=[a_{ij}]_{2\times2}=\begin{cases}0, when\, i=j\\1, when\, i\neq j\end{cases}$, then $A^2=$

(a) $\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$ (b) $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ (c) $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ (d) $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$

2. If $A=[a_{ij}]_{2\times2}=\begin{cases}0, when\, i=j\\1, when\, i\neq j\end{cases}$, then $A^{2025}=$

(a) $\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$ (b) $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ (c) $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ (d) $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$

3. If $A=\begin{pmatrix}1 & 0\\2 & 1\end{pmatrix}, B=\begin{pmatrix}x & 0\\1 & 1\end{pmatrix}$ and $A = B^2$, then x equals

(a) ±1 (b) 1 (c) –1 (d) 2

4. If $A=\begin{pmatrix}1 & x^2-2 & 3\\7 & 5 & 7\\3 & 7 & -5\end{pmatrix}$ be a symmetric matrix, then x equals

(a) ±3 (b) ±2 (c) $\pm\sqrt{2}$ (d) 0

5. If $A=\begin{pmatrix}0 & x^2+6 & 1\\-5x & x^2-9 & 7\\-1 & -7 & 0\end{pmatrix}$ be a skew -symmetric matrix, then x equals

(a) ±3 (b) 3 (c) –3 (d) 0

6. If $A=\begin{pmatrix}2y-7 & 0 & 0\\0 & x-3 & 0\\0 & 0 & 7\end{pmatrix}$ be a scalar matrix, then (x+y) equals

(a) 7 (b) 14 (c) 16 (d) 17

7. If A is matrix of order 2023 × 2024 and B is a matrix such that AB' and B'A both are defined, then the order of matrix B is

(a) 2023 × 2024 (b) 2023 × 2023 (c) 2024 × 2024 (d) 2024 × 2023

8. If A is matrix of order 2023 × 2024 and B is a matrix such that AB and BA both are defined, then the order of matrix B is

(a) 2023 × 2024 (b) 2023 × 2023 (c) 2024 × 2024 (d) 2024 × 2023

9. *If* $A = \begin{pmatrix} 2 & 0 & y-x \\ x+y-2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ be a diagonal matrix then (xy) equals

(a 1 (b) 2 (c) 3 (d) 4

10. If all entries of a square matrix of order 2 are either 3, –3 *or* 0, then how many Non-zero matrices are possible?

(a) 80 (b) 81 (c) 27 (d) 64

11. If all entries of a square matrix of order 3 are either 1 *or* 0, then how many Diagonal matrices are possible?

(a) 512 (b) 8 (b) 6 (d) 2

12. If all entries of a square matrix of order 3 are either 3 *or* 0, then how many Scalar matrices are possible?

(a) 1 (b) 8 (c) 6 (d) 2

13. If all entries of a square matrix of order 3 are either 5 *or* 0, then how many Identity matrices are possible?

(a) 1 (b) 8 (c) 2 (d) 0

14. If there are five one' s i.e. 1, 1, 1, 1, 1 & four zeroes i.e. 0, 0, 0, 0, then total number of symmetric matrices of order 3 ×3 possible?

(a) 10 (b) 12 (c) 3 (d) 9

15. If $x\begin{pmatrix} 1 \\ 2 \end{pmatrix} + y\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$, then

(a) x= 1, y = 2 (b) x= 2, y=1 (c) x= 1, y=–1 (d) x= 3, y=2

16. The product $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, is equal to

(a) $\begin{pmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{pmatrix}$ (b) $\begin{pmatrix} a^2+b^2 & 0 \\ a^2+b^2 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} a^2+b^2 & 0 \\ 0 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

17. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(a) I (b) A (c) $3A - I$ (d) $A - I$

18. If A and B are two non-zero matrices such that AB = A, BA = B and $(A + B)^3 = k\,(A + B)$, then k is equal to

(a) 1 (b) 2 (c) 4 (d) 8

19. If A is a square matrix such that $A^2 = A$, then $(A + I)^2 - 3A$ is equal to

(a) I (b) A (c) $2A$ (d) $3I$

20. If a matrix A = (1 2 3), then the matrix A.A' (where A' is the transpose of A) is

(a) $(1\ 2\ 3)_{1\times3}$ (b) $(14)_{1\times1}$ (c) $(6)_{1\times1}$ (d) $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}_{3\times1}$

21. If $A = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}$ and $2A + B$ is a null matrix, then B is equal to]

(a) $\begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -3 & -4 \\ -5 & -2 \end{pmatrix}$ (c) $\begin{pmatrix} -6 & -8 \\ -10 & -2 \end{pmatrix}$ (d) $\begin{pmatrix} -6 & -8 \\ -10 & -4 \end{pmatrix}$

22. If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then value of x is/are

(a) ±3 (b) $\pm\sqrt{7}$ (c) ±5 (d) 0

23. If $A = \begin{pmatrix} 2 & 8 \\ 6 & 4 \end{pmatrix} = p + Q,$ where P is a symmetric and Q is a skew-symmetric matrix, then Q is equal to

(a) $\begin{pmatrix} 2 & 6 \\ 8 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

ASSERTION-REASON BASED QUESTIONS (Q.24 & Q.25)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

24. *ASSERTION:* Matrix A $= \begin{pmatrix} 0 & 1 & -2 \\ -1 & 1 & 3 \\ 2 & -3 & 0 \end{pmatrix}$ is a skew-symmetric matrix.

REASONING: A matrix A is skew-symmetric if $A' = -A$.

25. *ASSERTION :* For matrices $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ *and* $B = \begin{pmatrix} 4 & 2 \\ 9 & 1 \end{pmatrix}$,

$$(A + B)(A - B) = A^2 - AB + BA - B^2$$

REASONING : Matrix multiplication is not commutative.

TWO MARKS QUESTIONS

26. If A is a square matrix, then show that

(a) $(A + A^T)$ is symmetric matrix.

(b) $(A - A^T)$ is symmetric matrix.

(c) (AA^T) is symmetric matrix.

27. Show that every square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix.

28. If A and B are two symmetric matrices of same order, then show that

(i) $(AB - BA)$ is skew-symmetric Matrix.

(ii) $(AB + BA)$ is symmetric Matrix.

29. (a) If $A = \begin{pmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}, C = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. Verify that $(A + B)C = AC + BC$.

(b) If $A + B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $A - 2B = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$ then show that $A = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

30. If $A = \begin{pmatrix} i & 0 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, show that $AB \neq BA$

31. Find a matrix X, for which $\begin{pmatrix} 5 & 4 \\ 1 & 1 \end{pmatrix} \times = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$

32. If A and B are symmetric matrices, show that AB is symmetric, if $AB = BA$.

33. Match the following:

Possible Number of Matrices (A_n) of order 3 × 3 with entry 0 or 1 which are

	Condition		**No. of matrices**
(1)	A_n is diagonal Matrix	P	2^0
(2)	A_n is upper triangular Matrix	Q	2^1
(3)	A_n is identity Matrix	R	2^3
(4)	A_n is scalar Matrix	S	2^6

34. If $A = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$ then prove that $A^3 = \begin{pmatrix} \cos 3x & -\sin 3x \\ \sin 3x & \cos 3x \end{pmatrix}$.

35. Express the following Matrices as a sum of a symmetric and skey-symmetric matrix. (Note: Part (b) and (c) can be asked for one marker, SO THINK ABOUT THIS!)

(a) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{pmatrix}$ (b) $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 5 & 7 \\ 5 & 7 & -5 \end{pmatrix}$ (c) $A = \begin{pmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{pmatrix}$

36. Show that the Matrix $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ satisfies the equation $A^2 - 4A + \mathbf{I} = 0$.

37. Find the values of x and y, if $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ satisfies the equation $A^2 + xA + yI = 0$.

38. Find $f(A)$, if $A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$ such that $f(x) = x^2 - 3x + 5$

39. Find A^2 if $A = \begin{pmatrix} 5 & 4 \\ 1 & 1 \end{pmatrix}$.

40. Find $2A^2$ when $x = \frac{\pi}{3}$ where $A = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$.

THREE MARKS QUESTIONS

41. Let $P = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{pmatrix}$ and $Q = [q_{ij}]$ be two 3 × 3 matrices such that $Q = P^5 + I_3$, then Prove that $\left(\dfrac{q_{21}+q_{31}}{q_{32}}\right) = 10$.

42. Construct a 3 × 3 matrix $A = [a_{ij}]$ such that

(a) $a_{ij} = \begin{cases} i+j; & i>j \\ \dfrac{i}{j}; & i=j \\ i-j; & i<j \end{cases}$ (b) $a_{ij} = \begin{cases} 2^i; & i>j \\ i.j; & i=j \\ 3^j; & i<j \end{cases}$

(c) $a_{ij} = \begin{cases} i^2+j^2; & i \neq j \\ 0; & i=j \end{cases}$ (d) $a_{ij} = \dfrac{|2i-3j|}{5}$

(e) $a_{ij} = \left[\dfrac{i}{j}\right]$, where [.] represents Greatest Integer Function.

43. If $A = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$, then prove that $A^2 = \begin{pmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{pmatrix}$, where $i = \sqrt{-1}$

44. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, evaluate $A^3 - 4A^2 + A$.

45. If $f(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then prove that $f(x).f(y) = f(x + y)$

46. If $f(x) = \dfrac{1}{\sqrt{1-x^2}}\begin{pmatrix} 1 & -x \\ -x & 1 \end{pmatrix}$, Prove that $f(x).f(y) = f\left(\dfrac{x+y}{1+xy}\right)$. Hence show that $f(x).f(-x) = 1$, where $|x| < 1$.

FIVE MARKS QUESTIONS

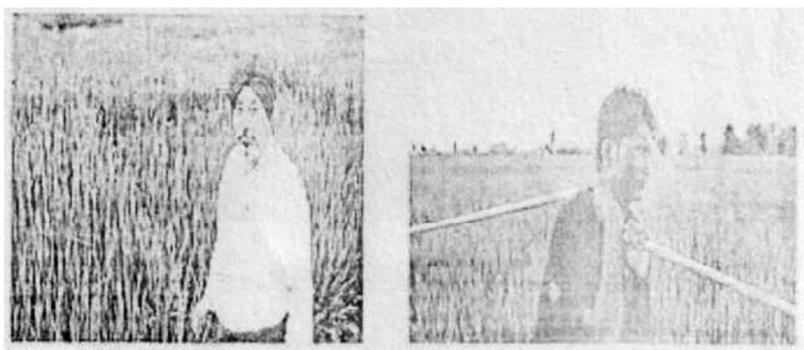
47. Find x, y and z if $A^T = A^{-1}$ and $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$. Also find how many triplets of (x, y, z) are possible. (NOTE: $A.A^{-1} = A^{-1}A = I$)

48. If A is a symmetric Matrix and B is skew-symmetric Matrix such that $A + B = \begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix}$ then show that $AB = \begin{pmatrix} 4 & -2 \\ -1 & -4 \end{pmatrix}$.

49. If $A = \begin{pmatrix} 4 & 1 \\ -9 & -2 \end{pmatrix}$ and $A^{50} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then show that $(a + b + c + d + 398) = 0$.

CASE STUDIES

50. (A) Two farmers Ramkishan and Gurcharan Singh cultivates only three varieties of rice namely Basmati, Permal and Naura. The Quantity of sale (in Kg) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.



$$A(\text{September sales}) = \begin{matrix} & \text{BASMATI} & \text{PERMAL} & \text{NAURA} \\ \end{matrix}$$
$$\begin{pmatrix} 1000 & 2000 & 3000 \\ 5000 & 3000 & 1000 \end{pmatrix} \begin{matrix} \text{RAMAKRISHAN} \\ \text{GURCHARAN SINGH} \end{matrix}$$

$$B(\text{October sales}) = \begin{matrix} & \text{BASMATI} & \text{PERMAL} & \text{NAURA} \\ \end{matrix}$$
$$\begin{pmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{pmatrix} \begin{matrix} \text{RAMAKRISHAN} \\ \text{GURCHARAN SINGH} \end{matrix}$$

If Ramakrishan sell the variety of rice (per kg)i.e. Basmati, Permal and Naura at Rs.30, Rs. 20 & Rs.10 respectively, While Gurcharan Singh sell the variety of rice (per kg) i.e. Basmati, Permal and Naura at Rs. 40, Rs. 30, & Rs.20 respectively.

Based on the above information answer the following:

(a) Find the Total selling Price received by Ramakrishan in the month of september.

(b) Find the Total Selling Price received by Gurcharan Singh in the month of september.

(c) Find the Total selling Price received by Ramakrishan in the month of september & october.

(B) A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below

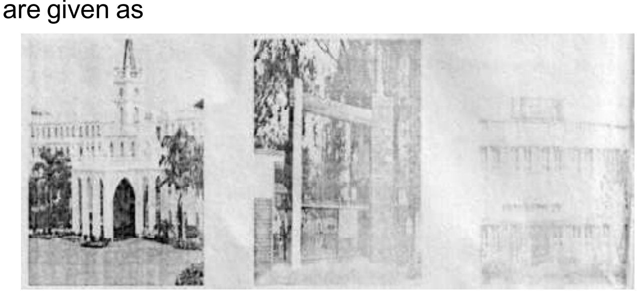
Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
A	10,000	2000	18,000
B	600	20,000	8,00

If the unit Sale price of Pencil, Eraser and Sharpener are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, and unit cost of the above three commodities are Rs. 2.00, Rs. 1.00 and Rs. 0.50 respectively, then, Based on the above information answer the following:

(a) Find the total Revenue of both the markets.

(b) Find the total Profit for both the markets.

(C) Three schools ABC, PQR and MNO decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs. 100 and Rs. 50 each respectively. The numbers of articles sold are given as



School/Article	ABC	PQR	MNO
Hand made fans	40	25	35
Marks	50	40	50
Plates	20	30	40

Based on the information given above, answer the following questions.

(a) What is the total amount of money (in Rs.) collected by all the three schools ABC, PQR & MNO?

(b) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. If A is a symmetric matrix then which of the following is not Symmetric matrix,

   (a) $A + A^T$ (b) $A.A^T$

   (c) $A - A^T$ (d) A^T

2. Suppose P, Q and R are different matrices of order 3×5, $a \times b$ and $c \times d$ respectively, then value of $ac + bd$ is, if matrix $P + Q - R$ is defined

   (a) 9 (b) 14

   (c) 24 (d) 34

3. If A and B are two square matrices of same order such that, $AB = A$ and $BA = B$, then $(A + B)(A - B) =$

   (a) O (b) A

   (c) $A^2 - B^2$ (d) B

4. If $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$, then $2x + y - z =$

   (a) 1 (b) 3

   (c) 5 (d) 7

5. If a matrix has 2022 elements, how many orders it can have?

   (a) 6 (b) 2

   (c) 4 (d) 8

SELF ASSESSMENT-2

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. If matrix $A = [a_{ij}]_{2\times 2}$ where

   $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$, then $A^{2021} =$

   (a) O (b) A

   (c) $-A$ (d) I

2. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then $A^4 =$

   (a) A (b) $3A$

   (c) $9A$ (d) $27A$

3. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $A^2 + pA + qI = 0$, then pq =

(a) 0 (b) 1

(c) –1 (d) 2

4. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then $a + b + c + d$ =

(a) 0 (b) 4

(c) 6 (d) 10

5. If A is a square Matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(a) $2A + I$ (b) $A + 2I$

(c) I (d) $A + I$

ANSWER

One Mark Questions

1. (b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
2. (c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
3. (b) + 1
4. (a) ±3
5. (b) 3
6. (d) 17
7. (a) 2023 × 2024
8. (d) 2024 × 2023
9. (a) 1
10. (a) 80
11. (b) 8
12. (d) 2
13. (d) 0
14. (b) 12
15. (b) x =2, y = 1
16. (a) $\begin{pmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{pmatrix}$
17. (b) A
18. (c) 4
19. (a) I
20. (b) (14)
21. (d) $\begin{pmatrix} -6 & -8 \\ -10 & -4 \end{pmatrix}$
22. (c) ±5
23. (b) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
24. (d) A si false but R is true.
25. (a) Both A and R are true and R is the correct explanation of A

Two Marks Questions

31. $X = \begin{pmatrix} -3 & -14 \\ 4 & 17 \end{pmatrix}$

33. $(1) \to R \quad (2) \to S \quad (3) \to P \quad (4) \to Q$

35. (a) $\begin{pmatrix} 1 & 2 & \frac{1}{2} \\ 2 & 5 & \frac{3}{2} \\ \frac{1}{2} & \frac{3}{2} & -5 \end{pmatrix} + \begin{pmatrix} 0 & 0 & \frac{5}{2} \\ 0 & 0 & \frac{11}{2} \\ \frac{-5}{2} & \frac{-11}{2} & 0 \end{pmatrix}$

35. (b) $\begin{pmatrix} 1 & 2 & 5 \\ 2 & 5 & 7 \\ 5 & 7 & -5 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

35. (c) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{pmatrix}$

37. $x = -2$, $y = 0$

38. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

39. $\begin{pmatrix} 29 & 24 \\ 6 & 5 \end{pmatrix}$

40. $\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$

Three Marks Questions

42. (a) $\begin{pmatrix} 1 & -1 & -2 \\ 3 & 1 & -1 \\ 4 & 5 & 1 \end{pmatrix}$

42. (b) $\begin{pmatrix} 1 & 9 & 27 \\ 4 & 4 & 27 \\ 8 & 8 & 9 \end{pmatrix}$

42. (c) $\begin{pmatrix} 0 & 5 & 10 \\ 5 & 0 & 13 \\ 10 & 13 & 0 \end{pmatrix}$

40. (d) $\begin{pmatrix} \frac{1}{5} & \frac{4}{5} & \frac{7}{5} \\ \frac{1}{5} & \frac{2}{5} & 1 \\ \frac{3}{5} & 0 & \frac{3}{5} \end{pmatrix}$

42. (e) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$

44. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

47. $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}};$

CASE STUDIES QUESTION

50. Case Study: A

(a) Rs. 1,00,000 (b) Rs. 3,10,000 (c) Rs. 5,10,000

50. Case Study: B

(b) Rs. 46,000 (For Market A)
Rs. 43,000 (Ror Market B)

(b) Rs. 15,000 (For Market A)
Rs. 17,000 (For Market A)

50. Case Study C:

(a) Rs. 21,000
Rs. 21,250

50. (iv) Option (d)

50. (v) Option (c)

SELF ASSESSMENT-1

1. (c)	2. (d)	3. (a)	4. (c)	5. (d)

SELF ASSESSMENT-2

1. (b)	2. (d)	3. (a)	4. (d)	5. (c)