CHAPTER-2

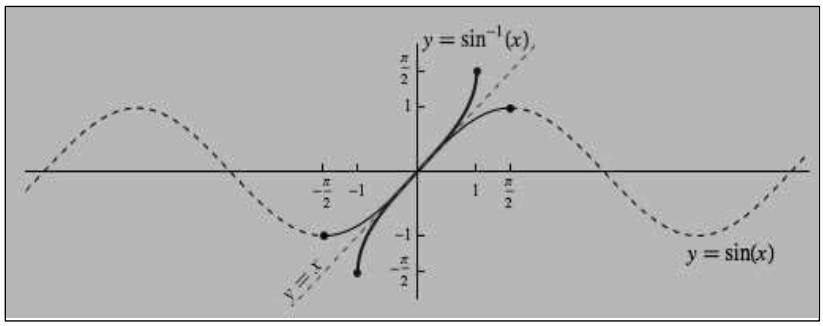
INVERSE TRIGONOMETRIC FUNCTIONS

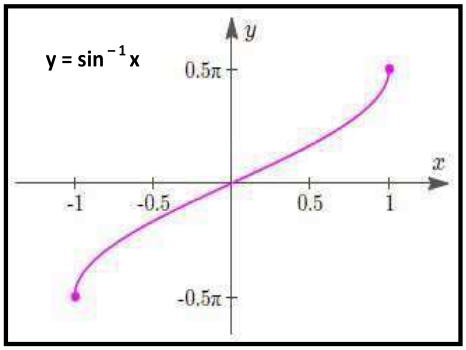
An example of people using inverse trigonometric functions would be builders such as construction workers, architects, and many others.

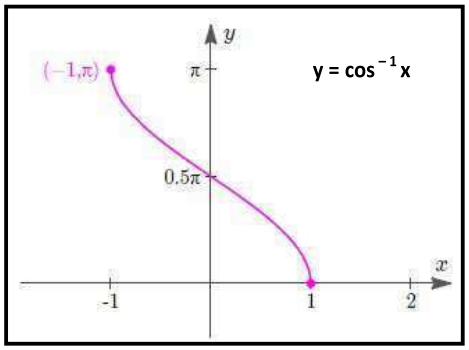
An example of the use would be the creation of bike ramp. You will have to find the height and the length. Then find the angle by using the inverse of sine. Put the ength over the height to find the angle. Architects would have to calculate the angle of a bridge and the supports when drawing outlines. These calculations are then applied to find the safest angle. The workers would then uses these calculations to build the bridge.

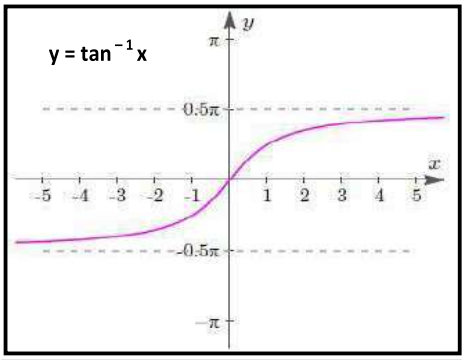
TOPIC TO BE COVERED AS PER CBSE LATEST CURRICULUM (2024-25)

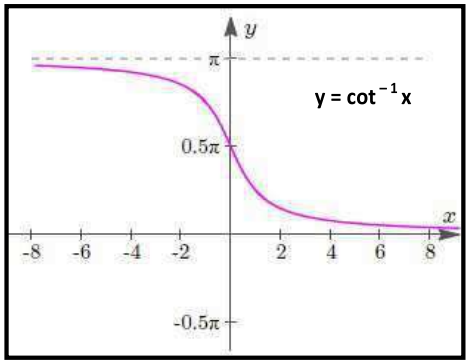
- Definition, range, domain, principal value branch.
- Graphs of inverse trigonometric functions.

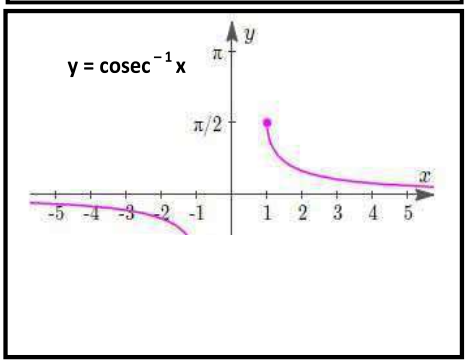

$y = \sin^{-1}(x)$
$y = \sin(x)$
$y = x$

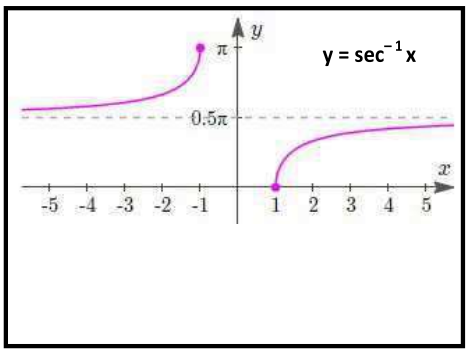

y = sin⁻¹x
y
x
0.5π
-0.5π
-1
-0.5
0.5
1


y = cos⁻¹x
y
x
(−1,π)
π
0.5π
-1
1
2


y = tan⁻¹x
y
x
π
0.5π
-0.5π
-π
-5 -4 -3 -2 -1 1 2 3 4 5


y = cot⁻¹x
y
x
π
0.5π
-0.5π
-8 -6 -4 -2 2 4 6 8


y = cosec⁻¹x
y
x
π
π/2
-5 -4 -3 -2 -1 1 2 3 4 5


y = sec⁻¹x
y
x
π
0.5π
-5 -4 -3 -2 -1 1 2 3 4 5

Function	Domain	Range
$y = \sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \operatorname{cosec}^{-1} x$	$R - (-1, 1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

- when $x \in [-1, 1]$

  $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.
- when $x \in R$

  $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$.
- when $x \in R - (-1, 1)$

  $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$.

- $\sin^{-1} (\sin x) = x$, when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- $\cos^{-1} (\cos x) = x$, when $x \in [0, \pi]$
- $\tan^{-1} (\tan x) = x$, when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\cot^{-1} (\cot x) = x$, when $x \in (0, \pi)$
- $\operatorname{cosec}^{-1} (\operatorname{cosec} x) = x$, when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
- $\sec^{-1} (\sec x) = x$, when $x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

- $\sin (\sin^{-1} x) = x$, when $x \in [-1, 1]$
- $\cos (\cos^{-1} x) = x$, when $x \in [-1, 1]$
- $\tan (\tan^{-1} x) = x$, when $x \in R$
- $\cot (\cot^{-1} x) = x$, when $x \in R$
- $\operatorname{cosec} (\operatorname{cosec}^{-1} x) = x$, when $x \in R - (-1, 1)$
- $\sec (\sec^{-1} x) = x$, when $x \in R - (-1, 1)$

- $\sin^{-1}(-x) = -\sin^{-1} x$, when $x \in [-1, 1]$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x$, when $x \in [-1, 1]$
- $\tan^{-1}(-x) = -\tan^{-1} x$, when $x \in R$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x$, when $x \in R$
- $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$, when $x \in R\ (-1, 1)$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x$, when $x \in R\ (-1, 1)$

Illustration:

Find the principal value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$.

Solution: As, $\sin^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}, \boxed{\frac{\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]}$

$\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3}, \boxed{\frac{\pi}{3} \in [0, \pi]}$

so, $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) = \frac{\pi}{6} + \pi - \frac{\pi}{3} = \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6}$

Illustration:

Find the principal value of $\sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(-\sqrt{3})$.

Solution: As, $\sec^{-1}(2) = \cos^{-1}\left(\frac{1}{2}\right)$

$\tan^{-1}(-\sqrt{3}) = -\tan^{-1}(\sqrt{3}) = -\tan^{-1}(\tan\frac{\pi}{3}) = -\frac{\pi}{3}, \boxed{-\frac{\pi}{3} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)}$

$\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(-\sqrt{3}) = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Illustration:

Find the range of the function $f(x) = \tan^{-1} x + \cot^{-1} x$.

Solution: As, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

so, $f(x) = \frac{\pi}{2}$ (A constant function)

Thus range of $f(x)$ is $\left\{\frac{\pi}{2}\right\}$.

Illustration:

If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then find the value of $\cos^{-1} x + \cos^{-1} y$.

Solution: As, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \Rightarrow \boxed{\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x}$

$\cos^{-1} x + \cos^{-1} y = \pi - (\sin^{-1} x + \sin^{-1} y) = \pi - \frac{2\pi}{3} = \boxed{\frac{\pi}{3}}$

Illustration:

If $a \le 2 \sin^{-1} x + \cos^{-1} x \le b$, then find the value a and b.

Solution: We know that, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and $\frac{-\pi}{2} \le \sin^{-1} x \frac{\pi}{2}$,

$\Rightarrow \quad 0 \le (\sin^{-1} x) + \frac{\pi}{2} \le \pi$

$\Rightarrow \quad 0 \le (\sin^{-1} x) + \sin^{-1} x + \cos^{-1} x \le \pi$

$\Rightarrow \quad 0 \le 2 \sin^{-1} x + \cos^{-1} x \le \pi$, but given, $a \le \sin^{-1}, x + \cos^{-1} x \le b$

Thus, $a = 0$ and $b = \pi$

Illustration:

If $\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1} x]$, then find x.

Solution: As, $\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1} x]$

$$\Rightarrow \quad \sin\left[\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right] = \cos\left[\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$$

$$\Rightarrow \quad x^2 + 2x + 2 = 1 + x^2$$

$$\Rightarrow \quad 2x = -1 \Rightarrow \boxed{x = -0.5}$$

Illustration:

If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then prove that $xy + yz + zx = 1$.

Solution: Let, $\tan^{-1} x = A$, $\tan^{-1} y = B$, $\tan^{-1} z = C$

so, $A + B + C = \frac{\pi}{2} \Rightarrow A + B = \frac{\pi}{2} - C$

$$\tan(A+B) = \tan\left(\frac{\pi}{2} - C\right) = \cot C$$

$$\frac{\tan A + \tan B}{1 - \tan A.\tan B} = \frac{1}{\tan C} \Rightarrow \frac{x+y}{1-xy} = \frac{1}{z}$$

$$\Rightarrow \quad xz + yz = 1 - xy$$

$$\Rightarrow \quad \boxed{xz + yz + zx = 1}$$

ONE MARK QUESTIONS

1. Principal Value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$ *is*

   (a) π (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$

2. Principal Value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ is

(a) $\frac{3\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{\pi}{2}$ (d) $\frac{-3\pi}{5}$

3. Principal value of $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$ is

(a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) π (d) $\frac{14\pi}{3}$

4. If the Principal value of $\tan\tan^{-1}(\tan\frac{7\pi}{6})$ is $\frac{a\pi}{b}$, Where *a* & *b* are co-prime numbers, then $(a + b)$=

(a) 13 (b) –13 (c) 7 (d) 5

5. If the Principal value of $\cos^{-1}(\cos\frac{2\pi}{3}) + \sin^{-1}(\sin\frac{2\pi}{3})$ is $\frac{a\pi}{b}$, then |a – b|=

(a) 0 (b) 1 (c) 2 (d) 4

6. If $\cos(\cos^{-1}\frac{1}{3} + \sin^{-1}x) = 0$, then (3x +1)=

(a) 0 (b) 1 (c) 2 (d) 4

7. If $\sin(\sin^{-1}\frac{3}{5} + \cos^{-1}x) = 1$, then (5x -2)=

(a) 0 (b) 1 (c) 2 (d) 4

8. Domain of the function $\cos^{-1}(2x - 1)$ is

(a) *R* (b) [–1,1] (c) [0,1] (d) [0,2]

9. Domain of the function $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) [1,2] (b) [–1,1] (c) [0,1] (d) [0,2]

10. Principal value of $\sec^{-1}(2) + \sin^{-1}(\frac{1}{2}) + \tan^{-1}(-\sqrt{3})$ is

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

11. Domain of the function $f(x) = \cos^{-1}\sqrt{x+1}$ is

(a) [1,2] (b) [–1,0] (c) [0,1] (d) [0,2]

12. Domain of the function f (x) = $\sin^{-1}(-x^2)$ is

(a) [1,2] (b) [–1,0] (c) [0,1] (d) [–1,1]

13. Domain of the function $f(x) = \sin^{-1}(2x+3)$ is

(a) [–2,2] (b) [–2,–1] (c) [0,1] (d) [–1,1]

14. If Domain of the function $f(x) = \sin^{-1}(x^2-4)$ is $[-b,-a] \cup [a,b]$ then the value of (a^2+b^2) is.

(a) 8 (b) 3 (c) 5 (d) 4

15. If $\sin^{-1} x_1 + \sin^{-1} x_2 = \pi$, then the value of $(x_1 + x_2)$ is

(a) 0 (b) 1 (c) –1 (d) 2

16. If $\cos^{-1} a + \cos^{-1} b = 2\pi$, then the value of $(a-b)^2$ is

(a) 0 (b) 1 (c) –1 (d) 4

17. $\cos^{-1}[\sin(\cos^{-1}\frac{1}{2})]=$

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

18. Principal value of $\sin^{-1}(\cos\frac{34\pi}{5})$ is

(a) $\frac{\pi}{5}$ (b) $\frac{-\pi}{10}$ (c) $\frac{3\pi}{10}$ (d) $\frac{-3\pi}{10}$

19. If $\cot(\cos^{-1}\frac{7}{25}) = x$, then $\sqrt{24x+2} =$

(a) 1 (b) 2 (c) 3 (d) 4

20. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$ & $\cot^{-1} x + \cot^{-1} y = \frac{k\pi}{5}$, then k =

(a) 1 (b) 2 (c) 3 (d) 4

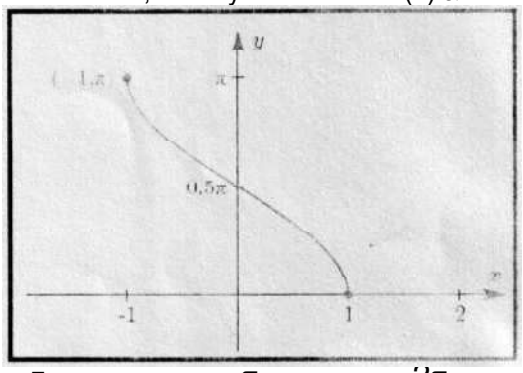
21. $\sum_{i=1}^{2023} \cos^{-1} x_1 = 0$, then the velue of $\sum_{i=1}^{2023} x_i$ is

(a) 0 (b) 1 (c) 2023 (d) –2023

22. If $\sum_{i=1}^{2024} \sin^{-1} x_i = 1012\,\pi$, then the value of $\sum_{i=1}^{2024} X_i$ is

(a) 1012 (b) 2024 (c) –1012 (d) –2024

23. If graph of $f(x)$ is shown below, identify the function $f(x)$ & find the value of $f\left(-\frac{1}{2}\right)$.



(a) $\frac{-\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{-\pi}{3}$ (d) $\frac{2\pi}{3}$

ASSERTION-REASON BASED QUESTIONS (Q.24 & Q.25)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

24. ASSERTION (A): The range of the function $f(x) = \sin^{-1}x + \frac{3\pi}{2}$, where

$$x \in [-1,1], \text{ is } [\frac{\pi}{2}, \frac{5\pi}{2}].$$

REASON (R): The range of the principal value branch of $\sin^{-1}x$ is $[0, \pi]$.

25. ASSERTION (A): All trigonometric function have their inverses over their respective domains.

REASON (R): The inverse of $\tan^{-1}x$ exists for some $x \in R$.

TWO MARKS QUESTIONS

26. Match the following:

If $\cos^{-1}a + \cos^{-1}b = 2\pi$ and $\sin^{-1}c + \sin^{-1}d = \pi$ then

	Column 1		Column 2
A	abcd	P	0
B	$a^2 + b^2 + c^2 + d^2$	Q	1
C	$(d - a) + (c - d)$	R	2
D	$a^3 + b^3 + c^3 + d^3$	S	4

27. Find the value of $\cos\left[\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right)\right]$

28. If $P = \tan^2(\sec^{-1}2) + \cot^2(\operatorname{cosec}^{-1}3)$, then find the value of $(P^2 + P + 11)$.

29. If $P = \sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3)$, then find the value of $(P^2 - 2P)$.

30. Find the value of $\sin\left(\frac{1}{2}\cot^{-1}\left(\frac{3}{4}\right)\right)$. $\left[\text{Hint : } \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}\right]$

31. Solve for x : $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$

32. Find the value of x, such that $\sin^{-1}x = \frac{\pi}{6} + \cos^{-1}x$.

33. Find x, if $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{2}$

34. If $\tan^{-1}(\cot x) = 2x$, find x.

35. Solve for x : $\cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \sin^{-1}\left(\sin\frac{3\pi}{4}\right) = x$

THREE MARKS QUESTIONS

36. Find the value of k, if $100 \sin(2 \tan^{-1}(0.75)) = k$ [**Hint:** $\sin 2\theta = 2\sin\theta\cos\theta$]

37. Prove that:

(a) $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2},\ x \in \left(0,\ \frac{\pi}{4}\right)$

(b) $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{x}{4} - \frac{1}{2}\cos^{-1} x$

(c) $\tan^{-1}\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

(d) $\sin^{-1}\left(2\tan^{-1}\left(\frac{2}{3}\right)\right) = \frac{12}{13}$

38. (a) Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

(b) Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

(c) Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{a}{b}\right) = \frac{2\sqrt{a^2+b^2}}{b}$.

(d) Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$

(e) Prove that : $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \frac{\pi}{4} + \frac{x}{2},\ x \in \left(0, \frac{\pi}{2}\right)$

(f) Prove that : $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

39. Solve for x :

(a) $\sin^{-1}(6x)+\sin^{-1}(6\sqrt{3}x)=\frac{\pi}{2}$

(b) Solve for x : $\sin^{-1}(6x)+\sin^{-1}(6\sqrt{3}x)=\frac{-\pi}{2}$

(c) $(\tan^{-1} x)^2+(\cot^{-1} x)^2=\frac{5\pi^2}{8}$.

40. Solve for x : $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right), x > 0$

FIVE MARKS QUESTIONS

Illustration: (For Solving Q.41)

If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, then prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

Solution: Let, $\cos^{-1} x = A$, $\cos^{-1} y = B$, $\cos^{-1} z = C$

so, $A + B + C = \pi \Rightarrow A + B = \pi - C$

Thus, $\cos(A + B) = \cos(\pi - C)$

$\Rightarrow \cos A \cos B - \sin A \sin B = -\cos C$

$\Rightarrow \cos A \cos B - \sqrt{1-\cos^2 A}\sqrt{1-\cos^2 B} = -\cos C$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$\Rightarrow (xy + z) = \sqrt{1-x^2}\sqrt{1-y^2}$

On squaring both the sides, we get

$(xy + z)^2 = (1 - x^2)(1 - y^2)$

$\Rightarrow$ ~~x^2y^2~~ $+ z^2 + 2xyz = 1 - x^2 - y^2 +$ ~~x^2y^2~~

$\therefore$ $\boxed{x^2 + y^2 + z^2 + 2xyz = 1}$

41. Prove the following:

(a) If $\cos^{-1}\left(\frac{x}{a}\right)+\cos^{-1}\left(\frac{y}{b}\right)=\alpha$, then prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\alpha=\sin^2\alpha$

(b) If $\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\theta$, then prove that $9x^2 + 4y^2 - 12xy\cos\theta = 36\sin^2\theta$.

42. Prove the following:

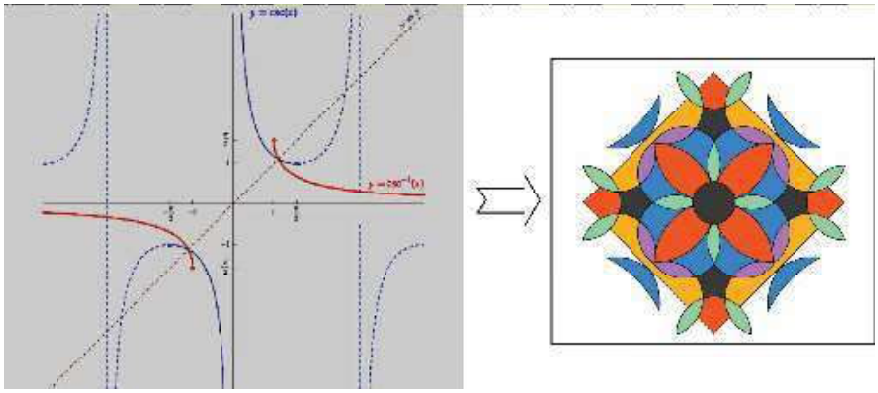
(a) If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, then prove that $x + y + z = xyz$

(a) If $\cot^{-1}x+\cot^{-1}y+\cot^{-1}z=\pi$, then prove that $xy + yz + zx = 1$

CASE STUDIES

43. On National Mathematics Day, December 22, 2020, Mathematics Teachers of DOE organized Mathematical Rangoli Competition for the students of all DOE schools to celebrate and remembering the contribution of Srinivasa Ramanujan to the field of mathematics. The legendary Indian mathematician who was born on this date in 1887.

Team A of class XI students made a beautiful Rangoli on Trigonometric Identities as shown in the figure Above, While Team B of class XII students make the Rangoli on the graph of Trigonometric and Inverse Trigonometric Functions. As shown in the following figure.



On the basis of above information, Teacher asked few questions from Team B. Now you try to answer. Those questions which are as follows:

(a) Write the domain & range (principal value branch) of the function f (x) = $\tan^{-1}$ x?

(b) If the principal branch of $\sec^{-1}x$ is $[0,\pi]-\{k\pi\}$, then find the value of k.

(c) Draw the graph of $\sin^{-1}$ x, where $x\in[-1,1]$. Also write its Principal branch Range.

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. If $\cos\left(\cos^{-1}\frac{2}{3}+\sin^{-1}x\right)=0$, then (3x – 1)

(a) 0 (b) 1

(c) –1 (d) 2

2. Domain of the function $\cos^{-1}\left(\frac{x}{2}-1\right)$ is

(a) [0, 2] (b) [–1, 1]

(c) [0, 1] (d) [0, 4]

3. If $\cos^{-1}a + \cos^{-1}b = 2\pi$ and $\sin^{-1}c + \sin^{-1}d = \pi$, then $a^2 + b^2 + c^2 + d^2$ =

(a) 0 (b) 1

(c) 2 (d) 4

4. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

(a) 0 (b) π

(c) 2π (d) $\frac{4\pi}{3}$

5. If $\cos^{-1}\left(\frac{1}{x}\right) = \theta$, then $\tan\theta$ =

(a) x (b) $x^2 + 1$

(c) $\sqrt{x^2+1}$ (d) $\sqrt{x^2-1}$

SELF ASSESSMENT-2

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE.

1. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then $(x^3 + y^3 + z^3 - 3xyz)$ =

(a) 0 (b) 1

(c) –1 (d) 2

2. Principal Range of the function $\sin^{-1}x$ is

(a) $[0, \pi]$ (b) $(0, \pi)$

(c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

3. If $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)= x$, then x =

(a) 0 (b) π

(c) $\frac{5\pi}{3}$ (d) $\frac{10\pi}{3}$

4. If $\sin^{-1}\left(\frac{x}{5}\right)+\text{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then x =

(a) 0 (b) 1

(c) 2 (d) 3

5. Range of $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ is

(a) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$ (B) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

(C) $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$ (D) $\left[\frac{\pi}{4},\frac{\pi}{2}\right)$

ANSWER

One Mark Questions

1. (*d*) $\frac{5\pi}{6}$ 2. (*b*) $\frac{2\pi}{5}$ 3. (*b*) $\frac{2\pi}{3}$

4. (*c*) 7 5. (*a*) 0 6. (c) 2

7. (*b*) 1 8. (*c*) [0, 1] 9. (a) [1,2]

10. (*d*) $\frac{\pi}{6}$ 11. (*b*) [–1,0] 12. (d) [-1,1]

13. (*b*) [–2,–1] 14. (*a*) 8 15. (*d*) 2

16. (*a*) 0 17. (*a*) $\frac{\pi}{6}$ 18. (*d*) $\frac{-3\pi}{10}$

19. (*c*) 3 20. (*a*) 1 21. (c) 2023

22. (*b*) 2024 23. (*d*) $\frac{2\pi}{3}$ 24. (c) A is true but R is false.

25. (d) A is false but R is true.

Two Marks Questions

26. $A \to Q$, $B \to S$, $C \to R$, $D \to P$ 27. 1

28. $(P^2 + P + 11) = 143$ 29. $(P^2 – 2P) = 195$ 30. $\frac{1}{\sqrt{5}}$

31. $x = 0$ or $–1$ 32. $\frac{\sqrt{3}}{2}$ 33. 1

34. $\frac{\pi}{6}$ 35. π

Three Marks Questions

36. 96 39. (a) $x = \frac{1}{12}$ (b) $x = \frac{-1}{12}$ (c) $x = -1$

40. $x = \frac{3}{4}$

CASE STUDIES BASED QUESTION

43. (a) Domain $= R = (-\infty, \infty)$, Range $= (\frac{-\pi}{2}, \frac{\pi}{2})$ (b) k = 0.5

SELF ASSESSMENT-1

1. (b) 2. (d) 3. (d) 4. (b) 5. (d)

SELF ASSESSMENT-2

1. (a) 2. (c) 3. (a) 4. (d) 5. (c)