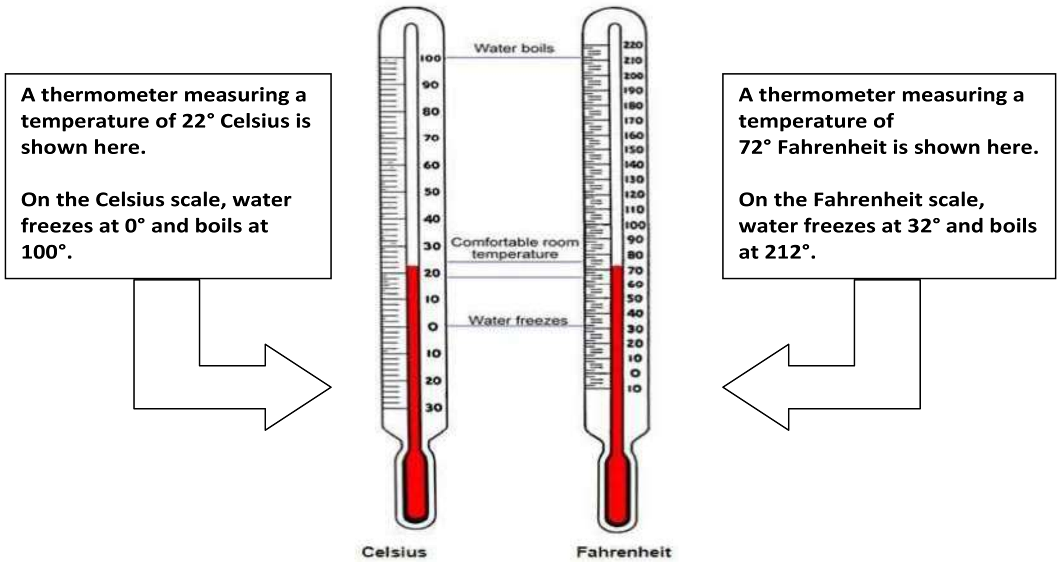
CHAPTER-1

RELATIONS AND FUNCTIONS



By looking at the the two thermometers shown, you can make some general comparisons between the scales. For example, many people tend to be comfortable in outdoor temperatures between 50°F and 80°F (or between 10°C and 25°C). If a meteorologist predicts an average temperature of 0°C (or 32°F), then it is a safe bet that you will need a winter jacket.

Sometimes, it is necessary to convert a Celsius measurement to its exact Fahrenhelt measurement or vice versa.

For example, what if you want to know the temperature of your child in Fahrenheit, and the only thermometer you have measures temperature in Celsius measurement? Converting temperature between the systems is a straightforward process. Using the function

$F = f(C) = \frac{9}{5} C + 32$, any temperature in Celsius can be converted into Fahrenheit scale.

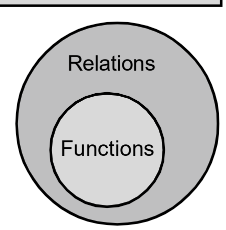
TOPIC TO BE COVERED AS PER CBSE LATEST CURRICULUM 2024-25
Types of relations: reflexive, symmetric, transitive and equivalence relations.
One to one and onto functions

A relation in a set A is a subset of A × A.

Thus, R is a relation in a set A = $R \subseteq A \times A$

If $(a, b) \varepsilon R$ then we say that a is related to b and write, a R b

If $(a, b) \notin$ in R then we say that a is not related to b and write, $a \not R b$.



If number of elements in set A and set B are p and q respectively, Means $n(A) = p$, $n(B) = q$, then

No. of Relation af A × A = 2^{p^2}

No. of Relation of B × B = 2^{q^2}

No. of Relation of A × B = No. of Relation of B × A = 2^{pq}

No. of **NON EMPTY** Relation of A × A = ($2^{p^2} - 1$),

No. of **NON EMPTY** Relation of B × B = ($2^{q^2} - 1$).

No. of **NON-EMPTY** Relation of A × B = No. of Relation of B × A = ($2^{pq} - 1$)

Q.1 If A = {a, b, c} and B = {1, 2} find the number of Relation R on (i) A × A (ii) B × B (iii) A × B

Ans. As $n(A) = 3$, $n(B) = 2$, so

No. of Relation R on A × A = $2^{3 \times 3} = 2^9 = 512$

No. of Relation R on B × B = $2^{2 \times 2} = 2^4 = 16$

No. of Relation R on A × B = $2^{3 \times 2} = 2^8 = 64$

Q.2 A = {d, o, e} and B = {22, 23} find the number of Non-empty Relation R on (i) A × A (ii) B × B

Ans. As $n(A) = 3$, $n(B) = 2$, so

No. of Relation Non-empty relations R on A × A = $2^{3 \times 3} - 1 = 2^9 - 1 = 511$

No. of Relation Non-empty R on B × B = $2^{2 \times 2} - 1 = 2^4 - 1 = 15$

Different types of relations

- **Empty Relation Or Void Relation**

  A relation R in a set A is called an empty relation, if no element of A is related to any element of A and we denote such a relation by ϕ.

  Example: Let A = {1, 2, 3, 4} and let R be a relation in A, given by R = {(a, b): $a + b = 20$}.

- **Universal Relation**

  A relation R in a set A is called an universal relation, if each element of A is related to every element of A.

  Example: Let A = {1, 2, 3, 4} and let R be a relation in A, given by R = {(a, b): $a + b > 0$}.

- **Identity Relation**

  A relation R in a set A is called an identity relation, where R = {(a, a), $a \in A$}.

  Example : Let A = {1, 2, 3, 4} and let R be a relation in A, given by R = {(1, 1), (2, 2), (3, 3), (4, 4)}.

- **Reflexive Relation**

  A relation R in a set A is called a Reflexive relation, if $(a, a) \in R$, for all $a \in A$.

  Example : Let A = {1, 2, 3, 4} and let R be a relation in A, given by

  R = {(1, 1), (2, 2), (3, 3), (4, 4)}.

  R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2)}.

  R = {(1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (1, 3), (3, 1)}.

- **Symmetric Relation**

  A relation R in a set A is called a symmetric relation, if $(a, b) \in R$, then $(b, a) \in R$ for all $a, b \in A$.

  Example : Let A = {1, 2, 3, 4} and let R be a relation in A, given by

  R = {(1, 1), (2, 2), (3, 3)}.

  R = {(1, 2), (2, 1), (3, 3)}.

  R = {(1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (1, 3), (3, 1), (3, 2)}.

- **Transitive Relation**

  A relation R in a set A is called a transitive relation,

  if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ for all $a, b, c \in A$

  Or

  $(a, b) \in R$ and $(b, c) \notin R$ for all $a, b, c \in A$

  Example : Let {1, 2, 3, 4} and let R be a relation in A, given by

  R = {(1, 1), (2, 2), (3, 3)}. (According to second condition)

  R = {(1, 2), (2, 1), (1, 1), (2, 2)}. (According to first condition)

  R = {(2, 3), (1, 3), (3, 1), (3, 2), (3, 3), (2, 2), (1, 1)}.

- **Equivalence Relation**

  A relation R in a set A is said to be an equivalence relation if it is reflexive, symmetric and transitive.

Illustration:

Let A be the set of all integers and let R be a relation in A, defined by R = {a, b}: $a = b$}, Prove that R is Equivalence Relation.

Solution: Reflexivity : Let R be reflexive $\Rightarrow (a, a) \in R \ \forall \ a \in A$

$\Rightarrow \quad a = a$, which is true

Thus, R is Reflexive Relation.

Symmetricity : Let $(a, b) \in R \ \forall \ a, b \in A$

$\Rightarrow \quad a = b$

$\Rightarrow \quad b = a$

so $(b, a) \in R$. Thus R is symmetric Relation.

Transitivity : Let $(a, b) \in$ R and $(b, c) \in R$ $\forall$ $a, b, c \in$ A

$\Rightarrow$ $a = b$ and $b = c$

$\Rightarrow$ $a = b = c$

$\Rightarrow$ $a = c$

so $(a, c) \in$ R. Thus R is transitive Relation.

As, R is reflexive, Symmetric and transitive Relation

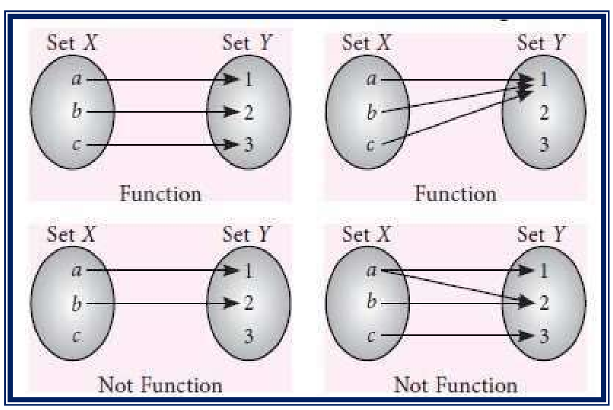
$\therefore$ R is an Equivalence Relation

FUNCTIONS

Functions can be easily defined with the help of concept mapping. Let X and Y be any two non-empty sets. "A function from X to Y is a rule or correspondence that assigns to each element of set X, one and only one element of set Y". Let the correspondence be 'f' then mathematically we write $f : X \rightarrow Y$.

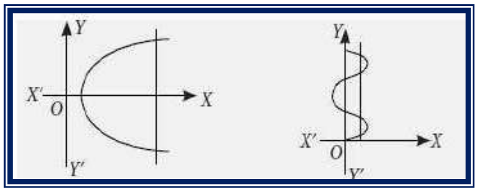
where $y = f(x)$, x ε X and y ε Y. We say that 'y' is the imagesof 'x' under f (or x is the pre image of y).

- A mapping f : X $\rightarrow$ Y is said to be a function if each element in the set X has its image in set Y. It is also possible that there are few elements in set Y which are not the images of any element in set X.
- Every element in set X should have one and only one image. That means it is impossible to have more than one image for a specific element in set X.
- Functions cannot be multi-valued (A mapping that is multi-valued is called a relation from X and Y) eg.

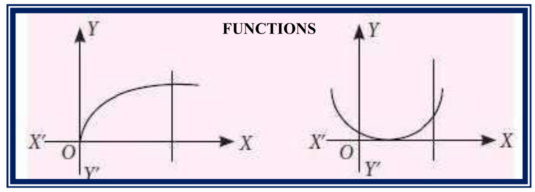


Testing for a function by Vertical line Test

A relation f : A $\rightarrow$ B is a function or not, it can be checked by a graph of the relation. If it is possible to draw a vertical line which cuts the given curve at more thatn one point then the given relation is not a function and when this vertical line means line parallel to Y-axis cuts the curve at only one point then it is a function. Following figures represents which is not a function and which is a function.



NOT FUNCTION NOT A FUNCTION



Number of Functions

Let X and Y be two finite sets having m and n elements respectively. Thus each element of set X can be associated to any one of n elements of set Y. So, total number of functions from set X to set Y is n^m.

Real valued function: if R, be the set of real numbers and A, B are subsets of R, then the function f : A $\to$ B is called a real function or real valued functions.

Domain, Co-Domain And Range of Function

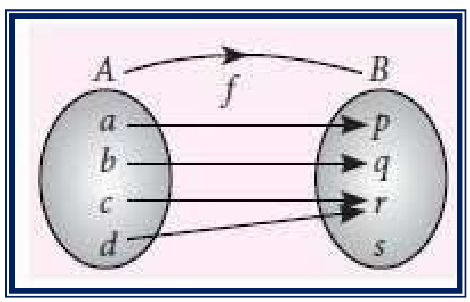
If a function *f* is defined from a set A to set B then (if : A $\to$ B) set A is called the domain of f and set B is called the co-domian of *f*.

The set of all *f*-images of the elements of A is called the range of *f*.

In other words, we can say

Domain = All possible values of x for which $f(x)$ exists.

Range = For all values of , all possible values of $f(x)$.



From the figure we observe that

Domain = A = $\{a, b, c, d\}$ Range = $\{p, q, r\}$, Co-Domain = $\{p, q, r, s\}$ = B

EQUAL FUNCTION

Two function *f* and *g* are said to be equal functions, if and only if

(i) Domain of f = Domain of g

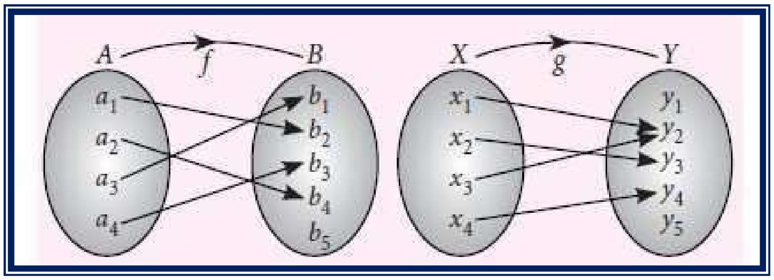
(ii) Co-domain of *f* = Co-domain of g

(iii) $f(x) = g(x)$ for all $x \varepsilon$ their common domain

TYPES OF FUNCTION

One-one function (injection): A function $f : A \to B$ is said to be a one-one function or an injection, if different elements of A have different images in B.

e.g. Let $f : A \to B$ and $g : X \to Y$ be two functions represented by the following diagrams.



Clearly, $f : A \to B$ is a one-one function. But $g : X \to Y$ is not one-one function because two distinct elements x_1 and x_3 have the same image under function g.

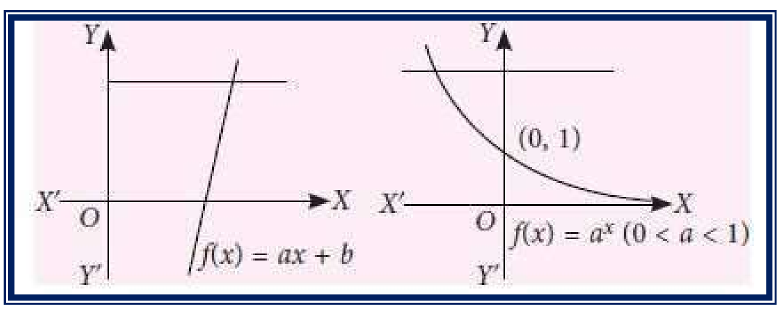
Method to check the injectivity (One-One) of a function

(i) Take two arbitrary elements x, y (say) in the domain of f.

(ii) Solve $f(x) = f(y)$. If $f(x) = f(y)$ give $x = y$ only, then $f : A \to B$ is a one-one function (or an injection). Otherwise not.

If function is given in the form of ordered pairs and if two ordered pairs do not have same second element then function is one-one.

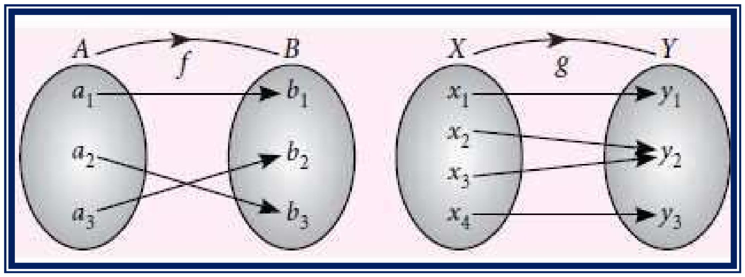
If the graph of the function $y = f(x)$ is given and each line parallel to x-axis cuts the given curve at maximum one point then function is one-one. (Strictly increasing or Strictly Decreasing Function). E.g.



Number of one-one functions (injections) : If A and B are finite sets having m and n elements respectively, then number of one-one functions from A and B = ${}^nP_m$ is $n \geq m$ and 0 if $n < m$.

If $f(x)$ is not one-one function, then its Many-one function.

Onto function (surjection) : A function $f : A \to B$ is onto if each element of B has its pre-image in A. In other words, Range of f = Co-domain of f. *e.g.* The following arrow-diagram shows onto function.

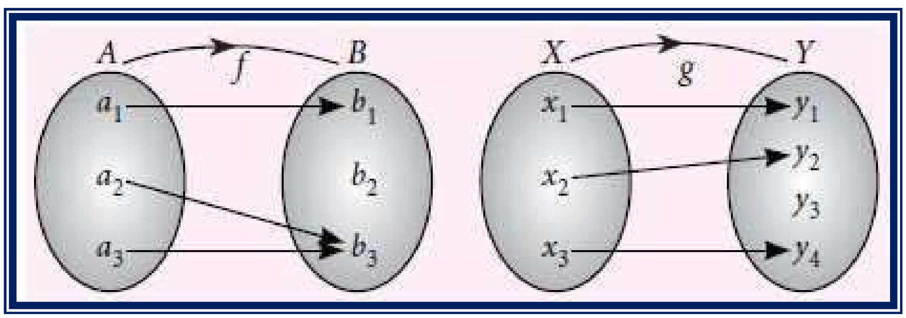


Number of onto function (surjection): If A and B are two sets having m and n elements respectively such that $1 \leq n \leq m$, then number of onto functions from A to B is $\sum_{r=1}^{n} (-1)^{n-r}\ {}^nC_r$

$. r^m$

Into function: A function $f : A \to B$ is an into function if there exists an element in B having no pre-image in A.

In other words, $f : A \to B$ is an into function if it is not an onto function e.g, The following arrow diagram shows into function.



Method to find onto or into function:

(i) Solve $f(x) = y$ by taking x as a function of y i.e., $g(y)$(say).

(i) Now if $g(y)$ is defined for each $y \varepsilon$ co-domain and $g(y)\ \varepsilon$ domain then f(x) is onto and if any one of the above requirements is not fulfilled, then *f(x)* is into.

One-one onto function (bijection) : A function $f : A \to B$ is a bijection if it is one-one as well as onto.

In other words, a function $f : A \to$ Bis a bijection if

(i) It is one-one ie., $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$.

(ii) It is onto i.e., for all $y \in B$, there exists $x \in A$ such that $f(x) = y$.

Clearly, f is a bijection since it is both injective as well as surjective.

Illustration :

Let $f : R \to R$ be defined as $f(x) = 7x - 5$, then show that function is one-one and onto Both.

Solution : Let $f(x) = f(y)\ \forall\ x, y \in R$

$\Rightarrow \quad 7x - 5 = 7y - 5$

$\Rightarrow \quad x = y$, so $\boxed{f(x) \text{ is one-one function}}$

Now, As $f(x) = 7x - 5$, is a polynomial function.

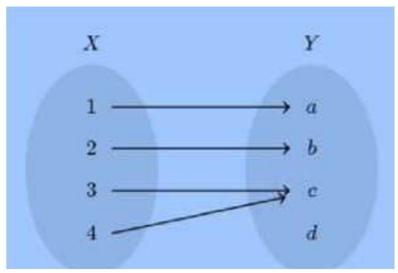
so it is defined everywhere. Thus, Range = R

As, Range = co-domain, so $\boxed{f \text{ is onto function.}}$

Alternative method : Graph of $f(x)$ is a line which is strictly increasing for all values of x, so its one-one function and Range of $f(x)$ is R which is equal to R so onto funclion.

ILLUSTRATION:

If $f: X \to Y$ is defined, then show that f is neither one-one nor onto function.



Solution : As for elements 3 and 4 from set X we have same image c in set Y, so f is not one-one function.

Further element d has no pre -image in set X,

so f is not onto function

ILLUSTRATION:

Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 2022$ is one-one.

SOLUTION : APPROACH-I

Let $f(x_1) = f(x_1) \;\forall\; x_1, x_2 \in N \Rightarrow = x_1^2 + x_1 + \cancel{2022} = x_2^2 + x_2 + \cancel{2022}$

$\Rightarrow \quad x_1^2 + x_1 = x_2^2 + x_2$

$\Rightarrow \quad (x_1^2 - x_2^2) + (x_1 - x_2) = 0$

$\Rightarrow \quad (x_1 - x_2) + (x_1 + x_2 + 1) = 0$

Thus,$(x_1 - x_2) = 0$ as $(x_1 + x_2 + 1) \neq 0 \;\forall\; x_1, x_2 \in N$

so, f is ONE-ONE function

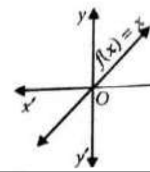
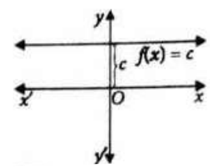
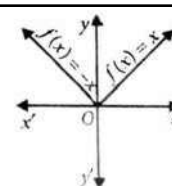
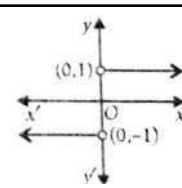
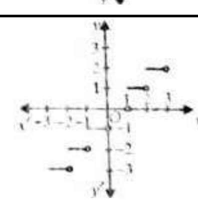
APPROACH-II

$f(x) = x^2 + x + 2022 \Rightarrow f'(x) = 2x + 1$

As, $x \in N$ so, $2x + 1 > 0 \Rightarrow f'(x) = 0$ (Strictly Increasing function)

so, f is ONE-ONE function

Type of Functions

Name of Function	Definition	Domain	Range	Graph
1. Identify Function	The function $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = x \ \forall \ x \in \mathrm{R}$	R	R	
2. Constant Function	The function $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = c \ \forall \ x \in \mathrm{R}$	R	(c)	
3. Polynomial Function	The function $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = p_0 + p_1x + p_2x^2 + ... + p_nx^n$, where $n \in \mathrm{N}$ and $p_0, p_1, p_2 ..., p_n \in \mathrm{R} \ \forall \ x \in \mathrm{R}$			
4. Rational Function	The function f defined by $f(x) = \dfrac{P(x)}{Q(x)}$, where P($x$) and Q($x$) are polynomial functions, $Q(x) \neq 0$			
5. Modulus Function	The function $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = \lvert x \rvert = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \ \forall \ x \in \mathrm{R}$	R	$[0, \infty)$	
6. Signum Function	The function $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = \begin{cases} \dfrac{\lvert x \rvert}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \\ 0, & x = 0 \end{cases}$	R	$\{-1, 0, 1\}$	
7. Greatest Integer Function	The function $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = \lvert x \rvert = \begin{cases} x, \ x \in \mathrm{Z} \\ \text{integer less than} \\ \textit{equal to } x, \ x \notin \mathrm{Z} \end{cases}$	R	Z	
8. Linear Function	The function $f: \mathrm{R} \to \mathrm{R}$ defined by $f(x) = mx + c$, $x \in \mathrm{R}$ where m and c are constants	R	R	

ONE–MARK QUESTIONS

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE

1. Consider the set A = {1, 2, 3}, then write smallest equivalence relation on A.

   (a) { } (b) {(1, 1)} (c) {(1, 1), (2, 2), (3, 3)} (d) {(3, 3)}

2. Consider the set A containing 5 elements, then the total number of injective functions from A onto itself are

   (a) 5 (b) 25 (c) 120 (d) 125

3. Let Z be the set of integers and R be the relation defined in Z such that aRb if $(a - b)$ is divisible by 4, then R partitions the set Z into how many Pairwise disjoint subsets.

   (a) 2 (b) 3 (c) 4 (d) 5

4. If A = {d, 0, e} then the number of relations on A × A are

   (a) 3 (b) 8 (c) 15 (d) 512

5. If A = {2023, 2024) then the number of non-empty relations on A × A are

   (a) 1 (b) 4 (c) 15 (d) 16

6. If A = {2023, 2024) then the number of Reflexive relations on A × A are

   (a) 2 (b) 4 (c) 8 (d) 16

7. If A = {s, u, v}, then the number of Symmetric relations on A × A are

   (a) 8 (b) 9 (c) 32 (d) 64

8. Let A be the set of the Letters of the name of our country the "INDIA". Then find the number of reflexive relations on A × A

   (a) 4096 (b) 2048 (c) 1024 (d) 16

9. Let A = $\{x : x^2 < 3, x \in W\}$, then the number of Symmtric relations on $A \times A$ are

   (a) 1 (b) 2 (c) 4 (d) 8

10. If there are 'p' elements in set A, such that number of Reflexive relation on A × A are 4096, then p =

    (a) 4 (b) 6 (c) 8 (d) 12

11. Let A = {d, 0, e}, then Find 'p' if the number of Symmetric relations on A × A are 2^p.

    (a) 4 (b) 6 (c) 8 (d) 12

12. Find the maximum number of equivalence relations on the set A = {1, 2, 3}.

    (a) 3 (b) 5 (c) 8 (d) 9

13. If the function $f : R — \{1, -1\} \rightarrow A$ defined by $f(x) = \dfrac{x^2}{1+x^2}$ is Surjective, then A =

    (a) R (b) R — {1, – 1} (c) [0, 1) (d) [0, ∞]

14. The number of injections possible from A = {1, 2, 3, 4} to B = {5, 6, 7} are

    (a) 0 (b) 3 (c) 6 (d) 12

15. If the number of one-one functions that can defined from A = {4, 8, 12, 16} to B is 5040, then $n(B)$ =

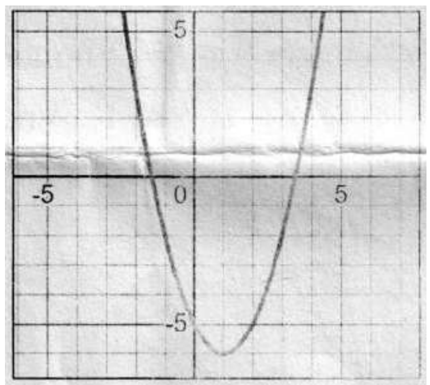
(a) 7 (b) 3 (c) 6 (d) 10

16. If the function $f : \mathrm{R} \rightarrow \mathrm{A}$ defined by $f(x) = 3 \sin x + 4\cos x$ is Surjective, then A =

(a) [–7, 7] (b) [–1, 1] (c) [1, 7] (d) [–5, 5]

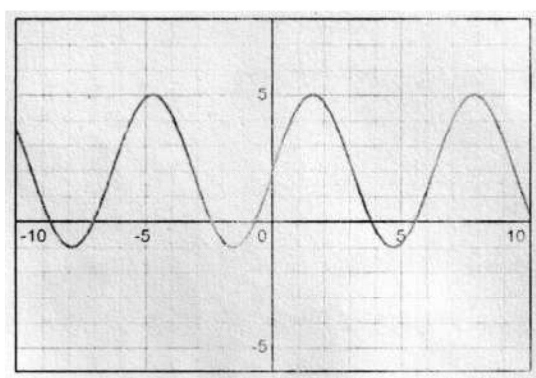
17. The Part of the graph of a Non-Injective function $f : \mathrm{R} \rightarrow$ Range defined by $f(x) = x^2 - 2x + a$ is given below. If the domain of $f(x)$ is modified as either $(-\infty, b]$ or $[b, \infty]$ then $f(x)$ becomes the Injective function. What must be the value of $(b - a)$.

(a) 6 (b) 5 (c) 4 (d) 0



18. The graph of the function $f : R \rightarrow \mathrm{A}$ defined by $y = f(x)$ is given below, then find A such that function $f(x)$ is onto function

(a) [–1, 5] (b) [–5, 5] (c) [–5, 1] (d) R



ASSERTION-REASON BASED QUESTIONS (Q.19 & Q.20)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are ture but R is not the correct explanation of A.

(c) A is true but R is false

(d) A is false but R is true

19. ASSERTION (A) : A relation R = {(a, b) : |a – b| < 1} defined on the set

    A = {1, 2, 3, 4} is Reflexive

    Reason (R) : A realtion R on the set A is said to be reflexive if for $(a, b) \in R$

    & $(b, c) \in R$, we have $(a, c) \in R$.

20. Assertion (A) : A function $f : R \to R$ given $f(x) = |x|$ is one-one function.

    Reason (R) : A function $f : A \to B$ is said to be Injective if

    $f(a) = f(b) \Rightarrow a = b$

TWO MARKS QUESTIONS

21. If A = {*a, b, c, d*} and *f* = {(*a, b*), (*b, d*), (*c, a*), (*d, c*)}, show that *f* is one-one from A to A.
22. Show that the relation R on the set of all real numbers defined as R = {(a, b) : $a \leq b^3$} is not transitive.
23. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \dfrac{x^2}{1 - x^2}$, is Surjective, then find A.
24. Give an example to show that the union of two equivalence relations on a set A need not be an equivalence relation on A.
25. How many reflexive relations are possible in a set A whose (A) = 4. Also find How many symmetric relations are possible on a set B whose *n*(B) = 3.
26. Let W denote the set of words in the English dictionary. Define the relation R by R {(*x, y*) ε W × W such that *x* and *y* have at least one letter in common). Show that this relation R is reflexive and symmetric, but not transitive.
27. Show that the relation R in the set of all real numbers, defined as R = {(*a, b*): $a \leq b^2$} is neither reflexive Nor symmetric.
28. Consider a function $f : R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R+ is the set of all positive real numbers. Show that function is one-one and onto both.
29. Let L be the set of all lines in a plane. A relation R in Lis given by R {(L_1, L_2): L_1 and L_2 intersect at exactly one point, L_1, L_2 ε L}, then show that the relation R is symmetric Only.
30. Show that a relation R on set of Natural numbers is given by R = {(*x, y*): *xy* is a square of an integer} is Transitive.

THREE MARKS QUESTIONS

31. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective.

    (i) {(*x, y*) : *x* is a person, *y* is the mother of *x*}.

(ii) $\{(a, b)$: a is a person, b is an ancestor of $a\}$.

32. Show that the function $f: R \to R$ defined by $f(x) = \dfrac{x^2}{x^2+1}$; $\forall\, x \in R$, is neither one-one nor onto.

33. Let R be the set of real numbers and $f: R \to R$ be the function defined by $f(x) = 4x + 5$. Show that f is One-one and onto both.

34. Show that the relation R in the set A = {3, 4, 5, 6, 7} given by $R = \{(a, b) : |a - b|$ is divisible by $2\}$ is an equivalence relation. Show that all the elements of {3, 5, 7} are related to each other and all the ements of {4, 6} are related to each other, but no element of {3, 5, 7} is related to any element {4, 6}.

35. Check whether the relation R in the set Z of integers defined as $R = ((a, b) : a + b$ is "divisible by 2"} is reflexive, symmetric, transitive or Equivalence.

36. Show that that following Relations R are equivalence relation in A.

    (a) Let A be the set of all triangles in a plane and let R be a relation in A, defined by $R = \{(T_1, T_2) : T_1$, is congruent $T_2\}$

    (b) Let A be the set of all triangles in a plane and let R be a relation in A, defined by $R = \{(T_1, T_2) : T_1$, is similar $T_2,\}$

    (c) Let A be the set of all lines in xy-plane and let R be a relation in A, defined by $R = \{(L_1, L_2,) : L_1$, is parallel to $L_2\}$

    (d) Let A be the set of all integers and let R be a relation in A, defined by $R = \{(a, b) : (a - b)$ is even$\}$

    (e) Let A be the set of all integers and let R be a relation in A, defined by $R = \{(a, b) : |a - b|$ is a multiple of $2\}$

    (f) Let A be the set of all integers and let R be a relation in A, defined by $R = \{(a, b) : |a - b|$ is a divisible by $3\}$

37. Check whether the following Relations are Reflexive, Symmetric or Transitive.

    (a) Let A be the set of all lines in xy-plane and let R be a relation in A, defined by $R = \{(L_1, L_2) : L_1$ is perpendicular to $L_2\}$

    (b) Let A be the set of all real numbers and let R be a relation in A defined by $R = \{(a, b): a \le b\}$

    (c) Let A be the set of all real numbers and let R be a relation in A defined by $R = \{(a, b): a \le b^2\}$

    (d) Let A be the set of all real numbers and let R be a relation in A defined by $R = \{(a, b) : a \le b^3\}$

    (e) Let A be the set of all natural numbers and let R be a relation in A defined by

R = (a, b) : a is a factor b}

OR

R {(a.b): b is divisible by a}

(f) Let A be the set of all real numbers and let R be a relation in A defined by

R {(a.b): (1+ ab) > 0}

38. Let S be the set of all real numbers. Show that the relation R = {a, b): $a^2 + b^2 = 1$} is symmetric but neither reflexive nor transitive.

39. Check whether relation R defined in R as R = {a, b): $a^2 - 4ab + 3b^2 = 0$, $a, b \in$ R} is reflexive, symmetric and transitive.

40. Show that the function $f : (-\infty, 0) \to (-1, 0)$ defined by $f(x) = \dfrac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto.

FIVE MARKS QUESTIONS

41. For real numbers x and y, define $x\,R\,y$ if and only if $x - y + \sqrt{2}$ is an irrational number. Then check the reflexivity, Symmetricity and Transitivity of the relation R.

42. Determine whether the relation R defined on the set of all real numbers as

R = {(a, b) : a, b ε R and $a - b + \sqrt{3}$ ε S}

(Where S is the set of all irrational Numbers) is reflexive, symmetric or transitive.

43. Let N be the set of all natural numbers and let R be a relation on N × N, defined by Show that R is an equivalence relation.

(i) $(a, b)\,R\,(c, d) \Leftrightarrow a + d = b + c$

(ii) $(a, b)\,R\,(c, d) \Leftrightarrow ad = bc$

(iii) $(a, b)\,R\,(c, d) \Leftrightarrow \dfrac{1}{a} + \dfrac{1}{d} = \dfrac{1}{b} + \dfrac{1}{c}$

(iv) $(a, b)\,R\,(c, d) \Leftrightarrow ad\,(b + c) = bc\,(a + d)$

44. Let A = R – {1}, f : A $\to$ A is a mapping defined by $f(x) = \dfrac{x-2}{x-1}$, show that f is one-one and onto.

45. Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of f, is One-One and Onto Function.

CASE STUDIES

A. A person without family is not complete in this world because family is an integral part of all of us Human deings are considered as the social animals living in group called as family. Family plays many important roles throughout the life.

Mr. D.N. Sharma is an Honest person who is living happily with his family. He has a son Vidya and a Daughter Madhulika. Mr. Vidya has 2 sons Tarun and Gajender and a daughter Suman while Mrs. Madhulika has 2 sons Shashank and Pradeep and 2 daughters Sweety and Anju. They all Lived together and everyone shares equal responsibilities within the family. Every member of the family emotionally attaches to each other in their happiness and sadness. They help each other in their bad times which give the feeling of security.

A family provides love, warmth and security to its all members throughout the life which makes it a complete family. A good and healthy family makes a good society and ultimately a good society involves in making a good country.

On the basis of above information, answer the following questions:

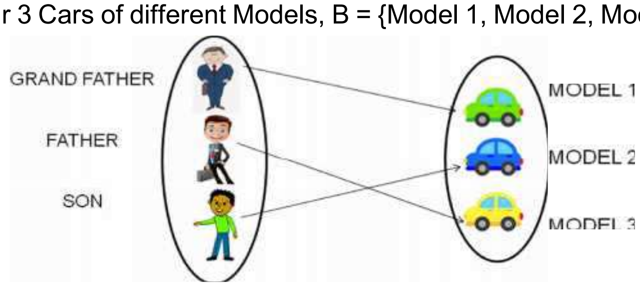
Consider Relation R in the set A of members of Mr. D. N. Sharma and his family at a particular time

(a) If R = {(x, y) : x and y live in the same locality), then R show that R is reflexive Relation.

(b) If R = {(x, y) : x is exactly 7 cm taller than y}, then R show that R is not Symmetric relation.

(c) If R = {(x, y) : x is wife of y}, then show that R is Transitive only.

B. Let A be the Set of Male members of a Family, A = (Grand father, Father, Son) and B be the set of their 3 Cars of different Models, B = {Model 1, Model 2, Model 3}



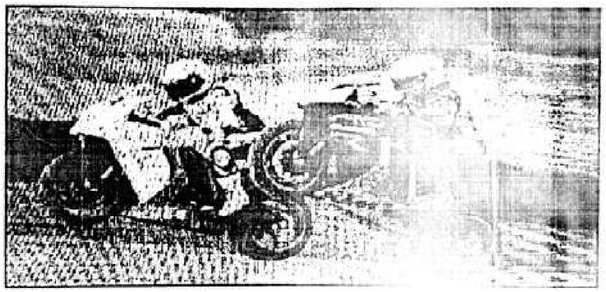
On the basis of The above Information, answer the following questions:

(a) If m & n represents the total number of Relations & functions respectively on A × B, then find the value of (m + n).

(b) If p & q represents the total number of Injective function & total numbers of Surjective functions respectively on A × B, then find the value of $|p – q|$.

C. An organization conducted bike race under two different categories—Boys & Girls. There were 28 participants in all. Among all of them finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these Participants for his college Project.

Let B = {b1, b2, b3} and G = {g1, g2}, represents the set of Boys selected & G the set of Girls selected for the final race.



(a) How many relations are possible from B to G?

(b) Among all possiblerelations form B to G, how many functions can be formed from B to G?

(c) Let R:B $\rightarrow$ B be defined by

R = {(x, y) : x & y are students of same sex}. Check R is equivalence Relation.

OR

A function f : B $\rightarrow$ G be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$

Check if f is bijective. Justify your answer.

SELF ASSESSMENT-1

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT, CHOOSE THE CORRECT ALTERNATIVE:

1. Consider the set A = {1. 2, 3} and R be the smallest equivalence relation on A.then R =

(a) {(1.1)} (b) {(1,1), (2,2)}

(c) {(1,1),(2,2),(3,3)} (d) ϕ

2. Consider the set A containing n elements. Then, the total number of injective functions from A onto itself is

(a) 2^n (b) n

(c) n (d) $n!$

3. The total number of injective nappingsfrom a set with m elements to a set with n elements, $m \leq n$ is

(a) $n!$ (b) n^m

(c) m^n (d) $\frac{n!}{(n-m)!}$

4. The number of injections possible from A = {1,3,5,6} to B = {2,8,11} is

(a) 12 (b) 22

(c) 3 (d) 0

5. The number of one-one functions that can defined from

A = {4,8,12,16} to B is 5040, then n(B) =

(a) 7 (b) 8

(c) 9 (d) 10

SELF ASSESSMENT-2

EACH OF THE FOLLOWING MCQ HAS ONE OPTION CORRECT CHOOSE THE CORRECT ALTERNATIVE.

1. A relation R in a set A is called .... if $(a_1, a_2) \in$ R implies $(a_2, a_1) \in$ R, for all a_1, a2 $\in$ A.

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

2. Let f : R – {0} $\to$ R – {0} be defined by $f(x) = \frac{1}{x}\ \forall\ x \in$ R. Then f is

(a) One-One (b) Many-One

(c) Not defined (d) None of these

3. Let P = $\{(x, y) \mid x^2 + y^2 = 1, x, y \in$ R$\}$. Then P is

(a) Re flexive (b) Symmetric

(c) Transitive (d) Equivalence

4. The function f : R $\to$ R defined by $f(x) = [x]$, where [.] is greatest integer function is

(a) One-One (b) Many -One

(c) Onto (d) None of these

5. The number of bijective functions (One-one and onto both) from set A to itself when A contains 2022 elements is

(a) 2022 (B) 2022!

(C) 2022^2 (D) 2022^{2022}

ANSWER

One Mark Questions

1. (c) {(1,1), (2,2), (3,3)}	2. (c) 120	3. (c) 4
4. (d) 512	5. (c) 15	6. (b) 4
7. (d) 64	8. (a) 4096	9. (d) 8
10. (a) 4	11. (b) 6	12. (b) 5
13. (c) [0,1)	14. (a) 0	15. (d) 10
16. (d) [-5,5]	17. (a) 6	18. (a) [-1,5]
19. (c)	20. (d)	
A is true but R is false	A is false but R is true	

Two Mark Questions

23. A = R – [–1, 0)

25. Reflexive Relations = 4096 Symmetric Relation = 64

Three Mark Questions

31. (a) Yes it's function, Not Injective but Surjective (b) No, its not a function

35. EQUIVALENCE RELATION

37. (a) Symmetric (b) Reflexive and Transitive
    (c) Neither Reflexive, Symmetric nor Transitive
    (d) Neither Reflexive, Symmetric nor Transitive
    (e) Reflexive and Transitive
    (f) Reflextive and Symmetric

39. Reflexive only

Four/Five Mark Questions

41. Reflexive only 42. Reflexive only

CASE STUDIES BASED QUESTION

B. (a) 512 +27 =539 B. (b) 0

C. (a) 64
(b) 8
(c) R is an Equivalence Relation OR (c) f is not Bijective

SELF ASSESSMENT-1

1. (c) 2. (d) 3. (d) 4. (d) 5. (d)

SELF ASSESSMENT-2

1. (b) 2. (a) 3. (b) 4. (b) 5. (b)