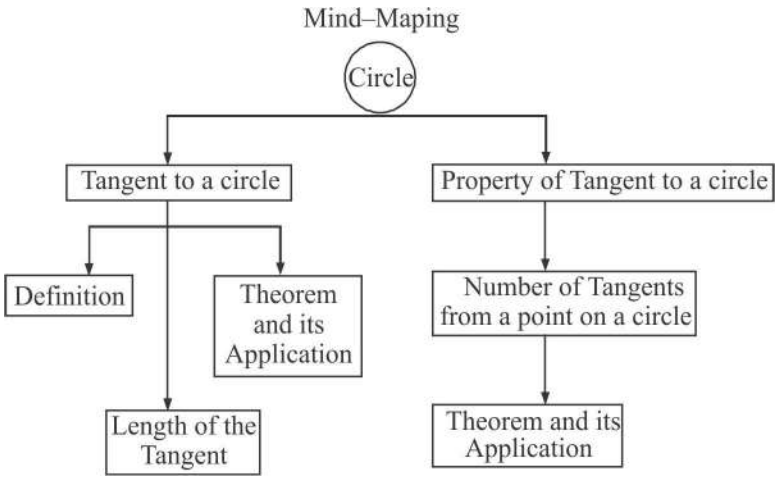
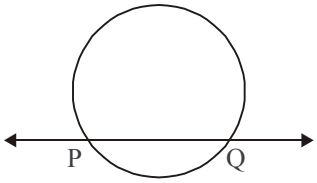
CHAPTER

10 Circles



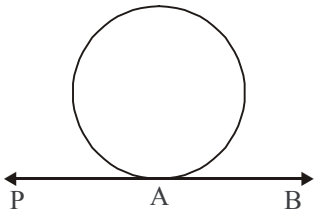
KEY POINTS

1. A **circle** is a collection of all those points in a plane which are at a constant distance from a fixed point. The fixed point is called the **centre** and fixed distance is called the **radius**.
2. **Secant:** A line which intesects a circle in two distinct points is called a secant of the circle.



3. **Tangent**: It is a line that intersects the circle at only one point. The point where tangent touches the circle is called the point of contact.

   Here A is the point of contact.



4. **Number of Tangent**: Infinitely many tangents can be drawn on a circle.

5. **Number of Secant**: There are infinitely many secants which can be drawn to a circle.

6. The proofs of the following theorems can be asked in the examination:–

(i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

(ii) The lengths of tangents drawn from an external point to a circle are equal.

7. The tangent to a circle is a special case of the secant.

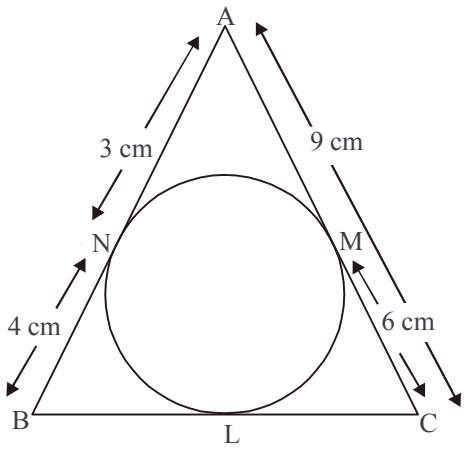
8. There is no tangent to a circle passing through a point lying inside the circle.

9. There is one and only one tangent to a circle passing through a point lying on the circle.

10. There are exactly two tangents to a circle through a point lying outside the circles.

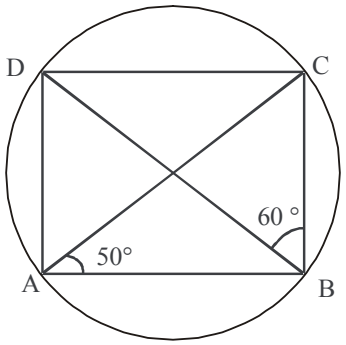
VERY SHORT ANSWER TYPE QUESTIONS

1. In fig., ΔABC is circumscribing a circle. Find the length of BC.

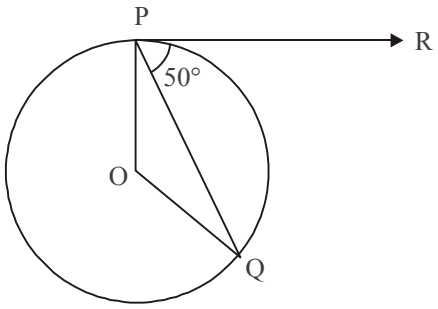


2. The length of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm. What is the radius of the circle.

3. In fig., ABCD is a cyclic quadrilatreral. If $\angle BAC = 50°$ and $\angle DBC = 60°$, then find $\angle BCD$.



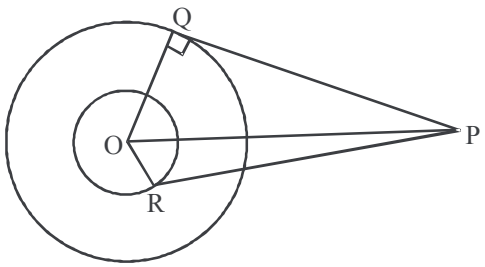
4. In figure, O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angles of 50° with PQ. Find $\angle POQ$.



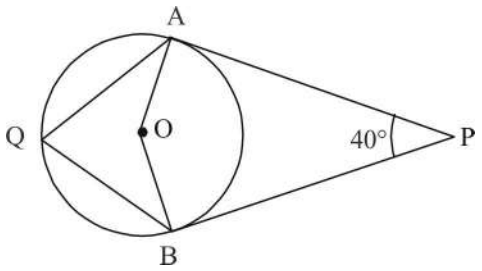
5. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

6. If radii of two concentric circles are 4 cm and 5 cm, then find the length of the chord of that circle which is tangent to the other circle.

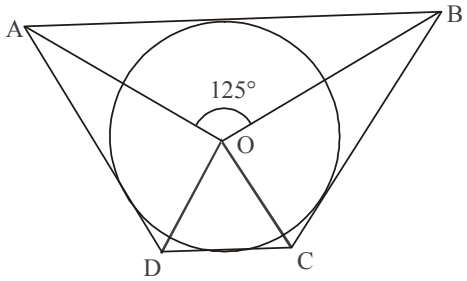
7. In the given figure, PQ is tangent to outer circle and PR is tangent to inner circle. If PQ = 4cm, OQ = 3 cm and OR = 2 cm then find the length of PR.



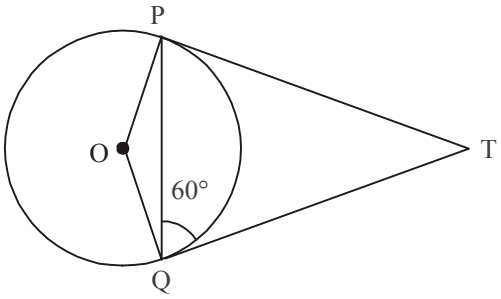
8. In the given figure, O is the centre of the circle, PA and PB are tangents to the circle then find $\angle AQB$. **(CBSE 2016)**



9. In the given figure, If $\angle AOB = 125°$ then find $\angle COD$.



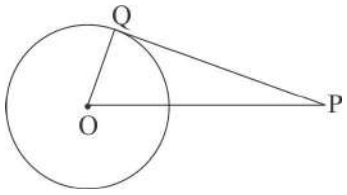
10. If two tangent TP and TQ are drawn from an external point T such that $\angle TQP = 60°$ then find $\angle OPQ$.



11. How many tangents can a circle have?

12. A tangent to a circle intersects it in _____ point.

13.



If PQ is a tangent then find the value of $\angle POQ + \angle QPO$.

14. Choose the correct Answer.

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(a) 12 cm (b) 13 cm (c) 8.5 cm (d) $\sqrt{119}$ cm

15. A circle can have _____ parallel tangents at the most.

16. The common point of a tangent to a circle and radius of the circle is called _____.

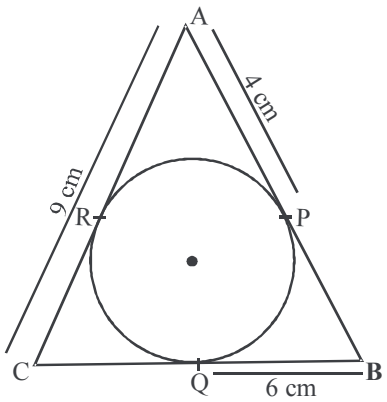
SHORT ANSWER TYPE QUESTIONS

17. If diameters of two concentric circles are d_1 and d_2 ($d_2 > d_1$) and c is the length of chord of bigger circle which is tangent to the smaller circle. Show that $d_2^2 = c^2 + d_1^2$.

18. The length of tangent to a circle of radius 2.5 cm from an external point P is 6 cm. Find the distance of P from the nearest point of the circle.

19. TP and TQ are the tangents from the external point T of a circle with centre O. If $\angle OPQ = 30°$ then find the measure of $\angle TQP$.

20. In the given fig. AP = 4 cm, BQ = 6 cm and AC = 9 cm. Find the semi perimeter of ΔABC.

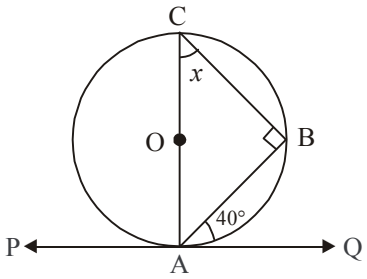


21. A circle is drawn inside a right angled triangle whose sides are a, b, c where c is the hypotenuse, which touches all the sides of the triangle. Prove $r = \dfrac{a + b - c}{2}$ where r is the radius of the circle.

22. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

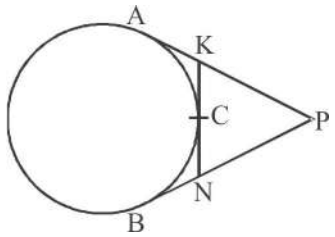
23. Prove that in two concentric circles the chord of the larger circle which is tangent to the smaller circle is bisected at the point of contact.

24. In the given Fig., AC is diameter of the circle with centre O and A is the point of contact, then find x.

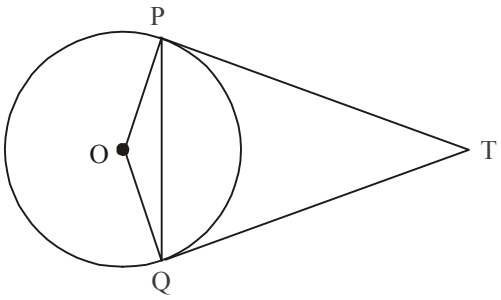


25. In the given fig. KN, PA and PB are tangents to the circle. Prove that:

KN = AK + BN.

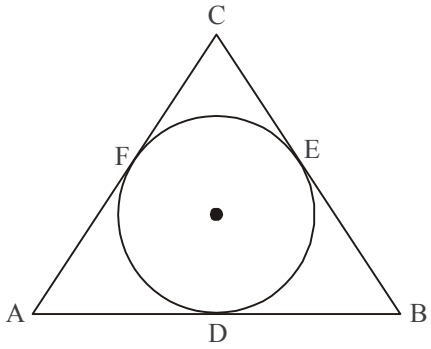


26. In the given fig. PQ is a chord of length 6 cm and the radius of the circle is 6 cm. TP and TQ are two tangents drawn from an external point T. Find $\angle PTQ$.

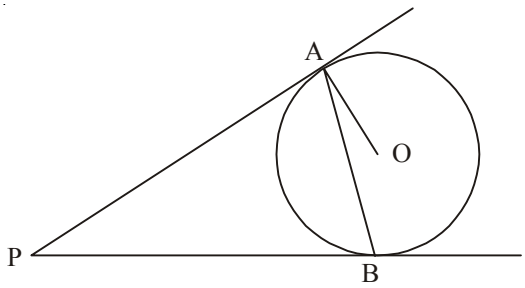


LONG ANSWER TYPE QUESTIONS

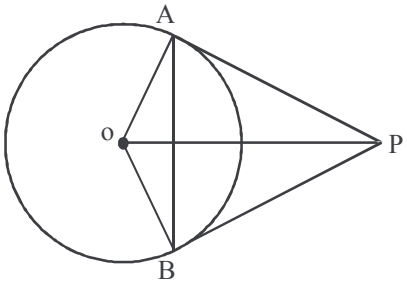
27. In the given figure find AD, BE, CF where AB = 12 cm, BC = 8 cm and AC = 10 cm.



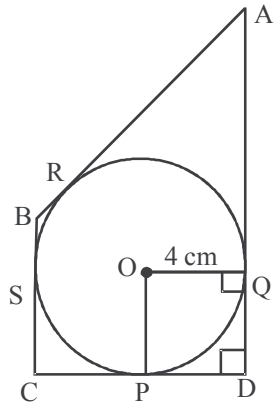
28. Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that ∠APB = 2 ∠OAB **(NCERT, Exemplar)**



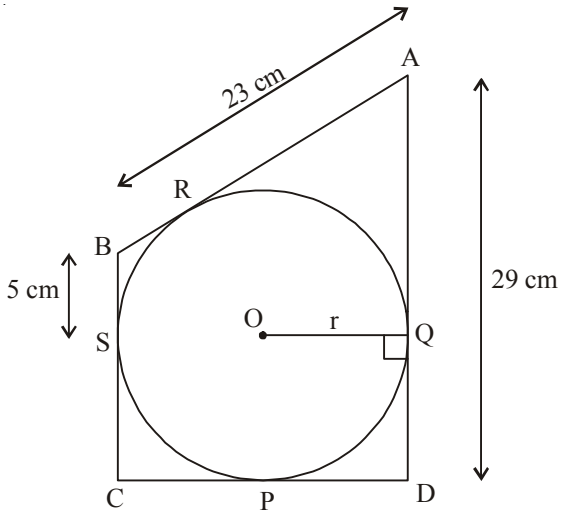
29. In the given fig. OP is equal to the diameter of the circle with centre O. Prove that ΔABP is an equilateral triangle.



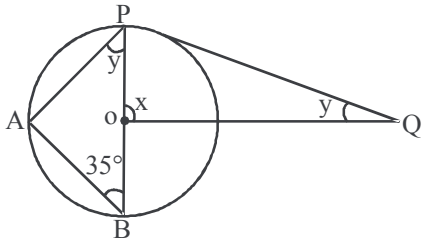
30. In the given fig., find PC. If AB = 13 cm, BC = 7 cm and AD = 15 cm.



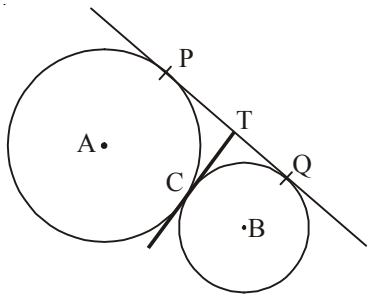
31. In the given figure, find the radius of the circle.



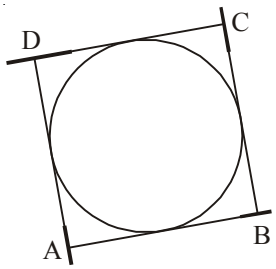
32. In the given fig. PQ is tangent and PB is diameter. Find the values of angles x and y.



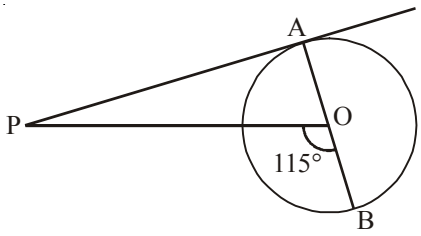
33. In given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



34. In the given figure, a circle touches all the four sides of a quadrilateral ABCD. If AB = 6 cm, BC = 9 cm and CD = 8 cm, then find the length of AD.



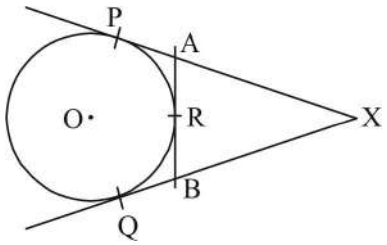
35. In figure, PA is a tangent from an external point P to a circle with centre O, If $\angle POB = 115°$. Find $\angle APO$.



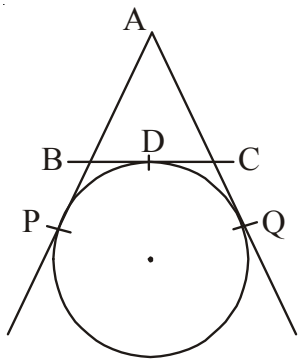
36. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

37. In figure, XP and XQ are tangents from X to the circle with centre O, R is a point on the circle and AB is tangent at R. Prove that :

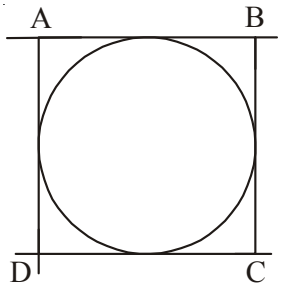
XA + AR = XB + BR



38. In the given figure, find the perimeter of ΔABC, if AP = 12 cm.



39. In the given figure, a quardrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = BC + AD



40. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

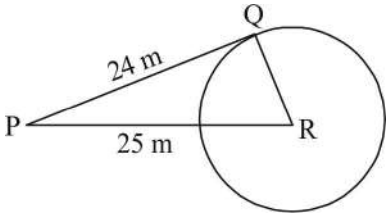
ANSWERS AND HINTS

1. Since length of both the tangents from a point outside the circle is equal, So

$$BN = BL,\ CM = CL$$

$$BL + CL = BC = 10 \text{ cm}$$

2.



By Pythagoras Theorem, QR = 7 cm.

3. Angle in the same segment are euqal.

- DC is the chord so $\angle DAC = \angle DBC = 60°$.
- The sum of the opposite angles of a cyclic quadrilateral is 180°.

So $\angle BCD = 70°$

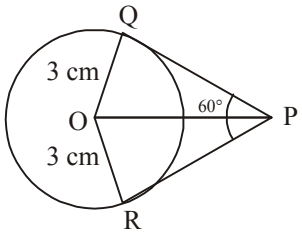
4. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

So, $\angle RPO = 90°$

$\angle OPQ = \angle OQP = 40°$

$\angle POQ = 100°$

5.



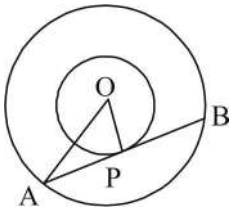
$$\Delta QPO \cong \Delta RPO$$

$$\Rightarrow \quad \angle QPO = \angle RPO = \frac{60°}{2} = 30°$$

In ΔQPO, $\angle OQP = 90°$ (Tangent is perpendicular at the point of contact).

$$\tan 30° = \frac{OQ}{QP} \quad \Rightarrow \quad QP = 3\sqrt{3} \text{ cm}$$

6.



In ΔAOP, right angled at P.

$$OA^2 = AP^2 + OP^2 \Rightarrow (5)^2 = AP^2 + 4^2 \Rightarrow AP^2 = 9$$

$$\Rightarrow \quad AP = 3$$

$$\therefore \quad AB = 6 \text{ cm} \quad (\because OP \perp AB \text{ so OP bisects AB})$$

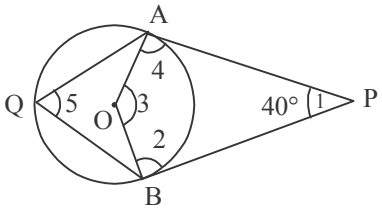
7. In $\Delta PQO \quad (4)^2 + (3)^2 = (OP)^2$

$$5 = OP$$

In $\Delta PRO, \quad (5)^2 = (2)^2 + (PR)^2$

$$PR = \sqrt{21} \text{ cm}$$

8.



In Quadrilateral OAPB

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$$

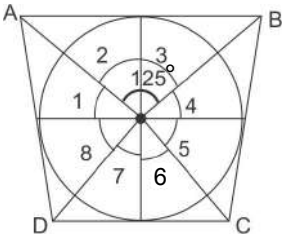
$$\angle 1 + \angle 3 = 180°$$

$$\angle 3 = 140°$$

Now, $\quad \angle 3 = 2 \angle 5$

$$\angle 5 = 70° \text{ or } \angle AQB = 70°$$

9. $\left.\begin{array}{l} \angle 1 = \angle 2 \\ \angle 3 = \angle 4 \\ \angle 5 = \angle 6 \\ \angle 7 = \angle 8 \end{array}\right]$ (CPCT) of their corresponding triangles.



$$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$$

$$\text{or } \angle AOB + \angle COD = 180°$$

$$\text{or } \angle COD = 55°$$

10. $\angle OQT = 90°$ (Angle between tangent & radius)

$\angle PQO = 30°$

$\angle PQO = \angle OPQ = 30°$

11. Infinitely many

12. One

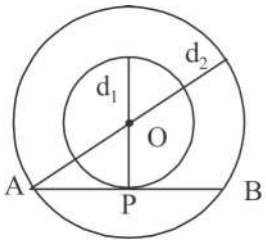
13. 90°

14. d($\sqrt{119}$ cm)

15. Two

16. Point of Contact

17.



AB = c

$AB = c$

$AO^2 = OP^2 + AP^2$

$\left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + AP^2$

$\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2 = AP^2$

$\sqrt{\frac{1}{4}\left[(d_2)^2 - (d_1)^2\right]} = AP$

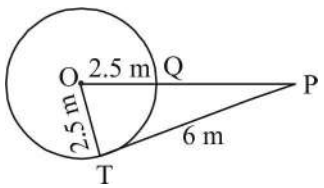
$2\sqrt{\frac{1}{4}\left[(d_2)^2 - (d_1)^2\right]} = AB$

$\sqrt{(d_2)^2 - (d_1)^2} = c$

$(d_2)^2 - (d_1)^2 = c^2$

$d_2{}^2 = c^2 + d_1{}^2$

18.



$(OP)^2 = (OT)^2 + (PT)^2$

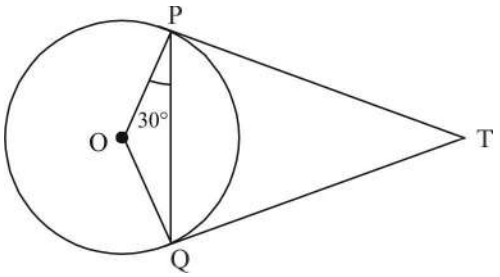
$(OP)^2 = (2.5)^2 + (6)^2$

$= 42.25$

$(OP)^2 = (6.5)^2 \quad \Rightarrow \quad OP = 6.5$ cm

$QP = 4$ cm

19.



$\angle OQP = \angle OPQ = 30°$

$\angle OQT = 90°$ (Angle between radius and tangent)

$\angle TQP = \angle OQT - \angle OQP$

$= 90° - 30° = 60°$

20.

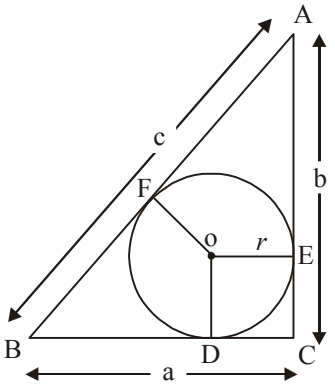
$AP = AR = 4$ cm

$CR = CQ = (9 - 4)$ cm $= 5$ cm

Semi perimeter $= \frac{1}{2}[AC + AB + BC]$

$= \frac{1}{2}[9 + 10 + 11] = 15$ cm

21.

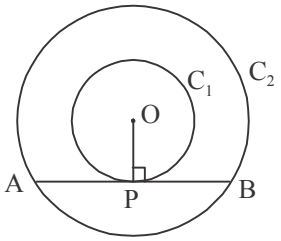


$$b - r = AF, \quad a - r = BF$$

or, $$AB = c = AF + BF = b - r + a - r$$

This gives, $$r = \frac{a + b - c}{2}$$

23. Join OP



AB is tangent to circle C_1 at P and OP is radius

$$OP \perp AB$$

AB is chord of circle C_2 and OP $\perp$ AB.

Therefore OP is the bisector of the chord AB as the perpendicular from the centre bisects the chord i.e,

$$AP = BP$$

24. $$\angle OAB = 50°$$

$$x + \angle B + \angle OAB = 180°$$

$$x + 90° + 50° = 180°$$

$$x = 40°$$

25. $$AK = KC$$

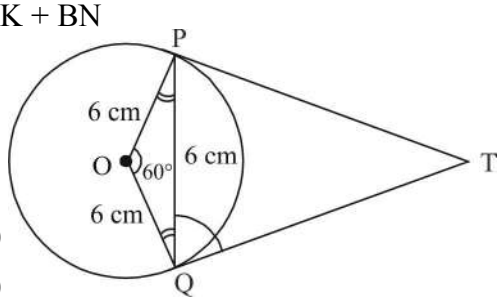
$$BN = NC$$

$\therefore$ $$KN = KC + NC = AK + BN$$

26. $$\angle POQ + \angle PTQ = 180°$$

$$60° + \angle PTQ = 180°$$

$$\angle PTQ = 120°$$



27. $$AC = AF + FC = 10 \text{ cm} \quad \ldots(1)$$

$$AB = AD + DB = 12 \text{ cm} \quad \ldots(2)$$

$$BC = BE + CE = 8 \text{ cm} \quad \ldots(3)$$

$$\begin{bmatrix} BD = BE \\ AD = AF \\ CF = CE \end{bmatrix} \quad \ldots(4)$$

$$AC = AD + FC = 10 \text{ cm} \quad \ldots(5)$$

$$AB = AD + DB = 12 \text{ cm} \quad \ldots(6)$$

$$BC = BD + CF = 8 \text{ cm} \quad \ldots(7)$$

Add (5, 6, 7)

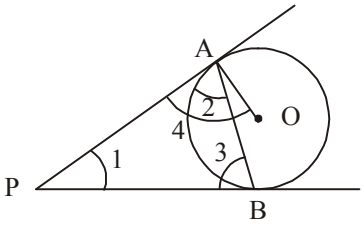
$2(AD + FC + DB) = 30$

$AD + FC + DB = 15$

Substitute values from (1), (2) & (3)

and find. AD = 7 cm, BE = 5 cm, CF = 3 cm.

28. $PA = PB$



So, $\angle 2 = \angle 3 = \frac{1}{2}(180° - \angle 1)$

$\angle 2 = \angle 3 = 90° - \frac{1}{2}\angle 1$

$\angle 4 = 90°$ (Angle between tangent & Radius)

$\angle OAB = \angle 4 - \angle 2$

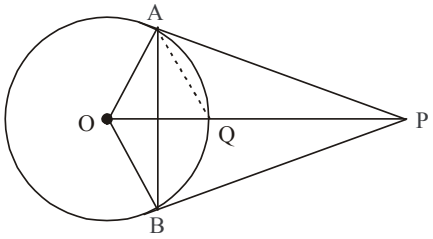
$= 90° - \left(90° - \frac{1}{2}\angle 1\right)$

$\therefore \quad \angle OAB = \frac{1}{2}\angle APB$

$2\angle OAB = \angle APB$

29. $OP = 2r$

$\Rightarrow \quad OQ = QP = r$



Consider ΔAOP in which $OA \perp AP$ and OP is the hypotenuse.

$OQ = AQ = OA$

(Mid point of hypotenuse is equidistance from the vertices).

$\Rightarrow$ OAQ is an equilitateral triangle.

$\Rightarrow \quad \angle AOQ = 60°$

Consider right angled triangle OAP

$\angle AOQ = 60°$

$\angle OAP = 90° \quad \Rightarrow \quad \angle APO = 30°$

$\angle APB = 2\angle APO = 2 \times 30° = 60°$

$PA = PB$ (tangents)

$\Rightarrow$ $\angle PAB = \angle PBA$

$\angle APB = 60°$

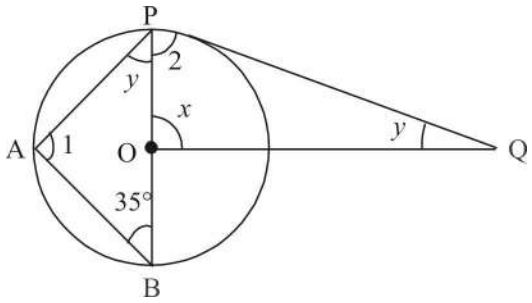
$$\angle PAB = \angle PBA = \frac{180° - 60°}{2} = 60°$$

$\therefore$ Δ ABP is an equilateral triangle.

30. PC = 5 cm

31. 11 cm

32.



In ΔABP, $\angle 1 = 90°$ (Angle in semi-circle)

$\angle 1 + 35° + \angle y = 180°$

$90° + 35° + \angle y = 180°$

$\angle y = 55°$

ΔOPQ, $\angle 2 = 90°$ (Angle between tangent and radius)

$\angle 2 + \angle x + \angle y = 180°$

$90° + \angle x + 55° = 180°$

$\angle x = 35°$

34. AD = 5 cm

35. 25°

38. 24 cm

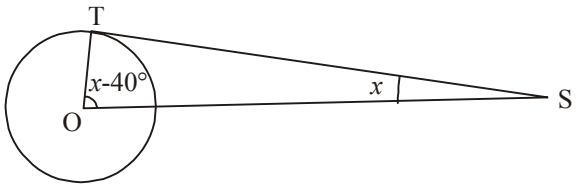
PRACTICE-TEST

CIRCLES

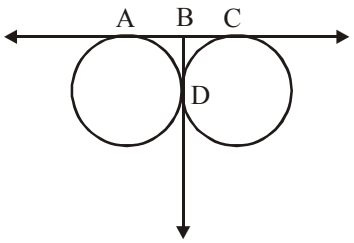
Time : 1 Hr. ***M.M.: 20***

SECTION-A

1. In the given figure find x, where ST is the tangent. **1**

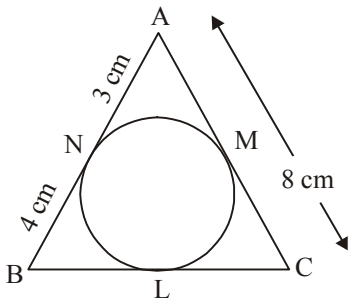


2. In the given figure if AC = 9 cm, find BD. **1**



3. In the given figure, ΔABC is circumscribing a circle, then find the length of BC.

1

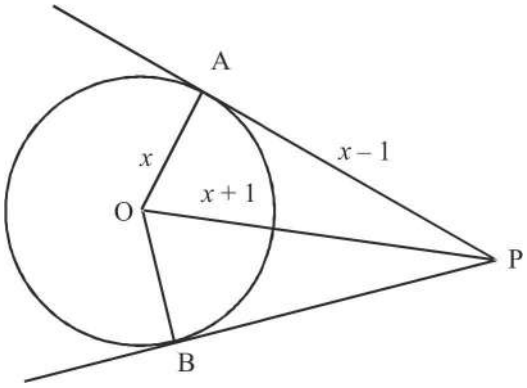


4. From the external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB. **1**

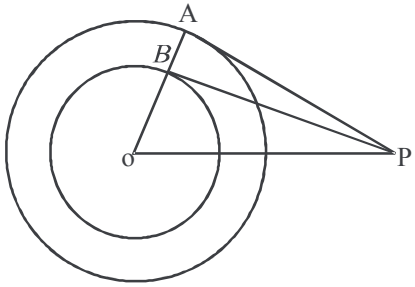
SECTION-B

5. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O is 60° then find the length of OP. **(All India 2017)** 2

6. In the following figure find x. **2**

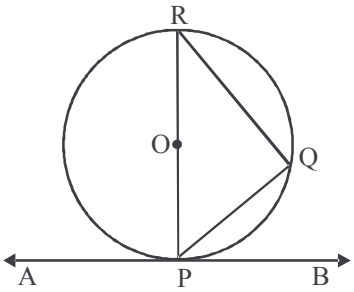


7. Two concentric circle with centre O are of radii 6 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circle as shown in the figure. If AP = 10 cm. Find BP **2**

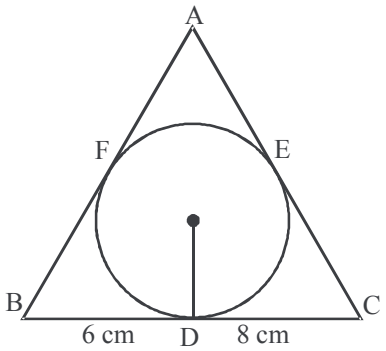


SECTION-C

8. In the given figure, AB is a tangent to a circle with centre O. Prove ∠BPQ = ∠PRQ. **3**



9. In the given figure ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segment BD and DC into which BC is divided by the point of contact D are of length 6 cm and 8 cm respectively, find side AB if the *ar*(ΔABC) = 63 cm² **3**



SECTION-D

10. AB is a diameter of a circle with centre O and AT is a tangent. If ∠AOQ = 58° find ∠ATQ. **4**