CHAPTER

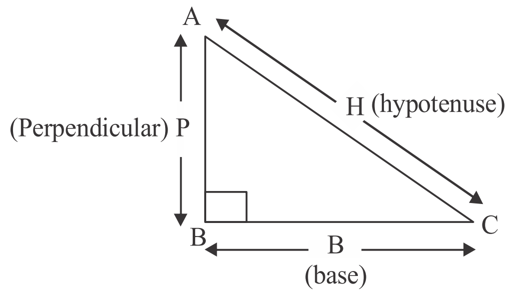
8 Introduction to Trigonometry

KEY POINTS

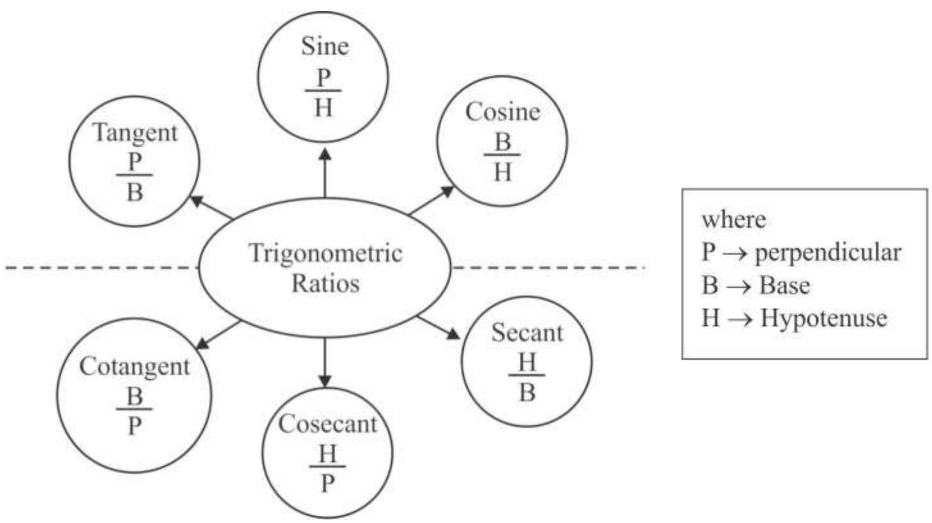
- A branch of mathematics which deals with the problems related to right angled triangles. It is the study of relationship between the sides and angles of a right angled triangle.

  Note : For ∠A, Perpendicular is BC and base is AB.

  For ∠C, Perpendicualr is AB and Base is BC.



Trigonometric Ratios of an acute angle in a right angled triangle express the relationship between the angle and the length of its sides.

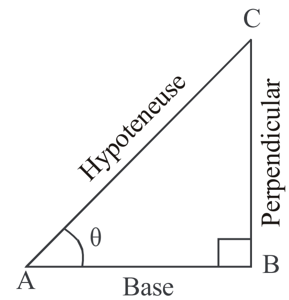


Mind Trick: To learn the relationship of sine, cosine and tangent follow this sentence.

Some **P**eople **H**ave **C**urly **B**rown **H**air **T**hrough **P**roper **B**rushing

$\sin A = \frac{P}{H}$ $\cos A = \frac{B}{H}$ $\tan A = \frac{P}{B}$

1. Trigonometric ratio : In ΔABC, $\angle B = 90°$. For $\angle A$,



$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

2. Opposites

$$\sin\theta = \frac{1}{\text{cosec}\,\theta}, \quad \text{cosec}\,\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

3. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

4. **Identities**

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

5. **Trigonometric ratios of some specific angles**

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

6. **Trigonometric ratios of complimentary angles**

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90° - \theta) = \sec\theta$

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\sin\theta = \cos\theta$, find the value of θ
2. If $\tan\theta = \cot(30° + \theta)$, find the value of θ
3. If $\sin\theta = \cos(\theta - 6°)$, find the value of θ
4. If $\tan\theta = \frac{4}{3}$ then find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$
5. If $3x = \operatorname{cosec}\theta$ and $\frac{3}{x} = \cot\theta$ then find $3\left(x^2 - \frac{1}{x^2}\right)$

6. If $x = a \sin \theta$ and $y = a \cos \theta$ then find the value of $x^2 + y^2$

7. Find the value of cosec 70° – sec 20°

8. Find the value of $9 \sec^2 A - 9 \tan^2 A$

9. Express sec θ in terms of cot θ

10. Find the value of $\cos \theta \cos (90° - \theta) - \sin \theta \sin (90° - \theta)$

11. If $\sin (20° + \theta) = \cos 30°$ then find the value of θ.

12. Find the value of $\dfrac{1+\tan^2 \theta}{1+\cot^2 \theta}$

13. Given $\tan \theta = \dfrac{1}{\sqrt{3}}$, find the value of $\dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$. (CBSE, 2010)

14. Express cosec 48° + tan 88° in terms of trigonometric ratios of angle between 0° and 45°.

15. If $5 \tan \theta - 4 = 0$, then value of $\dfrac{5 \sin \theta - 4 \cos \theta}{5 \sin \theta + 4 \cos \theta}$ is

(a) $\dfrac{5}{3}$ (b) $\dfrac{5}{6}$ (c) 0 (d) $\dfrac{1}{6}$

16. If A and B are complementary angles, then

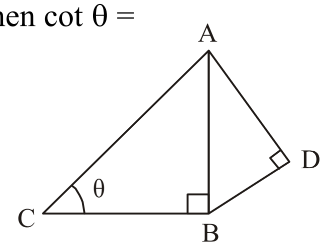
(a) sin A = sin B (b) cos A = cos B

(c) tan A = tan B (d) sec A = cosec B

17. In Fig. if $AD = 4$ cm, BD = 3 cm and CB = 12 cm. then cot θ =

(a) $\dfrac{12}{5}$ (b) $\dfrac{5}{12}$

(c) $\dfrac{13}{12}$ (d) $\dfrac{12}{13}$



18. The value of tan 1° x tan 2° x tan 3° x ... x tan 89° is.

(a) 1 (b) – 1 (c) 0 (d) None of these

19. If θ and 2θ – 45° are acute angles such that $\sin \theta = \cos (2\theta - 45°)$ then tan θ is

(a) 1 (b) – 1 (c) $\sqrt{3}$ (d) $\dfrac{1}{\sqrt{3}}$

SHORT ANSWER TYPE QUESTIONS (1)

Prove that :

20. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

21. $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \tan\theta + \text{Sec}\,\theta$

22. If $x = p\sec\theta + q\tan\theta$ & $y = p\tan\theta + q\sec\theta$ then prove that $x^2 - y^2 = p^2 - q^2$

23. If $7\sin^2\theta + 3\cos^2\theta = 4$ then show that $\tan\theta = \dfrac{1}{\sqrt{3}}$

24. If $\sin(A - B) = \dfrac{1}{2}$, $\cos(A + B) = \dfrac{1}{2}$ then find the value of A and B.

25. Find the value of $\dfrac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°}$.

26. If $3\cot A = 4$, find the value of $\dfrac{\text{cosec}^2 A + 1}{\text{cosec}^2 A - 1}$.

27. If $\tan(3x - 15°) = 1$ then find the value of x.

28. If A, B, C are interior angles of ΔABC, the prove that $\cos ec\left(\dfrac{A+B}{2}\right) = \sec\left(\dfrac{C}{2}\right)$.

(CBSE 2011)

29. In ΔABC, right angled at B, AB = 5 cm and $\angle ACB = 30°$. Find BC and AC.

30. Show that : $\dfrac{1-\sin 60°}{\cos 60°} = 2 - \sqrt{3}$. (CBSE, 2014)

31. Find the value of θ, if $\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 4$, $\theta \le 90°$. (CBSE, 2014)

SHORT ANSWER TYPE QUESTIONS

Prove that :

32. $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1 + \sin A}{\cos A}$

33. $\dfrac{1}{\sec x - \tan x} - \dfrac{1}{\cos x} = \dfrac{1}{\cos x} - \dfrac{1}{\sec x + \tan x}$

34. $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta + 1$ **(CBSE 2019)**

35. $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

36. $\sec A (1 - \sin A)(\sec A + \tan A) = 1$

37. If $\sec\theta = x + \dfrac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\dfrac{1}{2x}$

38. If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$

39. Without using trigonometric table, the value of

$\cot\theta \tan(90° - \theta) - \sec(90° - \theta)\operatorname{cosec}\theta + \sin^2 65° + \sin^2 25° + \sqrt{3} \tan 5° \tan 85°$.

40. **Prove that :** $\dfrac{\cot(90° - \theta)}{\tan\theta} + \dfrac{\operatorname{cosec}(90° - \theta)\sin\theta}{\tan(90° - \theta)} = \sec^2\theta$

41. **Find the value of :**

$\dfrac{\cos^2 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40} + 2 \operatorname{Cosec}^2 58° - 2 \operatorname{Cot} 58° \tan 32° - 4 \tan 13° \tan 37° \tan 77° \tan 45° \tan 53°$.

42. If A, B, C are the angles of ΔABC then prove that $\operatorname{cosec}^2\left(\dfrac{B+C}{2}\right) - \tan^2\dfrac{A}{2} = 1$

43. Find the value of $\sec^2 10° - \cot^2 80° + \dfrac{\sin 15° \cos 75° + \cos 15° \sin 75°}{\cos\theta \sin(90° - \theta) + \sin\theta \cos(90° - \theta)}$.

44. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

45. Evaulate : $\dfrac{\tan^2 60° + 4\cos^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\operatorname{cosec} 30° + \sec 60° - \cot^2 30°}$

46. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$ (CBSE, 2001 C)

Prove that : $a^2 + b^2 = m^2 + n^2$

LONG ANSWER TYPE QUESTIONS

Prove That:

47. $\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\cot^2\theta}\right)=\frac{1}{\sin^2\theta-\sin^4\theta}$

48. $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$

49. $(1+\cot A+\tan A)(\sin A-\cos A)=\sin A\tan A-\cot A\cos A$

50. If $\sin\theta+\cos\theta=m$ and $\sec\theta+\operatorname{cosec}\theta=n$ then show that $n(m^2-1)=2m$

51. find the value of :

$$\frac{\cot(90°-\theta)\tan\theta-\operatorname{cosec}(90°-\theta)\sec\theta}{\sin 12°\ \text{Cos}\,15°\ \sec 78°\ \operatorname{cosec}75°}+\frac{\sin^2(50°+\theta)+\sin^2(40°-\theta)}{\tan 15°\ \tan 37°\ \tan 53°\ \tan 75°}$$

52. Prove that :

$$\frac{1}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta-\cot\theta}$$

53. If $\frac{\cos\alpha}{\cos\beta}=m$ and $\frac{\cos\alpha}{\sin\beta}=n$, then prove that $(m^2+n^2)\ \text{Cos}^2\beta=n^2$

54. **Prove that :**

$$\sec^2\theta-\frac{\sin^2\theta-2\sin^4\theta}{2\cos^4\theta-\cos^2\theta}=1$$

55. $\cot\theta\tan(90°-\theta)-\sec(90°-\theta)\operatorname{cosec}\theta+\sqrt{3}\tan 12°\tan 60°\tan 78°$ find its value.

56. Find the value of —

$$\frac{\sec(90°-\theta)\operatorname{cosec}\theta-\tan(90°-\theta)\cot\theta+\cos^2 25°+\cos^2 65°}{3\tan 27°\tan 63°}$$

57. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$ **(CBSE 2020)**

58. Prove $\frac{\cot A-\cos A}{\cot A+\cos A}=\sec^2 A+\tan^2 A-2\sec A\tan A$ **(CBSE 2020 Basic)**

59. Prove $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ **(CBSE 2020 Basic)**

60. If $\cos(A + B) = \sin(A - B) = \dfrac{1}{2}$, $0 < A + B < 90°$ and $A > B$ then find the value of A and B. **(CBSE 2020 Basic)**

61. If $\tan\theta + \sin\theta = m$, $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$.

(CBSE 2020 Standard)

62. Find $\dfrac{\sec^2(90° - \theta) - \cot^2\theta}{2(\sin^2 25° + \sin^2 65°)} + \dfrac{2\cos^2 60° \tan^2 28° \tan^2 62°}{3(\sec^2 43° - \cot^2 47°)}$

(CBSE 2020 Standard)

63. Prove $\dfrac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \dfrac{1 - \sin\theta}{\cos\theta}$ **(CBSE 2020 Standard)**

64. Evaluate $\left(\dfrac{3\sin 43°}{\cos 47°}\right)^2 - \dfrac{\cos 37° \operatorname{cosec} 53°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°}$

(CBSE 2019)

65. Prove that $\dfrac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \dfrac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$ **(CBSE 2019)**

66. If $4\tan\theta = 3$ then find the value of $\dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}$ **(CBSE 2018)**

67. Prove $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$ **(CBSE 2018)**

ANSWERS AND HINTS

1. 45° **2.** 30°

3. 48° **4.** 7

5. $\dfrac{1}{3}$ **6.** a^2

7. 0 **8.** 9

9. $\sqrt{\dfrac{1+\cot^2\theta}{\cot\theta}}$

10. $0°$

11. $40°$

12. $\tan^2\theta$

13. $\dfrac{1}{2}$

14. $\sec 42° + \cot 2°$

15. (c)

16. (d)

17. (a)

18. (a)

19. (a)

20. LHS = $\sec^2\theta(\sec^2\theta - 1)$

RHS = $\tan^2\theta(\tan^2\theta + 1)$

Use $1 + \tan^2\theta = \sec^2\theta$

21. Relationalise and proceed in LHS

22. Squaring both sides of x and y and subtracting.

23. Divide both sides by $\cos^2\theta$

24. A = 45°, B = 15°

25. 1

26. $\dfrac{17}{8}$

27. 20°

28. Use (A + B + C = 180°)

29. AC = 10, BC = $5\sqrt{3}$, use Pythagoras theorem

30. Substitute values of sin 60° and cos 60° and solve

31. 60°

Note : 32 to 38 use trigonometric identities and prove (based on Ex. 8.4 of NCERT)

39. $\sqrt{3}$

40. Use $\cot(90-\theta) = \tan\theta$, $\text{cosec}(90-\theta) = \sec\theta$, $\tan(90-\theta) = \cot\theta$

41. – 1

42. Use A + B + C = 180°

43. 2

44. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Square both sides and get $1 + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow 2\cos\theta\sin\theta = 2\cos^2\theta - 1$ ...(1)

Now square $(\cos\theta - \sin\theta)^2$ and get

$(\cos\theta - \sin\theta)^2 = 1 - 2\cos\theta\sin\theta$ ...(2)

Substitute (1) in (2)

45. 9.

46. Find m^2 and n^2 and add

Note : Q47 to Q50 Use identities to prove

51. 0

52. Rationalise $\frac{1}{\operatorname{cosec}\theta + \cot\theta}$ in LHS and proceed, use $\frac{1}{\sin\theta} = \operatorname{cosec}\theta$.

Rationalise $\frac{1}{\operatorname{cosec}\theta - \cot\theta}$ on RHS and proceed, use $\frac{1}{\sin\theta} = \operatorname{cosec}\theta$.

53. Find m^2 and n^2 and substitute in LHS.

54. Take common $\sin^2\theta$ in Numerator and $\cos^2\theta$ in Denominator of 2nd term on LHS and replace 1 by $\sin^2\theta + \cos^2\theta$.

55. 0

56. $\frac{2}{3}$

57. $(\sin\theta + \cos\theta) = \sqrt{3}$

square both sides and get value of $\frac{1}{\sin\theta \times \cos\theta}$

Change $\tan\theta + \cot\theta$ into $\sin\theta$ and $\cos\theta$ proceed.

58. Change $\cot A = \frac{\cos A}{\sin A}$, take cos A common from Numerator and Denominator, Rationalise remaining term and change into sec A and tan A.

59. LHS = $\frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$, write 1 = $\sin^2\theta + \cos^2\theta$ and proceed.

60. $\cos(A + B) = \frac{1}{2} = \cos 60°$

$\sin(A - B) = \frac{1}{2} = \sin 30°$

$\Rightarrow \left.\begin{matrix} A+B=60° \\ A-B=30° \end{matrix}\right]$ Solve these equations

A = 45°, B = 15°

61. Find m^2 and n^2 substitute in $m^2 - n^2$ and substitute m and n in $4\sqrt{mn}$

62. $\frac{2}{3}$

63. Refer NCERT

64. (Use complementary form), 8

65. Convert cot θ and cosec θ into sin θ and cos θ
and use $\sin^2\theta = 1 - \cos^2\theta$

66. Divide Numerator and Denominator by cos θ, and use sec θ = $\sqrt{1+\tan^2\theta}$
or use pythagoras theorem and trigonometeric ratios,

Ans. $\frac{13}{11}$

67. Same as Q 59.

PRACTICE-TEST

Introduction to Trigonometry

Time : 1 Hrs. ***M.M.: 20***

SECTION-A

1. If $\sin\theta = \frac{4}{5}$ what is the value of $\cos\theta$. **1**

2. Write the value of $\sin(45° + \theta) - \cos(45° - \theta)$. **1**

3. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$, then the value of $\tan 5\alpha$ is **1**

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 1 (d) 0

4. If $\sin A + \sin^2 A = 1$, then the value of $(\cos^2 A + \cos^4 A)$ is **1**

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

SECTION-B

5. If $5\tan\theta = 4$ then find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ **2**

6. Find the value of $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$ **2**

7. Prove that $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha) = \sec\alpha + \operatorname{cosec}\alpha$ **2**

SECTION-C

8. Prove that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$ **3**

9. Prove that $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$ **3**

SECTION-D

10. Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1-\sin\theta}$. **4**