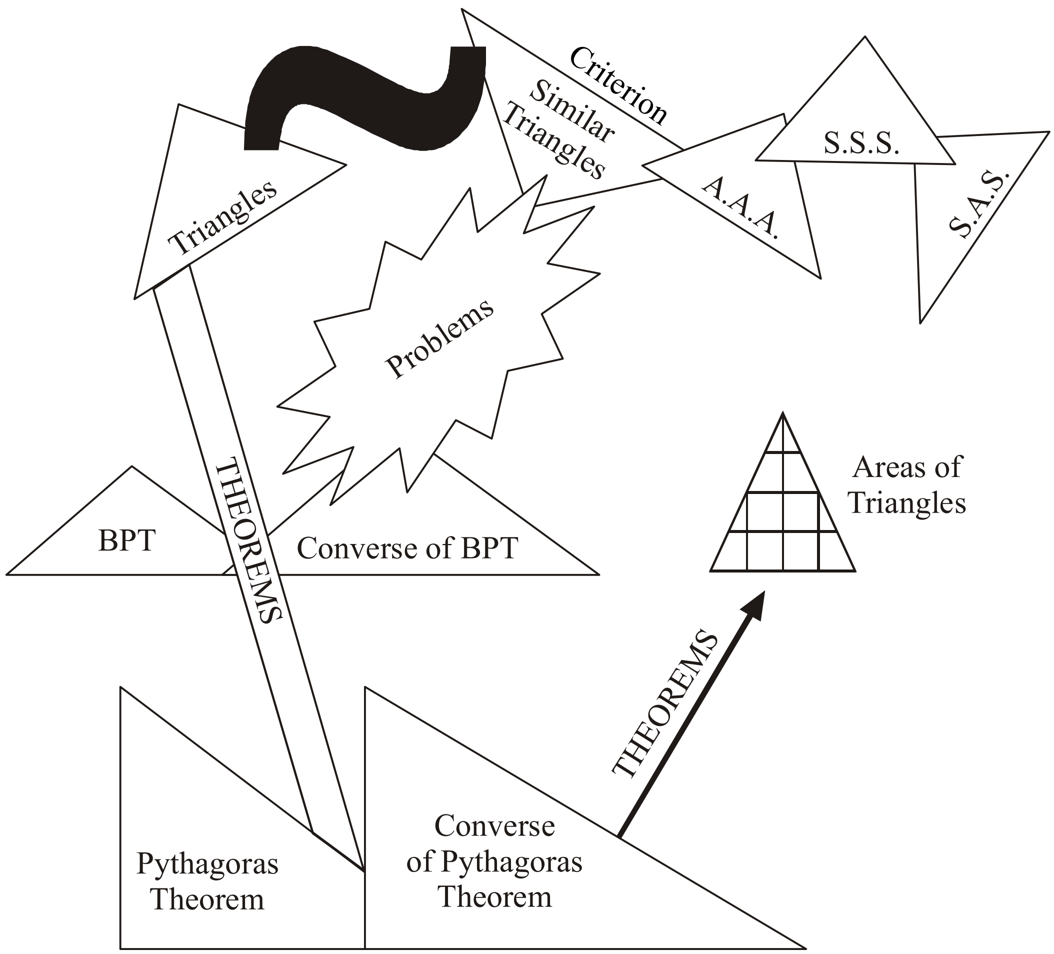
CHAPTER

6 Triangles



Key Points:

1. **Similar Triangles:** Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides ar proportional.

2. **Criteria for Similarity:**

   In ΔABC and ΔDEF

   (*i*) **AAA Similarity:** $\Delta ABC \sim \Delta DEF$ when $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

   (*ii*) **SAS Similarity :**

   $$\Delta ABC \sim \Delta DEF \text{ when } \frac{AB}{DE} = \frac{BC}{EF} \text{ and } \angle B = \angle E$$

   (*iii*) **SSS Similarity :** $\Delta ABC \sim \Delta DEF$, $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$

3. **The proof of the following theorems can be asked in the examination :**

   (*i*) Basic Proportionality Theorem : If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.

   (*ii*) The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

   (*iii*) Pythagoras Theorem: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

   (*iv*) Converse of pythagoras theorem : In a triangle, if the square of one side is equal to the sum of squares of other sides then the angle opposite to the first side is a right angle.

(Theorems without Proof)

(*i*) **Converse of BPT Theorem :** If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (without proof).

(*ii*) If a perpendicular is drawn from the vertex of the right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

VERY SHORT ANSWER TYPE QUESTIONS

1. Fill in the blanks :

(*i*) All equilateral triangles are __________ .

(*ii*) If ΔABC ~ ΔFED, then $\frac{AB}{\;} = \frac{\;}{ED}$.

(*iii*) Circles with equal radii are _________ .

(*iv*) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the _________ ratio.

(*v*) In __________ triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(*vi*) If two triangles are similar, their corresponding sides are __________.

(CBSE 2020)

(*vii*) In ΔABC, AB = $6\sqrt{3}$, AC = 12 cm and BC = 6 cm, then ∠B = _______ .

(*viii*) Let ΔABC ~ ΔDEF and their areas be respectively 81 cm^2 and 144 cm^2. If EF = 24 cm, then length of side BC is _________ cm.

2. State True or False :

(*i*) All the similar figures are always congruent.

(*ii*) The Basic Proportionality Theorem was given by Pythagoras.

(*iii*) The mid-point theorem can be proved by Basic Proprotionality Theorem.

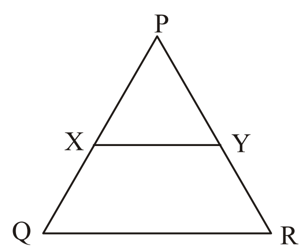
(*iv*) Pythagoras Theorem is valid for right angled triangle.

(*v*) If the sides of two similar triangles are in the ratio 4 : 9, then the areas of these triangles are in the ratio 16 : 81.

3. **Match the following :**

Column I	**Column II**
(*a*) If corresponding angles are equal in two triangles, then the two triangles are similar.	(*i*) SAS similarity criterion
(*b*) If sides of one triangle are proportional to the sides of the other triangle, then the two triangles are similar.	(*ii*) ASA similarity criterion
(*c*) If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.	(*iii*) AAA similarity criterion
	(*iv*) SSS similarity criterion

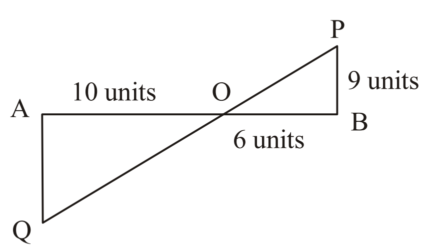
4. In the following figure, XY || QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, then



(*a*) $XY = QR$ (*b*) $XY = \frac{1}{3}QR$

(*c*) $XY^2 = QR^2$ (*d*) $XY = \frac{1}{2}QR$

5. In the following figure, QA ⊥ AB and PB ⊥ AB, then AQ is



(*a*) 15 units (*b*) 8 units

(*c*) 5 units (*d*) 9 units

6. The ratio of areas of two similar triangles is equal to the

(*a*) ratio of their corresponding sides.

(*b*) ratio of their corresponding altitudes.

(*c*) ratio of the square of their corresponding sides.

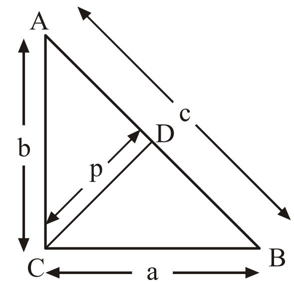
(*d*) ratio of their perimeter.

7. The areas of two similar triangles are 144 cm^2 and 81 cm^2. If one median of the first triangle is 16 cm, length of corresponding median of the second triangle is

(*a*) 9 cm (*b*) 27 cm

(*c*) 12 cm (*d*) 16 cm

8. In a right triangle ABC, in which $\angle C = 90°$ amd CD $\perp$ AB. If BC = a, CA = b, AB = c and CD = p, then



(*a*) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (*b*) $\frac{1}{p^2} \neq \frac{1}{a^2} + \frac{1}{b^2}$

(*c*) $\frac{1}{p^2} < \frac{1}{a^2} + \frac{1}{b^2}$ (*d*) $\frac{1}{p^2} > \frac{1}{a^2} + \frac{1}{b^2}$

9. If $\Delta ABC \sim \Delta DEF$, ar($\Delta$DEF) = 100 cm^2 and $\frac{AB}{DE} = \frac{1}{2}$, then ar($\Delta$ABC) is

(*a*) 50 cm^2 (*b*) 25 cm^2

(*c*) 4 cm^2 (*d*) 200 cm^2

10. If the three sides of a triangle are a, $\sqrt{3}a$ and $\sqrt{2}a$, then the measure of hte angle opposite to longest side is

(*a*) 45° (*b*) 30°

(*c*) 60° (*d*) 90°

11. A vertical pole of length 3 m casts a shadow of 7 m and a tower casts a shadow of 28 m at a time. The height of tower is

(*a*) 10 m (*b*) 12 m

(*c*) 14 m (*d*) 16 m

12. The length of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is **(NCERT Exempler)**

(*a*) 9 cm (*b*) 10 cm

(*c*) 8 cm (*d*) 20 cm

13. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true? **(NCERT Exempler)**

(*a*) BC.EF = AC.FD (*b*) AB.EF = AC.DE

(*c*) BC.DE = AB.EF (*d*) BC.DE = AB.FD

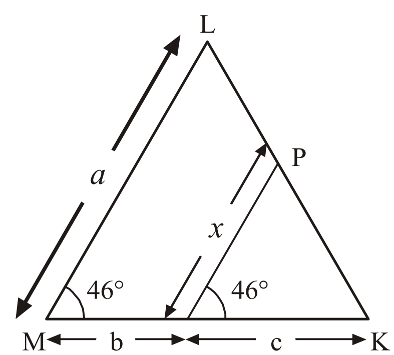
14. Write the statement of pythagoras theorem.

15. Write the statement of Basic Proportionality Theorem.

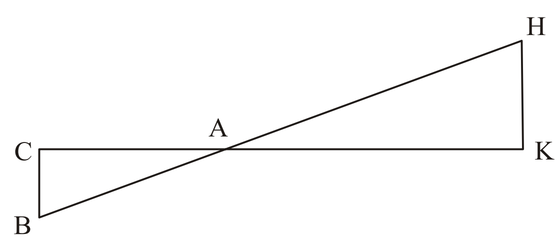
16. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle?

17. If $\Delta ABC \sim \Delta QRP$, $\frac{\text{Area }(\Delta\, ABC)}{\text{Area }(\Delta\, PQR)} = \frac{9}{4}$, AB = 18 cm, BC = 15 cm, then find the length of PR. **(CBSE 2018)**

18. In the given Fig., $\angle M = \angle N = 46°$, Express x in terms of a, b and c.



19. In the given Fig. $\Delta AHK \sim \Delta ABC$. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC. **(CBSE 2010)**



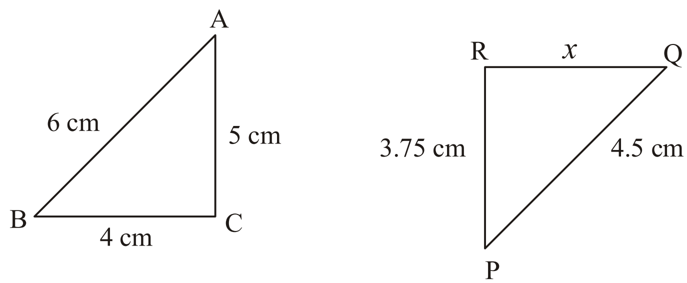
20. It is given that $\Delta DEF \sim \Delta RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$?

21. If the corresponding Medians of two similar triangles are in the ratio 5 : 7. Then find the ratio of their sides.

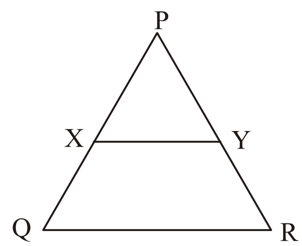
22. An aeroplane leaves an airport and flies due west at a speed of 2100 km/hr. At the same time, another aeroplane leaves the same place at airport and flies due south at a speed of 2000 km/hr. How far apart will be the two planes after 1 hour?

23. The areas of two similar triangles ΔABC and ΔDEF are 225 cm^2 and 81 cm^2 respectively. If the longest side of the larger triangle ΔABC be 30 cm, find the longest side of the smaller triangle DEF.

24. In the given figure, if ΔABC ~ ΔPQR, find the value of x?

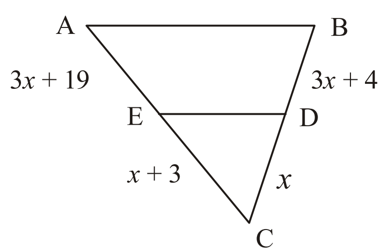


25. In the given figure, XY || QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, find XY : QR.



26. In the given figure, find the value of x which will make DE || AB ?

(NCERT Exempler)



27. If ΔABC ~ ΔDEF, BC = 3EF and ar (ΔABC) = 117cm^2 find area (ΔDEF).

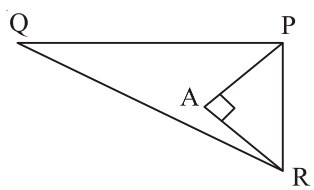
28. If ΔABC and ΔDEF are similar triangles such that ∠A = 45° and ∠F = 56°, then find the value of ∠C.

29. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then find the ratio of their corresponding attitudes.

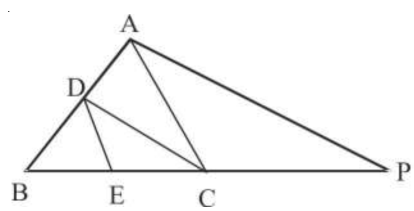
30. It is given that ΔABC ~ ΔPQR with $\frac{BC}{QR} = \frac{1}{3}$, then find the value of $\frac{ar(\Delta PRQ)}{ar(\Delta ACB)}$.

SHORT ANSWER TYPE QUESTIONS-I

31. In the given Fig. PQ = 24 cm, QR = 26 cm, ∠PAR = 90°, PA = 6 cm and AR = 8 cm, find ∠QPR.

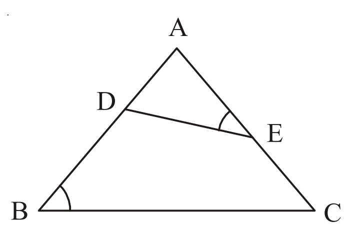


32. In the given Fig., DE || AC and DC || AP Prove that $\frac{BE}{EC} = \frac{BC}{CP}$ **(CBSE 2020)**

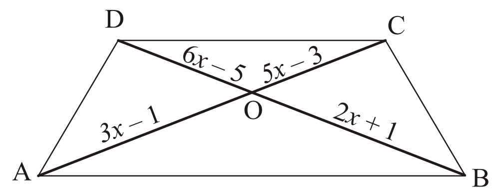


33. In ΔABC, AD ⊥ BC such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled triangle.

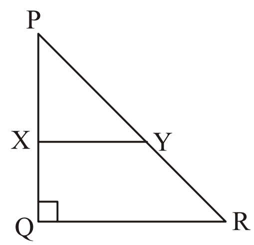
34. In the given Fig., D and E are points on sides AB and CA of ΔABC such that ΔB = ∠AED. Show that ΔABC ~ ΔAED.



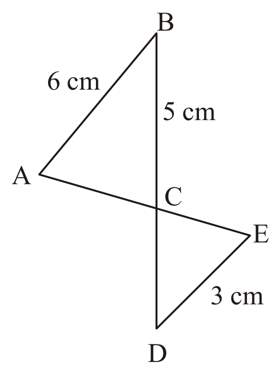
35. In the given fig., AB || DC and diagonals AC and BD intersects at O. If OA = $3x - 1$ and OB = $2x + 1$, OC = $5x - 3$ and OD = $6x - 5$, find the value of x.



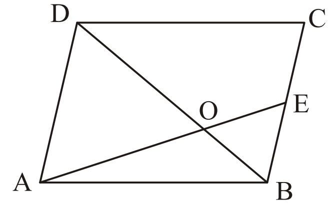
36. In the given Fig. PQR is a triangle, right angled at Q. If XY || QR, PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2. Calculate the lengths of PR and QR.



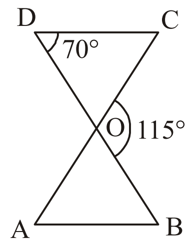
37. In the given figure, AB || DE. Find the length of CD.



38. In the given figure, ABCD is a parallelogram. AE divides the line segment BD in the ratio 1 : 2. If BE = 1.5 cm find BC.



39. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115°$ and $\angle CDO = 70°$. Find, (*i*) $\angle DOC$, (*ii*) $\angle DCO$, (*iii*) $\angle OAB$, (*iv*) $\angle OBA$.

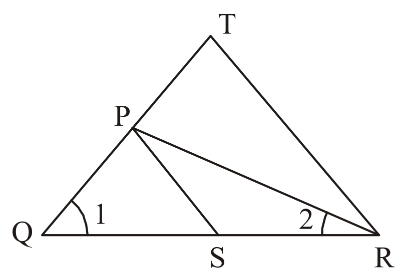


40. Perimeter of two equilateral triangles ΔABC and ΔPQR are 144 m and 96 m, Find ar (ΔABC) : ar (ΔPQR).

SHORT ANSWER TYPE QUESTIONS-II

41. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ then prove that $\Delta PQS \sim \Delta TQR$.

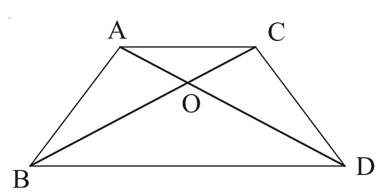
(NCERT)



42. In equilateral ΔABC, $AD \perp BC$. Prove that $3BC^2 = 4AD^2$.

43. In ΔABC, $\angle ACB = 90°$ and $CD \perp AB$. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$. **(HOTS)**

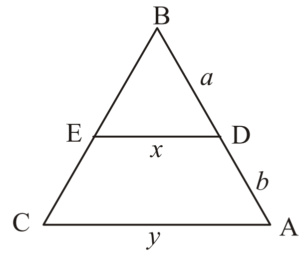
44. In the adjoining figure ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AO}{DO}$. **(CBSE 2020)**



45. If AD and PS are medians of ΔABC and ΔPQR respectively where $\Delta ABC \sim \Delta PQR$, Prove that $\frac{AB}{PQ} = \frac{AD}{PS}$.

46. In the given figure, DE || AC. Which of the following is correct?

$x = \frac{a+b}{ay}$ or $x = \frac{ay}{a+b}$



47. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

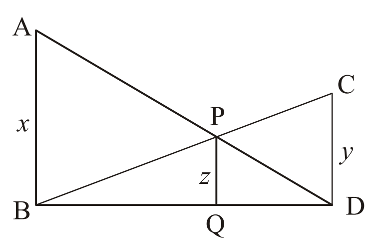
(NCERT, CBSE 2019, 2020)

48. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, find how far she is away from the base of the pole.

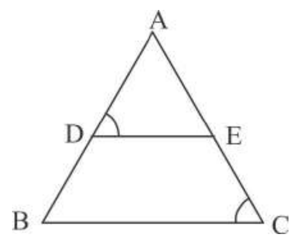
(NCERT Exempler)

49. Two poles of height a metrs and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

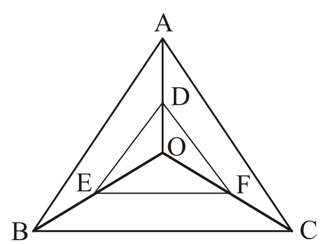
50. In the given figure AB || PQ || CD, AB = x, CD = y and PQ = z. Prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



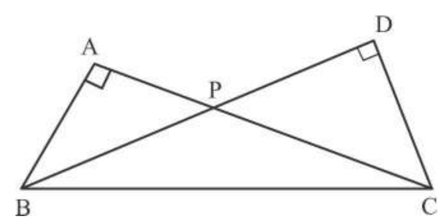
51. In the given figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$. Prove that ΔBAC is an isoscles triangle. **(CBSE 2020)**



52. In the figure, a point O inside ΔABC is joined to its vertices. From a point D on AO, DE is drawn parallel to AB and from a point E on BO, EF is drawn parallel to BC. Prove that DF || AC.



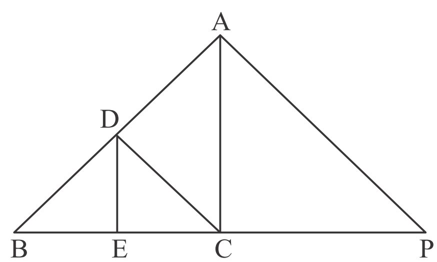
53. Two triangles ΔBAC and ΔBDC, right angled at A and D respectively are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P. PRove that $AP \times PC = DP \times PB$. **(CBSE 2019)**



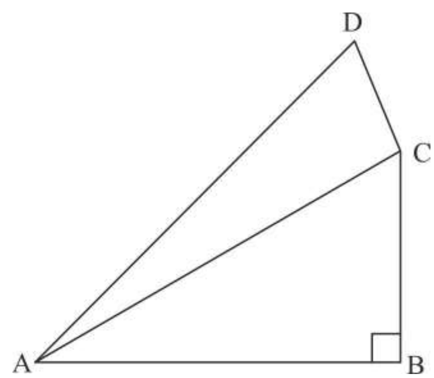
54. Hypotenuse of a right triangle is 25 cm and out of the remaining two sides, one is larger than the other by 5 cm, find the length of the other two sides.

(NCERT Exempler)

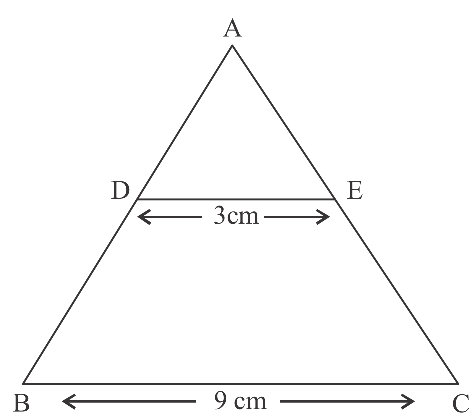
55. In the given figure DE ∥ AC and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that DC ∥ AP.



56. In a quadrilateral ABCD, ∠B = 90°, $AD^2 = AB^2 + BC^2 + CD^2$. Prove that ∠ACD = 90°.



57. In the given figure, DE ∥ BC, DE = 3 cm, BC = 9 cm and ar (ΔADE) = 30 cm^2. Find ar (BCED).



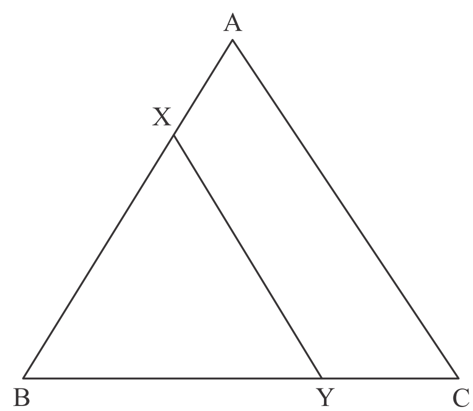
58. In an equilateral ΔABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$. **(NCERT, CBSE 2018, 2020)**

59. In ΔPQR, $PD \perp QR$ such that D lies on QR. If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$ and a, b, c, d are positive units. Prove that $(a + b)(a - b) = (c + d)(c - d)$.

(NCERT Exempler)

60. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squars of their corresponding sides. **(CBSE 2010, 2018, 2019)**

61. In the given figure, the line segment XY is Parallel to AC of ΔABC and it divides the triangle into two parts of equal areas. Prove that $\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$.



62. Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that $\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$.

63. Prove that if in a triangle, the square on one side is equal to the sum of the squares on the other two sides, then the angle opposite to the first side is a right angle.

(CBSE 2019, 2020)

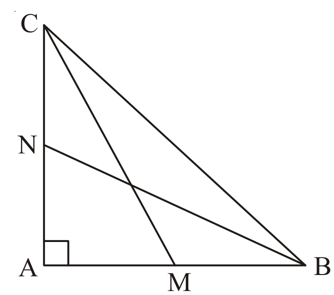
64. Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of the squares of other two sides. **(CBSE 2018, 2019, 2020)**

65. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

(CBSE 2019, 2020)

66. In an Obtuse ΔABC ($\angle B$ is obtuse), AD is perpendicular to CB produced. Then prove that $AC^2 = AB^2 + BC^2 + 2BC \times BD$.

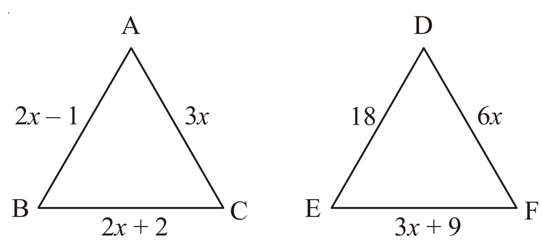
67. In figure BN and CM are medians of a ΔABC right angled at A. Prove that $4(BN^2 + CM^2) = 5\ BC^2$ **(CBSE (C) 2020)**



68. Sides AB and AC and median AD of ΔABC are respectively proportional to sides PQ and PR and median PM of ΔPQR. Show that $\Delta ABC \sim \Delta PQR$.

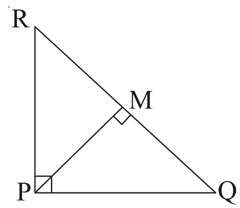
(CBSE 2020)

69. In figure if $\Delta ABC \sim \Delta DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle. **(CBSE 2020)**



70. The Perimeters of two similar triangles are 30 cm and 20 cm repsectively. If one side of the first triangle is 9 cm long. Find the length of the corresponding side of the second triangle. **(CBSE 2020)**

71. In figure ΔPQR is right-angled at P.M is a point on QR such that PM is perpendicular to QR. Show that $PQ^2 = QM \times QR$. **(CBSE 2020)**



ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. (*i*) Similar (*ii*) $\frac{AB}{FE} = \frac{BC}{ED}$ (*iii*) Congruent

(*iv*) Same (*v*) Right (*vi*) Proportional

(*vii*) 90° (*viii*) 18 cm

2. (*i*) False (*ii*) False (*iii*) True

(*iv*) True (*v*) True

3. (*a*) (*iii*) AAA similarity criterion.

(*b*) (*iv*) SSS similarity criterion.

(*c*) (*i*) SAS similarity criterion.

4. (B) $XY = \frac{1}{3}QR$

5. (A) 15 units

6. (C) Ratio of the square of their corresponding sides.

7. (C) 12 cm

8. (A) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

9. (B) 25 cm^2

10. (D) 90°

11. (B) 12 m

12. (B) 10 cm

13. (C) BC.DE = AB.EF

14. See point 3(*iii*) of Key Points

15. See point 3(*i*) of Key Points.

16. No, because $(12)^2 + (16)^2 \neq (18)^2$

17. 10 cm

18. $\Delta KPN \sim \Delta KLM$

$$\frac{x}{a} = \frac{c}{b+c}$$

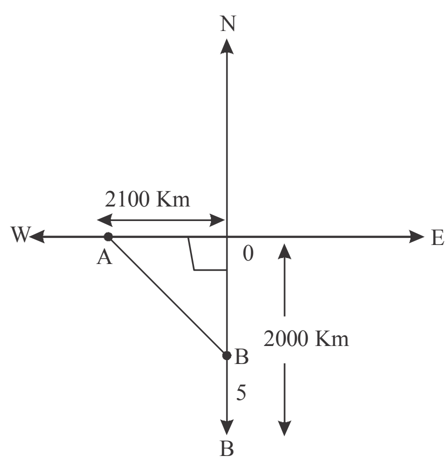
$$x = \frac{ac}{b+c}$$

19. $\frac{AK}{AC} = \frac{HK}{BC} \Rightarrow \frac{10}{AC} = \frac{7}{3.5} \Rightarrow AC = 5 \text{ cm}$

20. $\angle D = \angle R$ (True)

$\angle F = \angle P$ (False)

21. 5 : 7

22. 

$AB = \sqrt{(2100)^2 + (2000)^2} = 2900$ km

23. Let longest side of the ΔDEF be x cm.

$$\frac{225}{81} = \left(\frac{30}{x}\right)^2$$

$x = 18$ cm

24. $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{6}{4.5} = \frac{4}{x} \Rightarrow x = 3cm$

25. $\Delta PXY \sim \Delta PQR$

$\frac{PX}{PQ} = \frac{XY}{QR} = \frac{1}{3}$

$\therefore$ XY : QR = 1 : 3

26. $\frac{x+3}{3x+19} = \frac{x}{3x+4}$ (By B.P.T.)

$x = 2$

27. $\frac{\text{ar (ABC)}}{\text{ar (DEF)}} = \left(\frac{BC}{EF}\right)^2 = \left(\frac{3\,EF}{EF}\right)^2 = \left(\frac{3}{1}\right)^2$

$\frac{117}{\text{ar(DEF)}} = 9 \Rightarrow \text{ar(DEF)} = 13 \text{ cm}^2$

28. $\angle F = \angle C = 56°$

29. 2 : 3

30. 1/9

31. $PR = \sqrt{(6)^2 + (8)^2} = 10$ cm.

As $QR^2 = PQ^2 + PR^2$, therefore $\angle QPR = 90°$.

(By Converse of Pythagoras theorem)

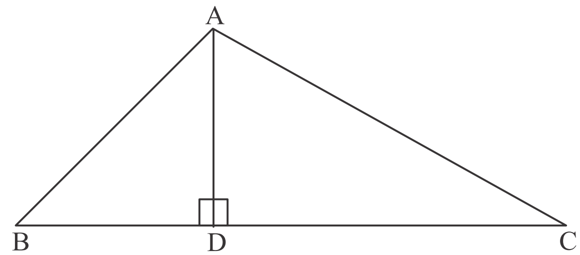
32. $DE \parallel AC, \frac{AD}{DB} = \frac{EC}{BE}$ ...(1) [$\because$ BPT]

$DC \parallel AP, \frac{AD}{DB} = \frac{CP}{BC}$ ...(2) [$\because$ BPT]

From (1) and (2), we get

$\frac{BE}{EC} = \frac{BC}{CP}$

33. In ΔADC, $AD^2 = AC^2 - DC^2$ ...(1)



In ΔADB, $AD^2 = AB^2 - BD^2$ ...(2)

Adding (1) and (2), we have

$2AD^2 = AC^2 + AB^2 - BD^2 - DC^2$

$2AD^2 + BD^2 + DC^2 = AC^2 + AB^2$

$2BD \times CD + BD^2 + DC^2 = AC^2 + AB^2$

$(BD + DC)^2 = AC^2 + AB^2$ (Using $(a + b)^2 = a^2 + b^2 + 2ab$)

$BC^2 = AC^2 + AB^2$

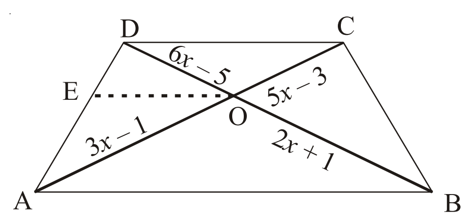
By converse of Pythagoras Theorem, ΔABC is a right angled triangle.

34. $\angle B = \angle AED$ (Given)

$\angle A = \angle A$ (Common)

$\therefore \Delta ABC \sim \Delta AED$ [AA similarity criterion]

35. Draw EO || AB, $\dfrac{DE}{EA} = \dfrac{DO}{OB}$ (In ΔADB) and $\dfrac{DE}{EA} = \dfrac{OC}{OA}$ (In ΔACD)



$$\frac{3x-1}{5x-3} = \frac{2x+1}{6x-5} \quad \Rightarrow \quad x = \frac{1}{2} \text{ or } 2$$

But $x = \dfrac{1}{2}$ is neglected because $(5x - 3)$ get negative value.

So, $x = 2$ is the required value.

36. $\frac{PX}{XQ} = \frac{PY}{YR} \Rightarrow \frac{1}{2} = \frac{4}{YR} \Rightarrow$ YR = 8 cm

$\therefore$ PR = 8 + 4 = 12 cm

QR = $\sqrt{(12)^2 - (6)^2} = 6\sqrt{3}$ cm

37. $\Delta ABC \sim \Delta EDC$ (AA Similarity criterion)

$\frac{6}{3} = \frac{5}{CD}$

CD = 2.5 cm

38. $\Delta BOE \sim \Delta DOA$ (AA Similarity criterion)

$\frac{BO}{DO} = \frac{BE}{DA}$

$\frac{1}{2} = \frac{1.5}{DA}$

DA = 3 cm

BC = DA = 3 cm (Opposite sides of a parallelogram)

39. (*i*) 65°

(*ii*) 45°

(*iii*) 45°

(*iv*) 70°

40. $\frac{ar\ (\Delta\ ABC)}{ar\ (\Delta\ PQR)} = \left(\frac{144}{96}\right)^2 = \frac{9}{4}$

$\therefore$ ar (ΔABC) : ar (ΔPQR) = 9 : 4

41. In ΔPQR, $\angle 1 = \angle 2$

PR = PQ [Opposite sides of equal angles]

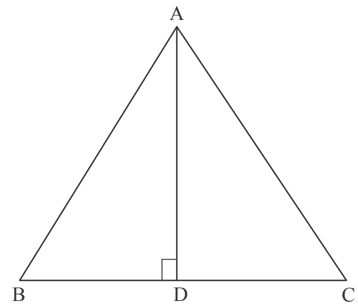
$\therefore \frac{QR}{QS} = \frac{QT}{PQ}$ and $\angle 1 = \angle 1$ (Common)

$\therefore \Delta PQS \sim \Delta TQR$ (SAS Similarity criterion)

42. $\Delta ADB \cong \Delta ADC$

$BD = DC$

$\therefore\ BD = \frac{1}{2} BC$ ...(1)



In right angled ΔADB,

$AB^2 = AD^2 + BD^2$

$BC^2 = AD^2 + \left(\frac{BC}{2}\right)^2$ [$\because$ AB = BC = CA and from (1)]

$3BC^2 = 4AD^2$

43. $\Delta ABC \sim \Delta CBD$

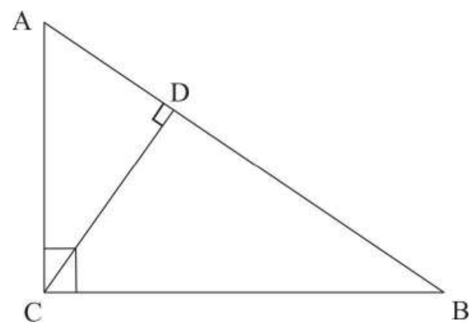
$\therefore\ BC^2 = AB.BD$ ...(1)

$\Delta ABC \sim \Delta ACD$

$\therefore\ AC^2 = AB.AD$ ...(2)

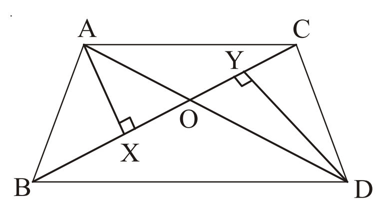
Divide (1) by (2), we get

$\frac{BC^2}{AC^2} = \frac{BD}{AD}$



44. Draw $AX \perp BC$ and $DY \perp BC$

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}\times BC\times AX}{\frac{1}{2}\times BC\times DY} = \frac{AX}{DY} \quad ...(1)$$



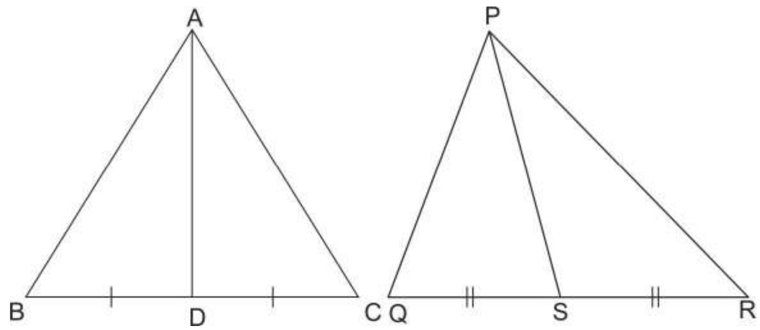
$\Delta AXO \sim \Delta DYO$ [AA similarity criterion]

$\frac{AX}{DY} = \frac{AO}{DO}$ ...(2) (C.P.S.T.)

From (1) and (2), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

45.



As $\Delta ABC \sim \Delta PQR$, Hence $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QS}$

In ΔABD and ΔPQS

$\frac{AB}{PQ} = \frac{BD}{QS}$ and $\angle B = \angle Q$.

$\therefore \Delta ABD \sim \Delta PQS$ (SAS Similarity criterion).

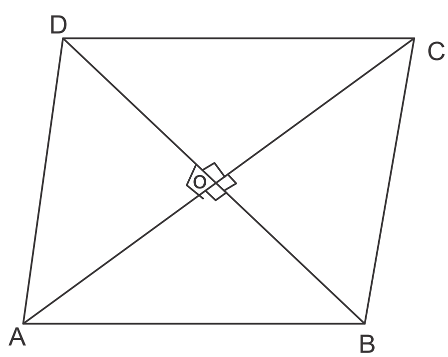
Hence, $\frac{AB}{PQ} = \frac{AD}{PS}$ (C.P.S.T.)

46. $\Delta BED \sim \Delta BCA$

$$\frac{x}{y} = \frac{a}{a+b}$$

$$\Rightarrow x = \frac{ay}{a+b}$$

47.



In right angled ΔAOB, $AB^2 = OA^2 + OB^2$ ...(1)

In right angled ΔBOC, $BC^2 = OB^2 + OC^2$ ...(2)

In right angled ΔCOD, $CD^2 = OC^2 + OD^2$ ...(3)

In right angled ΔDOA, $DA^2 = OD^2 + OA^2$ ...(4)

Adding (1), (2), (3) and (4), we get

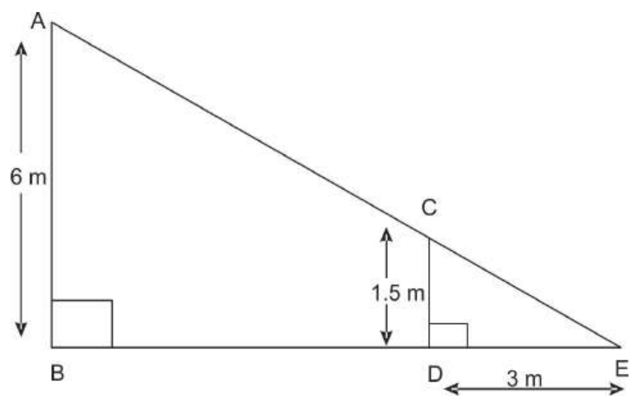
$$AB^2 + BC^2 + CD^2 + DA^2 = 2OA^2 + 2OB^2 + 2OC^2 + 2OD^2$$

$$= 2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2 + 2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2$$

[$\because$ Diagonals of rhombus $\perp$ bisect each other]

$= AC^2 + BD^2$

48. $\Delta ABE \sim \Delta CDE$



$$\frac{AB}{CD} = \frac{BE}{DE}$$

$$\frac{6}{1.5} = \frac{3+BD}{3}$$

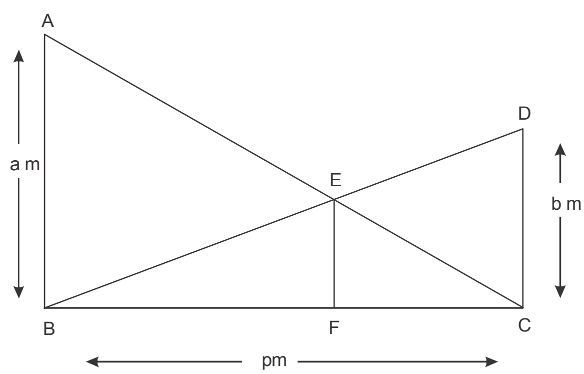
BD = 9m

49. **To prove :** $EF = \dfrac{ab}{a+b}$

Proof : AB || EF || DC

$\Delta EFC \sim \Delta ABC$

$$\frac{EF}{AB} = \frac{FC}{BC} \quad ...(1)$$



$\Delta BFE \sim \Delta BCD$

$$\frac{EF}{CD} = \frac{BF}{BC} \quad ...(2)$$

Adding (1) and (2), we get

$$\frac{EF}{AB} + \frac{EF}{CD} = \frac{FC+BF}{BC}$$

$$EF\left[\frac{1}{AB}+\frac{1}{CD}\right] = \frac{BC}{BC}$$

$$EF\left[\frac{1}{a}+\frac{1}{b}\right] = 1$$

$$EF = \frac{ab}{a+b}$$

50. Same as Q. 48.

51. $\frac{AD}{DB} = \frac{AE}{EC}$

By converse of BPT, DE || BC

$\therefore$ $\angle D = \angle B$ and $\angle E = \angle C$ (Corresponding Angles)

But $\angle D = \angle E$

So, $\angle B = \angle C$

$\therefore$ AB = AC

So, ΔABC is an isosceles triangle.

52. In ΔOAB, $\frac{OD}{DA} = \frac{OE}{EB}$ .... (1) ($\because$ BPT)

In ΔOBC, $\frac{OE}{EB} = \frac{OF}{FC}$ .... (2) ($\because$ BPT)

From (1) and (2), we get

$$\frac{OD}{DA} = \frac{OF}{FC}$$

By converse of BPT, DF || AC.

53. ΔAPB ~ ΔDPC (AA Similarity criterion)

$\frac{AP}{DP} = \frac{PB}{PC}$ ($\because$ C.P.S.T.)

AP.PC = DP.PB

54. Let sides of right angled triangle other than hypotenuse be x cm and $(x + 5)$ cm.

By Pythagoras theorem,

$(x)^2 + (x + 5)^2 = (25)^2$

$x = 15$ or -20

But side is always positive, So, $x = 15$.

$\therefore$ Length of two sides is 15 cm and 20 cm.

55. Same as Q.31.

56. In right angled ΔABC, $AC^2 = AB^2 + BC^2$ ...(1)

Given, $AD^2 = (AB^2 + BC^2) + CD^2$

$\Rightarrow AD^2 = AC^2 + CD^2$ [From (1)]

By converse of Pythagoras theorem, $\angle ACD = 90°$.

57. $\Delta ADE \sim \Delta ABC$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{30}{ar(\Delta ABC)} = \left(\frac{3}{9}\right)^2$$

$\therefore ar(\Delta ABC) = 270 \text{ cm}^2$

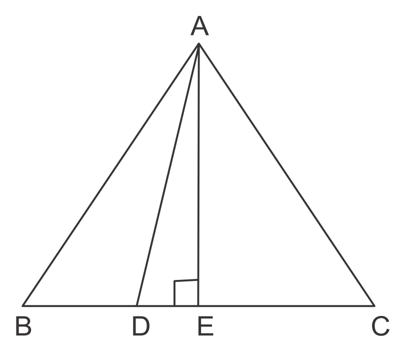
$ar(BCFD) = ar(\Delta ABC) - ar(\Delta ADE)$

$= 270 - 30 = 240 \text{ cm}^2$

58. Draw $AE \perp BC$

$\Delta ABE \cong \Delta ACE$

$\therefore BE = CE \Rightarrow BE = \frac{1}{2}BC$



In right angled ΔAED, $AE^2 = AD^2 - DE^2$ ...(1)

In right angled ΔAEB, $AE^2 = AB^2 - BE^2$ ...(2)

From (1) and (2), we have

$AD^2 - DE^2 = AB^2 - BE^2$

$AD^2 - (BE - BD)^2 = BC^2 - \left(\frac{1}{2}BC\right)^2$ $(\because AB = AC)$

$$AD^2 - \left[\frac{1}{2}BC - \frac{1}{3}BC\right]^2 = BC^2 - \frac{BC^2}{4}$$

$9AD^2 = 7AB^2$

59. In right angled ΔPDQ,

$PD^2 = a^2 - c^2$ ...(1)

In right angled ΔPDR

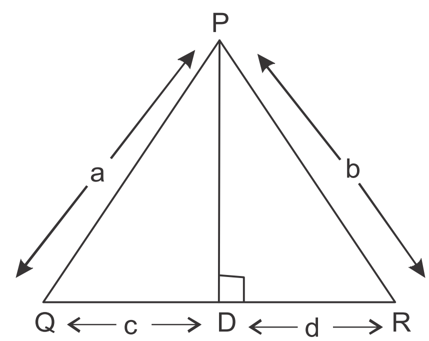
$PD^2 = b^2 - d^2$ ...(2)

From (1) and (2), we have

$a^2 - c^2 = b^2 - d^2$

$a^2 - b^2 = c^2 - d^2$

$(a - b)(a + b) = (c + d)(c - d)$



60. Theorem 6.6 of NCERT.

61. Given, ar ΔBXY = ar AXYC

ar (Δ ABC) = ar Δ BXY + ar AXYC

= 2 ar ΔBXY

$$\therefore \quad \frac{ar(\Delta\ ABC)}{ar(\Delta\ BXY)} = \frac{2}{1}$$

$\Delta ABC \sim \Delta XBY$

$$\left(\frac{AB}{XB}\right)^2 = \frac{ar\ (\Delta ABC)}{ar\ (\Delta BXY)}$$

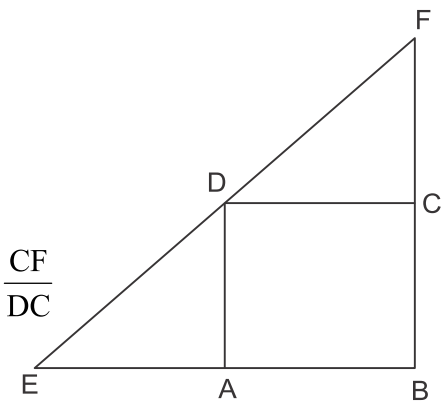
$$\frac{AB}{XB} = \sqrt{2}$$

$$\frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\frac{AB - XB}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$\frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$



62. $\Delta EAD \sim \Delta EBF$

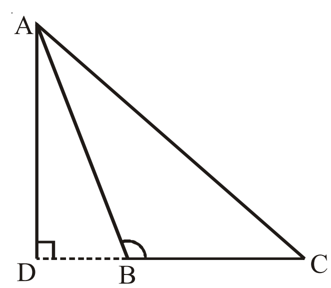
$$\frac{EA}{EB} = \frac{AD}{BF}$$

$$\Rightarrow \frac{BF}{BE} = \frac{AD}{AE} = \frac{BF - AD}{BE - AE} = \frac{BF - BC}{BA} = \frac{CF}{DC}$$

63. Theorem 6.9 of NCERT.

64. Theorem 6.8 of NCERT.

65. Theorem 6.9 of NCERT.



66. In ΔADB

$AB^2 = AD^2 + DB^2$ ...(*i*)

In ΔADC

$AC^2 = AD^2 + DC^2$

$= AD^2 + (DB + BC)^2$

$= AD^2 + DB^2 + BC^2 + 2BC.BD$

$= AB^2 + BC^2 + 2BC.BD$ (Using (*i*))

67. BN is median $\therefore\ AN = CN = \frac{1}{2} AC$

CM is median $\therefore\ AN = MB = \frac{1}{2} AB$

In ΔBAC, $BC^2 = AB^2 + AC^2$ ...(1)

In ΔBNC, $BC^2 = AB^2 + \left(\frac{AC}{2}\right)^2$

$4BN^2 = 4AB^2 + AC^2$ ...(2)

In ΔMAC, $(CM)^2 = (AM)^2 + (AC)^2$

$4CM^2 = AB^2 + 4AC^2$ ...(3)

Adding corresponding sides of (2) and (3)

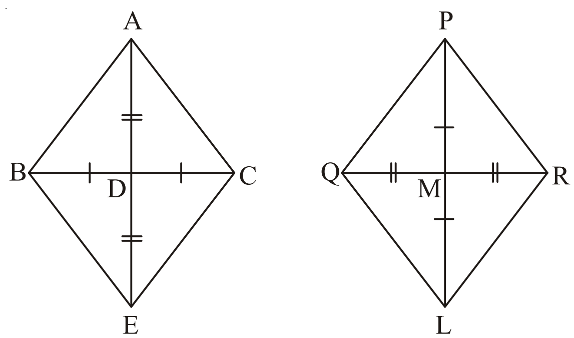
$4(BN^2 + CM^2) = 5AB^2 + 5AC^2$

$4(BN^2 + CM^2) = 5BC^2$

68. In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad ...(1)$$

Extend AD to a point E s.t. AD = DE and PM to point L s.t. PM = ML



$\therefore$ quadrilateral of ABEC and PQLR are parallelogram

($\because$ diagonals bisect each other)

$$\therefore \left.\begin{array}{l} AC = BE, AB = EC \\ PR = QL, PQ = LR \end{array}\right\} \quad ....(2)$$

From (1) and (2)

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM} = \frac{AE}{PL}$$

$\therefore \quad \Delta ABE \sim \Delta PQL$

$\therefore \quad \angle ABE = \angle PQL$ ...(3)

Similarly, $\Delta AEC \sim \Delta PLR$

$\Rightarrow \quad \angle CAE = \angle RPL$ ...(4)

$\Rightarrow \quad \angle CAB = \angle RPQ$ (from 3 and 4)

$\therefore \quad$ In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{ and } \angle CAB = \angle RPQ$$

$\therefore \quad \Delta ABC \sim \Delta PQR$

69. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ $\quad (\because \Delta ABC \sim \Delta DEF)$

$$\frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$$

Solving, we get $x = 5$

$\therefore \quad AB = 9$ cm $\quad BC = 12$ cm $\quad AC = 15$ cm

$\quad DE = 18$ cm $\quad EF = 24$ cm $\quad FD = 30$ cm

70. $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k$$

$\Rightarrow \quad AB = kDE, BC = kEF, AC = kDF$

$\therefore \quad AB + BC + AC = k\,(DE + EF + DF)$

$$\therefore \quad \frac{30}{20} = \frac{9}{x} \quad \Rightarrow \quad x = 6 \text{ cm}$$

71. In ΔPMR, $PR^2 = PM^2 + RM^2$ ...(1)

In ΔPMQ, $PQ^2 = PM^2 + MQ^2$ ...(2)

In ΔPQR, $RQ^2 = RP^2 + PQ^2$ ...(3)

$\Rightarrow \quad RM^2 + MQ^2 + 2RM.MQ = RP^2 + PQ^2$ ...(4) $(\because RQ = RM + MQ)$

Adding (1) and (2), $PR^2 + PQ^2 = 2PM^2 + RM^2 + MQ^2$ ...(5)

From (4) and (5)

$PM^2 = RM.MQ$

PRACTICE-TEST

Triangles

Time : 1 Hrs. ***M.M. : 20***

SECTION - A

1. If sides of two similar triangles are in the ratio of 8:10, then areas of these triangles are in the ratio __________ . **1**

2. If in two triangles ΔABC and ΔPQR, $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$, then **1**

(A) $\Delta PQR \sim \Delta CAB$ (B) $\Delta PQR \sim \Delta ABC$

(C) $\Delta CBA \sim \Delta PQR$ (D) $\Delta BCA \sim \Delta PQR$

3. ΔABC is an isosceles right triangle, right angled at C, then AB^2 = ...

(A) AC^2 (B) $2\ AC^2$ **(CBSE 2020)**

(C) $4\ AC^2$ (D) $3\ AC^2$ **1**

4. A line DE is drawn parallel to base BC of ΔABC, meeting AB in D and AC at E .

If $\frac{AB}{BD} = 4$ and CE = 2 cm, find the length of AE.

SECTION B

5. The lengths of diagonals of a rhombus field are 32 m and 24 m. Find the length of the side of the field. **2**

6. A man goes 24 m towards West and then 10 m towards North. How far is he from the starting point? **2**

7. Using converse of Basic Proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. **2**

SECTION C

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersect CD at F. Show that $\Delta ABE \sim \Delta DCB$. **3**

9. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitude. **3**

SECTION D

10. State and prove Basic Proportionality Theorem. **4**