5 Arithmetic Progression

Point to ponder

1. If a_n is given then
   $d = a_n - a_{n-1}$
2. If S_n is given then $a_n = S_n - S_{n-1}$
3. If a,b,c are in A.P then $2b = a + c$
4. In an AP n^{th} term from end $= l - (n-1)d$, $l \rightarrow$ last tam.
5. 3 terms in A.P if their sum is known
   $a - 3d, a, a + d$
6. 4 such terms are $a - 3d, a - d, a + d, a + d$
7. 5 such terms $a - 2d, a - d, a, a + d, a + 2d$

$S_n =$ sum of n terms of an AP $= \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}[a + l]$

$l \rightarrow$ last term
$l = an$

In other words A.P is a sequence
$a_1, a_2, a_3 \ldots a_n$ such that
$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$
$= \ldots = a_n - a_{n-1} = d$

$d \rightarrow$ is called common difference
$a \rightarrow$ 1st term
$an = a + (n-1)d$ is called general term of an A.P.

A.P.
ARITHMETIC PROGRESSION
A sequence in which the difference of each term from its succeeding term is constant throughout is called A.P

Sequence– It is a set of numbers arranged in a definite order and formed according to some rules

VERY SHORT ANSWER TYPE QUESTIONS

1. Find 5th term of an A.P. whose n^{th} term is $3n - 5$
2. Find the sum of first 10 even numbers.
3. Write the nth term of odd numbers.
4. Write the sum of first n natural numbers.
5. Write the sum of first n even numbers.
6. Find the nth term of the A.P. $- 10, - 15, - 20, - 25, ...$
7. Find the common difference of A.P. $4\frac{1}{9}, 4\frac{2}{9}, 4\frac{1}{3}, ...$
8. Write the common difference of an A.P. whose n^{th} term is $a_n = 3n + 7$
9. What will be the value of $a_8 - a_4$ for the following A.P.
   $4, 9, 14, ..., 254$
10. What is value of a_{16} for the A.P. $- 10, - 12, - 14, - 16, ...$
11. $3, k - 2, 5$ are in A.P. find k.
12. For what value of p, the following terms are three consecutive terms of an A.P.
    $$\frac{4}{5},$$
    $p, 2.$
13. **In the following A.Ps, find the missing terms in the boxes :** (NCERT)
    (a) $2, \square, 26$ (b) $\square, 13, \square, 3$
    (c) $5, \square, \square, 9\frac{1}{2}$ (d) $- 4, \square, \square, \square, \square, 6$
    (e) $\square, 38, \square, \square, \square, - 22$
14. **Multiple Choice Questions:**
    (a) 30th term of the A.P. $10, 7, 4 ....$ is
       (A) 97   (B) 77
       (C) -77 (D) -87
    (b) 11th term of an A.P. $- 3, -\frac{1}{2}, , ...$ is
       (A) 28   (B) 22
       (C) -38 (D) $-48\frac{1}{2}$

Mathematics-X

(c) In an A.P. if $d = -4$, $n = 7$, $a_n = 4$, then a is
    (A) 6   (B) 7
    (C) 120   (D) 28

(d) The first three terms of an A.P. respectively are $3y - 1$, $3y + 5$ and $5y + 1$ then y equals: **(CBSE 2014)**
    (A) −3   (B) 4
    (C) 5   (D) 2

(e) The list of numbers $- 10, - 6, - 2, 2, ...$ is
    (A) An A.P. with $d = - 16$ (B) An A.P. with $d = 4$
    (C) An A.P. with $d = - 4$ (D) Not an A.P.

(f) The 11th term from the last term of an A.P. 10, 7, 4, ..., − 62 is **(NCERT)**
    (A) 25   (B) −32
    (C) 16   (D) 0

(g) The famous mathematician associated with finding the sum of the first 100 natural numbers is
    (A) Pythagoras   (B) Newton
    (C) Gauss   (D) Euclid

(h) What is the common difference of an A.P. in which $a_{18} - a_{14} = 32$?
    (A) 8   (B) − 8
    (C) − 4   (D) 4

(i) The nth term of the A.P. $\left(1 + \sqrt{3}\right)$, $\left(1 + 2\sqrt{3}\right)$, $\left(1 + 3\sqrt{3}\right)$, ... is

    (A) $1 + n\sqrt{3}$ (B) $n + \sqrt{3}$

    (C) $n\left(1 + \sqrt{3}\right)$ (D) $n\sqrt{3}$

(j) The common difference of the A.P. $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$ ...... is

    (A) $\sqrt{2}$ (B) 1

    (C) $2\sqrt{2}$ (D) $-\sqrt{2}$

(k) The first term of an A.P. is p and the common difference is q, then its 10th term is
    (A) $a + 9p$ (B) $p - 9q$
    (C) $p + 9q$ (D) $2p + 9q$

15. Match the following :

Column A	Column B
(a) $a = -18$, $n = 10$, $d = 2$ then a_n of A.P.	(a) $\dfrac{a+c}{2}$
(b) a, b and c are in A.P. then their Arithmetic mean is	(b) 0
(c) If 2, 4, 6, are in A.P. then 4, 8, 12 will also be an	(c) -41
(d) If $a_n = 9 - 5n$ of an A.P. then a_{10} will be	(d) 8
(e) If $d = -2$, $n = 5$ and $a_n = 0$ in A.P. then a is	(e) A.P.

16. State True/False and justify

    (a) 301 is a term of an A.P. 5, 11, 17, 23 ....   (NCERT)

    (b) Difference of mth and n^{th} term of an A.P. $= (m - n)\,d$.

    (c) 2, 5, 9, 14, ... is an A.P.

    (d) Sum of first 20 natural numbers is 410.

    (e) n^{th} term of an A.P. 5, 10, 15, 20 .... n terms and n^{th} term of A.P. 15, 30, 45, 60, ... n terms are same.

SHORT ANSWER TYPE QUESTIONS-I

17. Is 144 a term of the A.P. 3, 7, 11, ... ? Justify your answer.

18. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b^2)$ are in A.P.

19. Which term of the A.P. 5, 15, 25, ... will be 130 more than its 31st term?

20. The first term, common difference and last term of an A.P. are 12, 6 and 252 respectively, Find the sum of all terms of this A.P.

21. Find the sum of first 15 multiples of 8.

22. Is the sequence formed in the following situations an A.P.

    (i) Number of students left in the school auditorium from the total strength of 1000 students when they leave the auditorium in batches of 25.

    (ii) The amount of money in the account every year when Rs. 100 are deposit annually to accumulate at compound interest at 4% per annum.

23. Find the sum of even positive integers between 1 and 200.

24. If $4m + 8$, $2m^2 + 3m + 6$, $3m^2 + 4m + 4$ are three consecutive terms of an A.P. find m.

25. How many terms of the A.P. 22, 20, 18, ... should be taken so that their sum is zero.

26. If 10 times of 10th term is equal to 20 times of 20th term of an A.P. Find its 30th term.

Mathematics-X

27. Solve $1 + 4 + 7 + 10 + ... + x = 287$ **(CBSE 2020)**

28. Find whether (-150) is a term of A.P. 11, 8, 5, 2, ... ?   **(NCERT)**

29. Find how many two digit numbers are divisible by 6?   **(CBSE 2011)**

30. If $\dfrac{1}{x+2}$, $\dfrac{1}{x+3}$ and $\dfrac{1}{x+5}$ are in A.P. find x.   **(CBSE 2011)**

31. Find the middle term of an A.P. $-6, -2, 2, ... 58$.   **(CBSE 2011)**

32. In an A.P. find S_n, where $a_n = 5n - 1$. Hence find the sum of the first 20 terms.

     (CBSE 2011)

33. Which term of A.P. 3, 7, 11, 15 .... is 79? Also find the sum $3 + 7 + 11 + ... + 79$.

     (CBSE 2011C)

34. Which term of the A.P. : 121, 117, 113 ... is the first negative terms ?

     (NCERT)

35. Find the 20th term from the last term of the A.P. 3, 8, 13, ... 253.   **(NCERT)**

SHORT ANSWER TYPE QUESTIONS-II

36. Find the middle terms of the A.P. 7, 13, 19, ..., 241.

37. Find the sum of integers between 10 and 500 which are divisible by 7.

38. The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th term is 97. Find the A.P.

39. If the mth term of an A.P. be $\dfrac{1}{n}$ and n^{th} term be $\dfrac{1}{m}$, show that its $(mn)^{th}$ is 1.

40. If the pth term A.P. is q and the qth term is p, prove that its nth term is $(p + q - n)$.

41. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

42. The sum of 5th and 9th terms of an A.P. is 30. If its 25th term is three times its 8th term, find the A.P.

43. If S_n, the sum of first n terms of an A.P. is given by $S_n = 5n^2 + 3n$, then find its nth term and common difference.

44. Which term of the A.P. 3, 15, 27, 39 .... will be 120 more than its 21st term?

     (CBSE 2018)

45. If S_n, the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$, find the nth term.

(CBSE 2018)

46. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed? **(NCERT)**

47. For what value of n, are the nth term of two A.P.'s 63, 65, 67 .... and 3, 10, 17 ..... are equal ? **(NCERT)**

48. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, then find the nth term of the A.P. **(CBSE 2020)**

(NCERT)

49. If the sum of the first 14 terms of an A.P. is 1050 and its fourth term is 40, find its 20th term. **(CBSE 2020)**

50. Find the sum of odd numbers between 0 and 50. **(NCERT)**

51. If $S_n = 4n - n^2$ in an A.P. find the A.P. **(NCERT)**

52. How many terms of the A.P. 9, 17, 25, ... must be taken to give a sum of 636? **(NCERT)**

LONG ANSWER TYPE QUESTIONS

53. The sum of third and seventh terms of an A.P. is 6 and their product is 8. Find the sum of first 16 terms of the A.P.

54. Determine the A.P. whose 4th term is 18 and the difference of 9th term from the 15th term is 30.

55. The sum of first 9 terms of an A.P. is 162. The ratio of its 6th term to its 13th term is 1:2. Find the first and fifteenth terms of the A.P.

56. The sum of the first 9 terms of an A.P. is 171 and the sum of its first 24 terms is 996. Find the first term and common difference of the A.P. **(CBSE 2020)**

57. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 term is 161. Find the 28th term of this A.P.

58. The sum of first 20 terms of an A.P. is one third of the sum of next 20 term. If first term is 1, find the sum of first 30 terms of this A.P.

59. If the sum of the first four terms of an AP is 40 and the sum of the first fourteen terms of an AP is 280. Find the sum of first n terms of the A.P. **(CBSE 2018)**

60. Ramkali required ₹ 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly savings by ₹ 20 every week. Find wheather she will be able to send her daughter to school after 12 weeks.

(CBSE 2015)

61. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of last 15 terms is 2565. Find the A.P. **(CBSE 2014)**

62. The sum of first n terms of an A.P. is $5n^2 + 3n$. If the mth term is 168, find the value of m. Also find the 20th term of the A.P. **(CBSE 2013)**

63. If the sum of the first seven terms of an A.P. is 49 and the sum of its first 17 terms is 289. Find the sum of first n terms of an A.P. **(CBSE 2016)**

64. If the 4th term of an A.P. is zero, prove that the 25th term of the A.P. is three times its 11th term. **(CBSE 2016)**

65. In an A.P. if $S_5 + S_7 = 167$ and $S_{10} = 235$. Find the A.P., where S_n denotes the sum of its first n terms. **(CBSE 2015)**

66. In an AP prove $S_{12} = 3 (S_8 - S_4)$ where S_n represent the sum of first n terms of an A.P. **(CBSE 2015)**

67. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

68. Find the sum of first 16 terms of an Arithmetic Progression whose 4th and 9th terms are -15 and -30 respectively. **(CBSE 2020)**

ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. $a_n = 3n - 5 \qquad a_5 = 10$

2. $S_n = \dfrac{10}{2} [2 \times 2 + 9 \times 2] = 110$

3. 1, 3, 5, ...
$a_n = 1 + (n - 1)2 = 2n - 1.$

4. $1 + 2 + ... + n = \dfrac{n}{2} [1 + n]$

Mathematics-X

5. $2 + 4 + 6 + ... + 2n = \dfrac{n}{2}[2 + 2n] = n(n + 1)$

6. $a_n = a + (n - 1)d = -5(n + 1)$

7. $d = a_2 - a_1 = \dfrac{1}{9}$

8. $a_1 = 3 + 7 = 10$, $a_2 = 6 + 7 = 13$, $d = 3$

9. $(a + 7d) - (a + 3d) = 4d = 20$

10. $a_{16} = a + 15d = -40$

11. $3, k - 2, 5$ are in A.P.

$$\therefore \quad k - 2 = \dfrac{3 + 5}{2} = 4 \qquad k = 6$$

12. $p = \dfrac{7}{5}$ (same as Q.11)

13. (a) $\boxed{14}$ (b) $\boxed{18}, \boxed{8}$

    (c) $\boxed{6\dfrac{1}{2}}, \boxed{8}$ (d) $\boxed{-2}, \boxed{0}, \boxed{2}, \boxed{4}$

    (e) $\boxed{53}, \boxed{23}, \boxed{8}, \boxed{-7}$

14. (a) C   (b) B

    (c) D   (d) C

    (e) B   (f) B

    (g) C   (h) A

    (i) A   (j) A

    (k) C

15. (a) $\rightarrow$ (b)   (b) $\rightarrow$ (a)

    (c) $\rightarrow$ (e)   (d) $\rightarrow$ (c)

    (e) $\rightarrow$ (d)

16. (a) False, $301 = 5 + (n - 1)\,6$

Solving we get $n = \dfrac{151}{3}$ which is not a natural number.

$\therefore$ 301 is not a term of this A.P.

(b) True $[a + (m - 1)\,d] - [a + (n - 1)\,d] = (m - n)\,d$

Mathematics-X

(c) False $\because a_2 - a_1 = 5 - 2 = 3$

$\quad \because a_3 - a_2 = 9 - 5 = 4$

(d) False $\because S_n = \dfrac{n(n+1)}{2} = \dfrac{20 \times 21}{2} = 210$

(e) False (If $a, b, c, d \ldots$ are in AP then $ka, kb, kc, kd \ldots$ are in AP)

$\quad k \neq 0$, n^{th} term $= k$ times n^{th} term of original A.P. of new A.P.

17. $144 = 3 + (n - 1)\,4$

$\dfrac{141}{4} + 1 = n$ which is not possible

18. $a_1 = (a - b)^2 \quad a_2 = a^2 + b^2 \quad a_3 = (a + b)^2$

$\quad\quad a_2 - a_1 = a^2 + b^2 - (a - b)^2$

$\quad\quad\quad\quad = 2ab$

$\quad\quad a_3 - a_2 = (a + b)^2 - (a^2 + b^2)$

$\quad\quad\quad\quad = 2ab$

$\quad\quad a_2 - a_1 = a_3 - a_2$

$\therefore \quad$ in A.P.

19. Let $a_n = 130 + a_{31}$

Solve to get $n = 44$

Ans. 44th term

20. $a = 12, d = 6, a_n = 252 \quad \Rightarrow \quad n = 41$

Find $S_{41} = 5412$, use $S_n = \dfrac{n}{2}\,[2a + (n - 1)\,d]$

21. $S_{15} = \dfrac{15}{2}\,[2a + 14d]$

where $a = 8, d = 8$

Ans. 960

22. (i) Yes $\rightarrow$ (1000, 975, 950, 925 .....)

(ii) No $\rightarrow$ (104, 108.16, 112.48 .....)

23. $2 + 4 + 6 + \ldots + 198$

$a = 2, d = 2, a_n = 198 \Rightarrow n = 99$

$S_n = \dfrac{n}{2}\,[a + l] = 9900$

24. $b = \dfrac{a+c}{2}$

$\therefore 2m^2 + 3m + 6 = \dfrac{4m + 8 + 3m^2 + 4m + 4}{2}$

Solve to get $m^2 - 2m = 0$

$m = 0, 2$

25. $S_n = 0 \Rightarrow \dfrac{n}{2} [44 + (n-1)(-2)] = 0.$

Solve $n = 23$

26. ATQ $10\, a_{10} = 20\, a_{20}$

$\Rightarrow a_{10} = 2a_{20}$

$a + 9d = 2a + 38d$

$a = -29d \;...(1)$

$a_{30} = a + 29d$

Substitute a from (1)

Ans. $a_{30} = 0$

27. a $= 1, d = 3, a_n = x$

$S_n = 287$

$287 = \dfrac{n}{2}\big[2 \times 1 + (n-1)3\big]$

$\Rightarrow 3n^2 - n - 574 = 0$

$n = 14, \dfrac{-41}{3}$ (rejected)

$\therefore n = 14$

$\therefore x = a_{14} = 40$

28. Let $a_n = -150$

$11 + (n-1)(-3) = -150$

Solve and get n is not a natural number. $\left(n = \dfrac{164}{3} \right)$

$\therefore$ **Ans.** No.

Mathematics-X

29. Two digit numbers divisible by 6 are 12, 18, 24, ... 96.

$a_2 - a_1 = a_3 - a_2 = 6$

$\therefore$ A.P., $a_n = 96 \Rightarrow n = 15$

30. $\dfrac{2}{x+3} = \dfrac{1}{x+2} + \dfrac{1}{x+5}$ $\quad (2b = a + c)$

Solve to get $x = 1$.

31. $a_n = a + (n-1) d$

$58 = -6 + (n-1) 4$

find $n = 17$

Find Middle term using concept of median

$= \left(\dfrac{n+1}{2}\right)^{th}$ term $=$ 9th term

$a_9 = -6 + 8(4) = 26$

32. $a_n = 5n - 1$

Find A.P. $a_1 = 4,\ a_2 = 9,\ a_3 = 14$

4, 9, 14, ...

$a_2 - a_1 = 5 = a_3 - a_2$

$S_n = \dfrac{n}{2}[2a + (n-1)d] = \dfrac{n}{2}[8 + (n-1)\ 5]$

$= \dfrac{n}{2}[5n + 3]$

$S_{20} = \dfrac{20}{2}[100 + 3] = 10 \times 103 = 1030$

33. $79 = 3 + (n-1) 4$

$n = 20$

$S_{20} = \dfrac{20}{2}[3 + 79] = 10[82]$

$S_{20} = 820$

34. Let $a_n < 0$

$121 + (n-1)(-4) < 0$

$121 - 4n + 4 < 0$

$125 < 4n$

$$n > \frac{125}{4}$$

$\therefore n = 32$

32nd term will be first negative term.

35. 20th term from end using $[l - (n-1)\,d]$

$= 253 - 19 \times 5$

$= 253 - 95 = 158$

SHORT ANSWER TYPE QUESTIONS-II

36. Same as Q.27, $\quad n = 40$ Middle terms are a_{20}, a_{21}

Ans. 121, 127

37. Numbers between 10 and 500 which are divisible by 7, 14, 21, 28 ..., 497

Find n, using $a_n = a + (n-1)\,d$, then use $S_n = \dfrac{n}{2}\,[2a + (n-1)\,d]$

Ans. $S_n = 17885.$ $(n = 70)$

38. $a_5 + a_9 = 72$

$a_7 + a_{12} = 97$

Solve these equations to get a and d, $a = 6, d = 5$

$\therefore$ A.P., 6, 11, 16, 21, 26, ...

39. $a_m = \dfrac{1}{n} \quad \Rightarrow a + (m-1)d = \dfrac{1}{n}$

$a_n = \dfrac{1}{m} \quad \Rightarrow a + (n-1)d = \dfrac{1}{m}$

$$(m-n)\,d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

$\therefore d = \dfrac{1}{mn}$, find $a = \dfrac{1}{mn}$

$a_{mn} = a + (mn - 1)\,d$

$\quad = \dfrac{1}{mn} + (mn - 1)\dfrac{1}{mn}$

$a_{mn} = 1.$

40. $a_p = q$, $a_q = p$

Solve to get a and d, $a = q + p - 1$, $d = -1$

$a_n = p + q - n$

41. Numbers divisible by both 2 and 5

$\Rightarrow$ Numbers divisible by 10.

Numbers between 101 and 999 divisible by 2 and 5 both 110, 120, 130, 140, ..., 990.

Use $a_n = 990$ to get $n = 89$.

42. ATQ $a_5 + a_9 = 30$

$a_{25} = 3\ a_8$

Solve to get $a = 3$, $d = 2$

A.P. 3, 5, 7, 9, ...

43. $S_n = 5n^2 + 3n$

Find $a_n = S_n - S_{n-1} = 10\ n - 2$

Use it to get $d = 10$

44. Let $a_n = 120 + a_{21}$

$3 + (n - 1)d = 120 + [3 + 20d]$

$3 + (n - 1)12 = 120 + [3 + 20 \times 12]$

$= 120 + 243$

$(n - 1)12 = 363 - 3 = 360$

$n = 31$

45. $S_n = 3n^2 - 4n$

$a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - [3(n - 1)^2 - 4(n - 1)]$

$= (3n^2 - 4n) - [3n^2 + 3 - 6n - 4n + 4]$

$= -[7 - 6n]$

$a_n = 6n - 7$

46. 23, 21, 19, ... 5

$a_n = a + (n - 1)\ d$

$5 = 23 + (n - 1)\ (-2)$

$n = 10$

47. 63, 65, 67, ...

$a_n = 63 + (n – 1)\,2 = 61 + 2n$

3, 10, 17, ...

$a_n = 3 + (n – 1)\,7$

$= 7n – 4$

$61 + 2n = 7n – 4$

$65 = 5n$

$n = 13$

48. ATQ,

$a_{17} = 5 + 2 \times a_8$

$a + 16d = 5 + 2a + 14d$

$a – 2d = – 5$ 　　　　　...(1)

$a_{11} = a + 10d = 43$ 　　　....(2)

Solving (1) & (2), we get

$a = 3, d = 4$

$\therefore\ a_n = 4n – 1$

49. $S_{14} = 1050, a_4 = 40$

$S_{14} = \dfrac{14}{2}\,[2 \times a + 13d]$

$\dfrac{1050}{7} = 2a + 13d$

Solve $2a + 13d = 150$ and $a + 3d = 40$ to get $a = 10, d = 10$

$a_{20} = a + 19d = 10 + 190 = 200$

50. Odd numbers between 0 to 50

1, 3, 5, 7, ..., 49

$a_n = 49$

$a + (n – 1)d = 49$

$1 + (n – 1)2 = 49$

$n = 25$

$S_n = \dfrac{n}{2}\,[a + l]$

$S_{25} = \dfrac{25}{2}\,[1 + 49] = 25 \times 25 = 625$

Mathematics-X

51. $S_n = 4n - n^2$

$S_1 = a_1 = 4 - 1 = 3$

$S_2 = a_1 + a_2 \Rightarrow a_2 = 1$ A.P. 3, 1, – 1, ...

$S_3 = a_1 + a_2 + a_3 \Rightarrow a_3 - 1$

52. $n = 12, n = -\dfrac{53}{4}$ **(NCERT)**

LONG ANSWER TYPE QUESTIONS

53. $a_3 + a_7 = 6, \quad a_3 \times a_7 = 8$

On Solving

$a = 1, \quad d = \dfrac{1}{2} \quad S_n = 16$

$a = 5, \quad d = \dfrac{-1}{2} \quad S_n = 20$

Ans. 76, 20

54. **ATQ** $a_4 = 18 \quad$ ...(1), $\quad a_{15} - a_9 = 30 \quad$ ...(2)

equation (2) will give $d = 5$

Substitute $d = 5$ in (1) to get $a = 3$

A.P. 3, 8, 13, ...

55. **ATQ** $S_9 = 162 \Rightarrow \dfrac{9}{2} [2a + 8d] = 162 \quad$ ...(1)

ATQ $\dfrac{a_6}{a_{13}} = \dfrac{1}{2}$ solve and get $a = 2d$

Sub $a = 2d$ in (1) to get $d = 3, a = 6$

$a_{15} = a + 14d$

Ans. $a_{15} = 48, a = 6$

56. $S_9 = 171, \quad S_{24} = 996$

$a + 4d = 19, \quad 2a + 23d = 83$

Solve to get,

$d = 3, a = 7$

57. **ATQ** $S_7 = 63,$ ...(1)

Sum of next 7 terms = $S_{14} - S_7 = 161$ ...(2)

Use $S_n = \dfrac{n}{2} [2a + (n - 1) d]$

Solve (1) and (2) to get a and d then find a_{28} using $a_n = a + (n-1)\,d$.

$a = 3, d = 2$

Ans. $a_{28} = 57$

58. ATQ $S_{20} = \dfrac{1}{3}(S_{40} - S_{20})$, $a = 1$

Use $S_n = \dfrac{n}{2}\,[2a + (n-1)\,d]$ and $a = 1$ to find d, $d = 2$
then find S_{30}.
Ans. 900

59. $S_4 = 40 \Rightarrow \dfrac{4}{2}[2a + 3d] = 40$

$S_{14} = 280 \Rightarrow \dfrac{14}{2}[2a + 13d] = 280$

Solve to get $a = 7, d = 2$

Ans. $S_n = n^2 + 6n$ (using $S_n = \dfrac{n}{2}\,[2a + (n-1)\,d]$)

60. $a = 100, d = 20, n = 12$

$S_{12} = \dfrac{12}{2}[200 + 220] = 6 \times 420$

$= 2520 > 2500$

∴ Ram kali will be able to send her daughter to school after 12 weeks.

61. $S_{10} = 210 \Rightarrow 5\,[2a + 9d] = 210$

   $2a + 9d = 42$ ...(1)

   $S_{50} - S_{35} = 2565 \Rightarrow \dfrac{50}{2}[2a + 49d] - \dfrac{35}{2}[2a + 34d] = 2565$

   $\dfrac{15}{2}\,(2a) + d\,[25 \times 49 - 35 \times 17] = 2565$

   $15a + d\,[1225 - 595] = 2565$

   or $\quad 15a + 630\,d = 2565$

   or $\quad 3a + 126d = 513$ ...(2)

   Solve (1) and (2) $d = 4, a = 3$.

62. $S_n = 5n^2 + 3n$
   $S_1 = a_1 = 8$
   $S_2 = a_1 + a_2$

Mathematics-X

$26 = 8 + a_2 \Rightarrow a_2 = 18$

$d = 18 - 8 = 10$

$a_m = 168 \quad \Rightarrow \quad a + (m - 1)d = 168$

$\qquad\qquad\qquad 8 + (m - 1)10 = 168 \Rightarrow m = 17$

$a_{20} = a + 19d = 8 + 190 = 198$

63. $S_7 = 49$, $S_{17} = 289$ (Solve just like Q 53.)

64. $a_4 = 0 \quad \Rightarrow \quad a + 3d = 0 \quad \Rightarrow \quad a = -3d$

$a_{25} = a + 24d = -3d + 24d = 21d$

$a_{11} = a + 10d = -3d + 10d = 7d \qquad \therefore a_{25} = 3a_{11}$

65. Use $S_n = \dfrac{n}{2} [2a + (n - 1)\, d]$

$S_5 + S_7 = 167 \qquad S_{10} = 235$

Solve to get $a = 1$, $d = 5$

A.P. = 1, 6, 11, 16, 21, ...

Solve just like Q.53.

66. L.H.S. $= S_{12} = \dfrac{12}{2} [2a + 11d] = 6 [2a + 11d]$

R.H.S. $= 3\left[\dfrac{8}{2}(2a + 7d) - \dfrac{4}{2}(2a + 3d) \right] = 3[4a + 22d] = 6[2a + 11d]$

$\therefore$ L.H.S. = R.H.S.

67. Four consecutive terms are :

$a - 3d, a - d, a + d, a + 3d$

$d = 8$

$\dfrac{\text{Product of Extremes}}{\text{Product of means}} = \dfrac{(a - 3d)(a + 3d)}{(a - d)(a + d)} = \dfrac{7}{15}$

Put $a = 8$ and solve to get

$\Rightarrow d^2 = 4$

$\qquad d = \pm 2$

$\therefore \quad$ for $a = 8$, $d = 2$ terms are 2, 6, 10, 14

$\qquad$ for $a = 8$, $d = -2$ terms are 14, 10, 6, 2

68. $a_4 = -15$, $a_9 = -30$

$a + 3d = -15$, $a + 8d = -30$

Solve to get $a = -6$, $d = -3$

$S_{16} = -456 \left[S_n = \dfrac{n}{2} \{2a + (n - 1)\, d\} \right]$

Practice Test

Arithmetic Progression

Time: 1 Hr. M.M. : 20

Section-A

1. Find the sum of first 10 natural numbers. **1**

2. What is the common difference of an A.P. $8\dfrac{1}{8}, 8\dfrac{2}{8}, 8\dfrac{3}{8}, \ldots\ldots$ **1**

3. If k, $2k-1$ and $2k+1$ are in A.P. them value of k is **1**

4. The 10th term from the end of the AP 8, 10, 12, ..., 126 is **1**

Section-B

5. How many 2 digit number are there in between 6 and 102 which are divisible by 6. **2**

6. The sum of n terms of an A.P. is $n^2 + 3n$. Find its 20th term. **2**

7. Find the sum $(-5) + (-8) + (-11) + \ldots + (-230)$ **2**

Section-C

8. Find the five terms of an A.P. whose sum is $12\dfrac{1}{2}$ and first and last term ratio is 2 : 3. **3**

9. Find the middle term of an A.P. $20, 16, 12, \ldots\ldots, -176$. **3**

Section-D

10. The sum of three numbers in A.P. is 24 and their product is 440. Find the numbers. **4**