PRACTICE-TEST

Real Number

Time : 1 Hr. ***M.M. : 20***

SECTION A

1. Check whether $17 \times 19 \times 21 \times 23 + 7$ is a composite number. 1

2. In Euclid's Division Lemma, when $a = bq + r$ where a, b are positive integers then what values r can take? 1

3. HCF of x^4y^5 and x^8y^3 . 1

4. LCM of 14 and 122 . 1

SECTION B

5. Show that 9^n can never ends with unit digit zero. 2

6. Without actual division find the type of decimal expansion of $\frac{805}{10500}$, if terminating, after how many places. 2

7. Show that the square of any odd integer is of the form $4m + 1$, for some integer m.

2

SECTION C

8. Prove that $\frac{1}{3-2\sqrt{5}}$ is an irrational number. 3

9. Find the HCF of 36, 96 and 120 by Euclid's Lemma. 3

SECTION D

10. Once a sports goods retailer organized a campaign "Run to remember" to spread awareness about benefits of walking. In that Soham and Baani participated. There was a circular path around a sports field. Soham took 12 minutes to drive one round of the field, while Baani took 18 minutes for the same. Suppose they started at the same point and at the same time and went in the same direction. After how many minutes have they met again at the starting point? 4

CHAPTER

2 Polynomials

KEY POINTS

1. **Polynomial :** If x is a variable, n is a natural number and $a_0, a_1, a_2, a_3, \ldots\ldots\ldots a_n$ are real numbers, then $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots + a_1 x + a_0$, $(a_n \neq 0)$ is called a polynomial in x. The definition of $p(x)$ is n as $a_n \neq 0$.

2. Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.

3. A quadratic polynomial is an algebraic expression of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.

4. Zeros of a polynomial $p(x)$ are precisely the x – coordinates of the points where the graph of $y = p(x)$ intersects the x–axis, *i.e.,* $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$ which means $(x - a)$ is a factor of $p(x)$.

5. A polynomial can have at most the same number of zeros as the degree of the polynomial.

6. (*i*) If one zero of a quadratic polynomial $p(x)$ is negative of the other, then coefficient of x is 0.

   (*ii*) If zeroes of a quadratic polynomial $p(x)$ are reciprocal of each other, then coefficient of x^2 = constant term.

7. Relationship between zeros and coefficients of a polynomial

   If α and β are zeros of $p(x) = ax^2 + bx + c$ $(a \neq 0)$, then

   Sum of zeros = $\alpha + \beta = -\frac{b}{a}$

   Product of zeros = $\alpha\beta = \frac{c}{a}$

8. If α, β are zeros of a quadratic polynomial $p(x)$, then

$p(x) = k\,[x^2 - (\text{sum of zeros})\,x + \text{product of zeros}]$

$\Rightarrow\ p(x) = k\,[x^2 - (\alpha + \beta)x + \alpha\beta\ ]$; where k is any non-zero real number.

9. Graph of linear polynomial $p(x) = ax + b$ is a straight line.

10. Division Algorithm states that given any polynomials $p(x)$ and $g(x)$, there exist polynomial $q(x)$ and $r(x)$ such that:

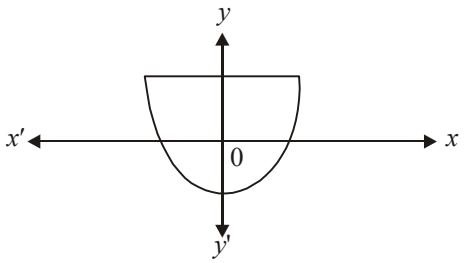
$$p(x) = g(x).\ q(x) + r(x)\ ;\ g(x) \neq 0,$$

[where either $r(x) = 0$ or degree $r(x) <$ degree $g(x)$]

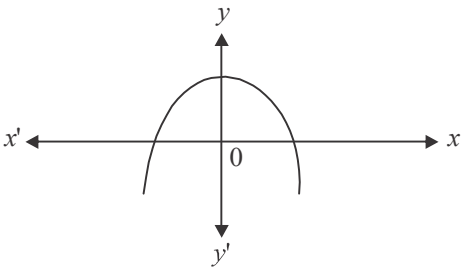
Graph of different types of polynomials:

- **Linear Polynomial :** The graph of a linear polynomial $ax + b$ is a straight line, intersecting x-axis at one point.
- **Quadratic Polynomial:**

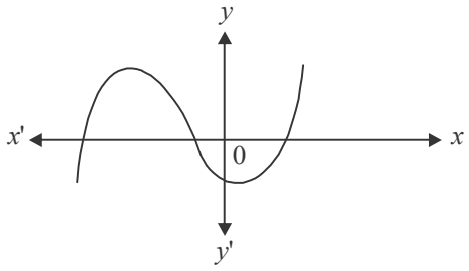
  (*i*) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like U, if $a > 0$ and intersect x-axis at maximum two distinct points.



  (*ii*) Graph of a quodratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like $\cap$, if $a < 0$ and intersect x-axis at maximum two distinct points.



(*iii*) Polynomial and its graph : In general a polynomial $p(x)$ of degree n crosses the x-axis at most n points.



VERY SHORT ANSWER TYPE QUESTIONS

1. If one zero of the polynomial $P(x) = 5x^2 + 13x + K$ is reciprocal of the other, then value of k is

(a) 0 (b) 5 (c) $\frac{1}{6}$ (d) 6

2. If α and β are the zeroes of the polynomial $p(x) = x^2 - p(x + 1) - c$ such that $(\alpha + 1)(\beta + 1) = 0$, the c = _____ .

3. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) – 10 (c) 5 (d) – 5

4. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$

(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

5. What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the resulting polynomial:

(a) 1 (b) 2 (c) 4 (d) 5

6. If α and β are the zeros of the polynomial

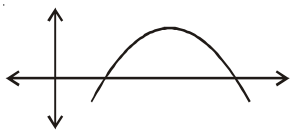
$f(x) = x^2 + x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

7. If a quadratic polynomial $f(x)$ is not factorizable into linear factors, then it has no real zero. (True/False)

8. If a quadratic polynomial $f(x)$ is a square of a linear polynomial, then its two zeros are coincident. (True/False).

9. If $p(x) = x^3 - 2x^2 - x + 2 = (x + 1)(x - 2)(x - d)$ then what is the value of d?

10. The quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ is represented by this graph then a is



(a) Natural no. (b) Whole no. (c) Negative Integer (d) Irrational no.

11. What will be the number of zeros of a linear polynomial $p(x)$ if its graph (*i*) passes through the origin. (*ii*) doesn't intersect or touch x-axis at any point?

12. Find the quadratic polynomial whose zeros are

$\left(5+2\sqrt{3}\right)$ and $\left(5-2\sqrt{3}\right)$

13. If one zero of $p(x) = 4x^2 - (8k^2 - 40k)\,x - 9$ is negative of the other, find values of k.

14. What number should be subtracted to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of polynomial so obtained.

15. How many (*i*) maximum (*ii*) minimum number of zeroes can a quadratic polynomial have?

16. What will be the number of real zeros of the polynomial $x^2 + 1$?

17. If α and β are zeros of polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial where zeros are 2α and 2β (CBSE)

18. If α and $\frac{1}{\alpha}$ are zeros of $4x^2 - 17x + k - 4$, find the value of k.

19. What will be the number of zeros of the polynomials whose graphs are parallel to (*i*) y-axis (ii) x-axis?

20. What will be the number of zeros of the polynomials whose graphs are either touching or intersecting the axis only at the points:

(*i*) (–3, 0), (0, 2) & (3, 0) (*ii*) (0, 4), (0, 0) and (0, –4)

SHORT ANSWER TYPE (I) QUESTIONS

21. For what value of k, $x^2 - 4x + k$ touches x-axis.

22. If the product of zeros of $ax^2 - 6x - 6$ is 4, find the value of a. Hence find the sum of its zeros.

23. If zeros of $x^2 - kx + 6$ are in the ratio 3 : 2, find k.

24. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is reciprocal of the other, find k.

25. If α and β are the zeros of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m. (CBSE)

26. If the sum of squares of zeros of the polynomial $x^2 - 8x + k$ is 40, find the value of k.

27. If α and β are zeros of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

28. What should be added to the polynomial $x^3 - 3x^2 + 6x - 15$, so that it is completely divisible by $x - 3$? (CBSE 2016)

29. If m and n are the zeros of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$. (CBSE, 2012)

30. Find a quadratic polynomial whose zeros are $\frac{3+\sqrt{5}}{5}$ and $\frac{3-\sqrt{5}}{5}$.

(CBSE, 2013)

SHORT ANSWER TYPE (II) QUESTIONS

31. If $(k + y)$ is a factor of each of the polynomials $y^2 + 2y - 15$ and $y^3 + a$, find the values of k and a.

32. Obtain zeros of $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.

33. If $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by $(x^2 + 5)$, remainder comes out to be $(px + q)$, find values of p and q.

34. –5 is one of the zeros of $2x^2 + px - 15$, zeroes of $p(x^2 + x) + k$ are equal to each other. Find the value of k.

35. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of zeros as half of their product.

36. If zeros of the polynomial $ax^2 + bx - c$, $a \neq 0$ are additive inverse of each other then what is the value of b?

37. If α and β are zeros of $x^2 - x - 2$, find a polynomial whose zeros are $(2\alpha + 1)$ and $(2\beta + 1)$

38. Find values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

39. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$?

40. What must be added to $4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $x^2 - 2x - 3$?

LONG ANSWER TYPE QUESTIONS

41. Find all zeros of the polynomial $2x^3 + x^2 - 6x - 3$ if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

42. If $\sqrt{2}$ is a zero of $(6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2})$, find its other zeroes.

43. If two zeros of $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $(2 \pm \sqrt{3})$, find other zeroes.

44. On dividing the polynomial $x^3 - 5x^2 + 6x - 4$ by a polynomial $g(x)$, quotient and remainder are $(x - 3)$ and $(-3x + 5)$ respectively. Find $g(x)$

45. Obtain all zeros of the polynomial $2x^4 - 2x^3 - 7x^2 + 3x + 6$ if two factors of this polynomial are $\left(x \pm \sqrt{\frac{3}{2}}\right)$.

46. If the polynomial $x^4 - 3x^3 - 6x^2 + kx - 16$ is exactly divisible by $x^2 - 3x + 2$, then find the value of k. **(CBSE, 2014)**

47. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $x^2 - 2x + k$, the reaminder is $(x + a)$ then find the value of k and a. **(CBSE)**

48. If α and β are zeros of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeros are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$. **(CBSE)**

49. Find K, so that $x^2 + 2x + K$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeros of the two polynomials: **(Exempler, HOTS)**

50. If $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, then find all the zeros of the polynomial.

51. If zeros of $x^2 - 5kx + 24$ are in the ratio 3 : 2, find k.

52. Form a quadratic polynomial one of whose zero is $2 + \sqrt{5}$ and sum of the zeros is 4.

53. Form a polynomial whose zeros are the reciprocal of the zeros of $p(x) = ax^2 + bx + c$, $a \neq 0$.

54. If $(x + 2)$ is a factor of $x^2 + px + 2q$ and $p + q = 4$ then what are the values of p and q?

55. What should be subtracted from $x^3 - 3x^2 + 6x - 15$, so that it is completely divisible by $(x - 3)$?

56. If $x^2 + 1$ is a factor of $x^4 + x^3 + 8x^2 + ax + b$ then what are the values of a and b.

57. If sum of the zeros of $5x^2 + (p + q + r)x + pqr$ is zero, then find $p^3 + q^3 + r^3$.

58. If the zeros of $x^2 + px + q$ are double in value to the zeros of $2x^2 - 5x - 3$ find p and q.

ANSWERS AND HINTS

1. (b) 5

2. 1

3. (b) –10

4. (d) a = 0, b = –6

5. (b) 2

6. – 1

7. True

8. True

9. 1

10. (c) Negative Integer

11. (i) 1 (ii) 0

12. $x^2 - 10x + 13$

13. $k = 0, 5$

14. (– 2)

15. (i) 2 (ii) 0

16. 0

17. $[3x^2 - 7x - 6]\ k$

18. $k = 8$

19. (i) 1 (ii) 0

20. (i) 2 (ii) 1

21. 4

22. $a = -\frac{3}{2}$, sum of zeroes = – 4

23. – 5, 5

24. 5

25. 6

26. 12

27. $4t^2 + t - 1$

28. On dividing $x^3 - 3x^2 + 6x - 15$ by x – 3, remainder is + 3, hence – 3 must be added to $x^3 - 3x^2 + 6x - 15$.

29. $\frac{m}{n}+\frac{n}{m}=\frac{m^2+n^2}{mn}=\frac{(m+n)^2-2mn}{mn}=\frac{\left(-\frac{11}{3}\right)^2-2\left(-\frac{4}{3}\right)}{-\frac{4}{3}}=-\frac{145}{12}$

30. $\alpha+\beta=\frac{6}{5}, \quad \alpha\beta=\frac{4}{25},$
$25x^2-30x+4$

31. $k=-3, 5$ and $a=-27, 125$

32. $-\frac{2}{\sqrt{3}}, \quad \frac{\sqrt{3}}{4}$

33. $p=2, \ q=3$

34. $\frac{7}{4}$

35. 1

36. $b=0$

37. x^2-4x-5

38. $a=1, b=7$

39. $14x-10$

40. $61x-65$

41. $\sqrt{3}, \ -\sqrt{3}, \ -\frac{1}{2}$

42. $-\frac{\sqrt{2}}{2}, \ \frac{-2\sqrt{2}}{3}$

43. $-5, \ 7$

44. x^2-2x+3

45. $2, \ -1, \ \mp\sqrt{\frac{3}{2}}$

46. $x^2-3x+2=(x-2)(x-1)$
$P(1)=0, K=24.$

47. On dividing $x^4-6x^3+16x^2-25x+10$ by x^2-2x+k we get remainder $(2k-9)x+(10-8k+k^2)$
Given remainder $=x+a$

$$2k-9=1 \quad \Rightarrow \quad k=5$$
$$10-8k+k^2=a \quad \Rightarrow \quad a=10-40+25=-5$$
$$a=-5, \quad k=5$$

48. $x^2-\frac{16}{3}x+\frac{16}{3}$ or $\frac{1}{3}(3x^2-16x+16)$

49. On dividing $2x^4 + x^3 - 14x^2 + 5x + 6$ by $x^2 + 2x + k$

We get $(7k + 21)x + 2k^2 + 8k + 6$ as remainder is zero.

$\Rightarrow$ $7k + 21 = 0$ and $2k^2 + 8k + 6 = 0$

$\Rightarrow$ $k = -3$ and $k = -1$ or -3

$\Rightarrow$ $k = -3$

quotient $= 2x^2 - 3x - (2k + 8)$

$= 2x^2 - 3x - 2$

Zeros of $x^2 + 2x - 3$ are $1, -3$ and $2x^4 + x^3 - 14x^2 + 5x + 6$ are $1, -3, 2, -\frac{1}{2}$

50. $\sqrt{5}, \sqrt{5} + \sqrt{2}, \sqrt{5} - \sqrt{2}$

51. $k = 2$

52. $2 - \sqrt{5}$

53. $k\left[x^2 + \frac{b}{c}x + \frac{a}{c}\right]$

54. $p = 4, q = 0$

55. 3

56. $a = 1, b = 7$

57. Product of the zeros $= 3pqr$

58. $p = -\frac{5}{4}$ and $q = -\frac{3}{8}$

PRACTICE-TEST

Polynomials

Time : 1 Hr. ***M.M. : 20***

SECTION-A

1. If α and β are zeros of a quadratic polynomial $p(x)$, then factorize $p(x)$. **1**

2. If α and β are zeros of $x^2 - x - 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. **1**

3. If one of the zeros of quadratic polynomial $(K-1)x^2 + kx + 1$ is -3 then the value of K is, **1**

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

4. A quadratic polynomial, whose zeros are -3 and 4, is **1**

(*a*) $x^2 - x + 12$ (b) $x^2 + x + 12$

(*c*) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

SECTION-B

5. If α and β are zeros of $x^2 - (k+6)x + 2(2k-1)$. find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$. **2**

6. Find a quadratic polynomial one of whose zeros is $(3+\sqrt{2})$ and the sum of its zeroes is 6. **2**

7. If zeros of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$ then find the value of a. **2**

SECTION-C

8. Find values of a and b if $(x^2 + 1)$ is a factor of the polynomial $x^4 + x^3 + 8x^2 + ax + b$. **3**

9. If truth and lie are zeros of the polynomial $px^2 + qx + r$, $(p \neq 0)$ and zeros are reciprocal to each other, Find the relation between p and r. **3**

SECTION-D

10. On dividing the polynomial $x^3 + 2x^2 + kx + 7$ by $(x - 3)$, remainder comes out to be 25. Find quotient and the value of k. Also find the sum and product of zeros of the quotient so obtained. **4**