SAMPLE PAPER TEST 05 FOR BOARD EXAM 2025

SUBJECT: MATHEMATICS (041) **MAX. MARKS : 80**
CLASS : XII **DURATION: 3 HRS**

General Instructions:
1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. The relation R in the set A = {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)} is
(a) reflexive and symmetric but not transitive
(b) reflexive and transitive but not symmetric
(c) symmetric and transitive but not reflexive
(d) an equivalence relation

2. The function $f(x) = \tan x - x$:
(a) always increases (b) always decreases
(c) never increases (d) sometimes increases and sometimes decreases

3. Solution of the differential equation $x\frac{dy}{dx} + y = xe^x$ is
(a) $xy = e^x (1 - x) + C$ (b) $xy = e^x (x + 1) + C$
(c) $xy = e^y (y - 1) + C$ (d) $xy = e^x (x - 1) + C$

4. The value of $\int_4^5 e^x dx$ is
(a) $e^4 (e + 1)$ (b) $e^4 (e - 1)$ (c) $e^2 (e - 1)$ (d) $e^2 (e + 1)$

5. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is
(a) 1 (b) 2 (c) 3 (d) not defined

6. The co-ordinates of the foot of the perpendicular drawn from the point (2, –3, 4) on the y-axis is
(a) (2, 3, 4) (b) (– 2, – 3, – 4) (c) (0, –3, 0) (d) (2, 0, 4)

7. If $y = \log_e\left(\frac{x^2}{e^2}\right)$ then $\frac{d^2y}{dx^2}$ equals to
(a) $-\frac{1}{x}$ (b) $-\frac{1}{x^2}$ (c) $\frac{2}{x^2}$ (d) $-\frac{2}{x^2}$

8. The function f : R → R given by f(x) = – |x – 1| is
(a) continuous as well as differentiable at x = 1
(b) not continuous but differentiable at x = 1
(c) continuous but not differentiable at x = 1
(d) neither continuous nor differentiable at x = 1

9. $\tan^{-1}3+\tan^{-1}\lambda=\tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ is valid for what values of λ?

(a) $\lambda\in\left(-\frac{1}{3},\frac{1}{3}\right)$ (b) $\lambda>\frac{1}{3}$ (c) $\lambda<\frac{1}{3}$ (d) All real values of λ

10. If the equation of a line AB is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$, find the direction ratios of a line parallel to AB.
(a) 1, 2, 4 (b) 1, 2, –4 (c) 1, –2, –4 (d) 1, –2, 4

11. If y = $\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then value of dy/dx at x = π/6 is:
(a) 1/2 (b) −1/2 (c) 1 (d) -1

12. Area of parallelogram, whose diagonals are along vectors $\hat{i}+2\hat{k}$ and $2\hat{j}-3\hat{k}$ is

(a) $\sqrt{29}$ (b) $-4\hat{i}+3\hat{j}+2\hat{k}$ (c) $\frac{1}{2}\sqrt{29}$ (d) none of these

13. The value of $\int_0^{2\pi}\frac{dx}{e^{\sin x}+1}$ is

(a) π (b) 0 (c) 3 π (d) π/2

14. If $|\vec{a}\times\vec{b}|=4$ and $|\vec{a}.\vec{b}|$ = 2 then $|\vec{a}|^2|\vec{b}|^2$ is equal to
(a) 2 (b) 6 (c) 8 (d) 20

15. $\int_1^3\frac{3\cos(\log x)}{x}dx$ is equal to
(a) sin (log 3) (b) cos (log 3) (c) 1 (d) π/4

16. If y = $\sqrt{a^2-x^2}$, then $y\frac{dy}{dx}$ is:
(a) 0 (b) x (c) -x (d) 1

17. If $\begin{bmatrix}2x+y & 4x\\ 5x-7 & 4x\end{bmatrix}=\begin{bmatrix}7 & 7y-13\\ y & x+6\end{bmatrix}$, then the value of x + y is
(a) x = 3, y = 1 (b) x = 2, y = 3 (c) x = 2, y = 4 (d) x = 3, y = 3

18. The integral $\int\frac{x^9}{(4x^2+1)^6}dx$ is equal to

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+C$ (b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+C$ (c) $\frac{1}{10x}(5)^{-5}+C$ (d) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+C$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, then P(E|F) = 2/3

Reason (R): Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, then P(E|F) = 1/3

20. Assertion(A): $\int_{-3}^{3} (x^3 + 5)dx = 30$

Reason(R): f(x) = x^3 + 5 is an odd function.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Find the values of k so that the function f $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & if \quad x \neq \dfrac{\pi}{2} \\ 3, & if \quad x = \dfrac{\pi}{2} \end{cases}$ is continuous at point $x = \dfrac{\pi}{2}$

OR

If $e^y (x + 1) = 1$, show that $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$

22. Find the value of $\cos^{-1}\left(\cos\dfrac{2\pi}{3}\right) + \sin^{-1}\left(\sin\dfrac{2\pi}{3}\right)$

23. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stich 6 shirts and 4 pants per day while B can stich 10 shirts and 4 pants per day. Form a linear programming problem to minimise the labour cost to produce at least 60 shirts and 52 pants.

24. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

25. Find the general solution of the differential equation $\dfrac{dy}{dx} + 2y = e^{3x}$

OR

Show that differential equation $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$ is a homogenous equation.

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. A random variable x has the following probability distribution.

X	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	$2k^2$	$7k^2 + k$

Determine: (i) k, (ii) P(x < 3); (iii) P(x > 6); (iv) P(0 < x < 3)

27. Find the general solution of differential equation:

$$\left[x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right]ydx=\left[y\sin\left(\frac{y}{x}\right)-\cos\left(\frac{y}{x}\right)\right]xdy$$

OR

Solve the differential equation: $ydx + (x - y^2)\, dy = 0$

28. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

29. If the position vectors of the vertices of a triangle are $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $3\hat{i}+\hat{j}+2\hat{k}$, show that triangle is equilateral.

OR

Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

30. Evaluate: $\int(\sqrt{\cot x}+\sqrt{\tan x})dx$

OR

Evaluate: $\int_1^3 |x^2-2x|\,dx.$

31. Minimise Z = 13x – 15y subject to the constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0$ and $y \ge 0$.

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

OR

Find the area of the region $\{(x, y) : x^2 + y^2 \le 4, x + y \ge 2\}$.

33. Prove that the volume of the largest cone that can be inscribed in a sphere of radius 'a' is 8/27 of the volume of the sphere.

34. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

35. Find the equation of the line which intersects the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ passes through the point (1, 1, 1).

OR

Find the shortest distance between the following lines :

$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1:

Soumya was doing a project related to the average number of hours spent on study by students selected at random. At the end of the survey, she prepared the report related to the data.

Let X denotes the average number of hours spent on study by students.

The probability that X can take the values x, has the following form, where k is some unknown constant

$$P(X=x)=\begin{cases} k, & if \quad x=0 \\ 2k, & if \quad x=1 \\ 3k, & if \quad x=2 \\ 0, & otherwise \end{cases}$$

Based on the above information, answer the following questions:

(i) What is the value of k? [1]

(ii) What is the value P(X = 2)? [1]

(iii) What is the probability that average study time of students is atleast 1 hours. [2]

OR

(iii) Find the mean of the given data. [2]

37. Case-Study 2:

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes.

A = {S, D}, B = {1, 2, 3, 4, 5, 6}

(i) Show that relation R : B ∈ B be defined by R = {(x, y) : y is divisible by x} is reflexive and symmetric but not transitive.
(ii) Let R be a relation on B defined by R = {(1, 2), (2,2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)}. Then R is show that R is neither reflexive nor symmetric nor transitive.

38. Case-Study 3:

Two schools A and B want to award their selected students on the values of Honesty, Hard work and Punctuality. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of ₹ 2200. School B wants to spend ₹ 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school A). The total amount of award for one prize on each value is ₹ 1200.

Using the concept of matrices and determinants, answer the following questions.
(i) What is the award money for Honesty? [1]
(ii) What is the award money for Punctuality? [1]
(iii) What is the award money for Hard work? [1]
(iv) If a matrix P is both symmetric and skew-symmetric, then find |P|. [1]

SAMPLE PAPER TEST 05 FOR BOARD EXAM 2025
(ANSWERS)

SUBJECT: MATHEMATICS (041) **MAX. MARKS : 80**
CLASS : XII **DURATION: 3 HRS**

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5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. The relation R in the set A = {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)} is
(a) reflexive and symmetric but not transitive
(b) reflexive and transitive but not symmetric
(c) symmetric and transitive but not reflexive
(d) an equivalence relation
Ans: (b) reflexive and transitive but not symmetric

2. The function $f(x) = \tan x - x$:
(a) always increases (b) always decreases
(c) never increases (d) sometimes increases and sometimes decreases
Ans: (a) always increases
We have, $f(x) = \tan x - x$
On differentiating with respect to x, we get
$f'(x) = \sec^2 x - 1$
$\Rightarrow f'(x) > 0, \forall x \in R$
So, $f(x)$ always increases.

3. Solution of the differential equation $x\dfrac{dy}{dx} + y = xe^x$ is
(a) $xy = e^x(1 - x) + C$ (b) $xy = e^x(x + 1) + C$
(c) $xy = e^y(y - 1) + C$ (d) $xy = e^x(x - 1) + C$
Ans: (d) $xy = e^x(x - 1) + C$

4. The value of $\int_4^5 e^x dx$ is
(a) $e^4(e + 1)$ (b) $e^4(e - 1)$ (c) $e^2(e - 1)$ (d) $e^2(e + 1)$
Ans: (b) $e^4(e - 1)$

5. The degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^2 = x\sin\left(\dfrac{dy}{dx}\right)$ is
(a) 1 (b) 2 (c) 3 (d) not defined
Ans: (d) not defined

6. The co-ordinates of the foot of the perpendicular drawn from the point (2, –3, 4) on the y-axis is
(a) (2, 3, 4) (b) (– 2, – 3, – 4) (c) (0, –3, 0) (d) (2, 0, 4)
Ans: (c) (0, –3, 0)

7. If $y = \log_e\left(\frac{x^2}{e^2}\right)$ then $\frac{d^2y}{dx^2}$ equals to

(a) $-\frac{1}{x}$ (b) $-\frac{1}{x^2}$ (c) $\frac{2}{x^2}$ (d) $-\frac{2}{x^2}$

Ans: (d) $-\frac{2}{x^2}$

Given $y = \log_e\left(\frac{x^2}{e^2}\right)$

$\Rightarrow \quad y = 2\log_e x - \log_e e^2 = 2\log_e x - 2$

$\Rightarrow \frac{dy}{dx} = \frac{2}{x} \Rightarrow \frac{d^2y}{dx^2} = \frac{-2}{x^2}$

8. The function f : R → R given by f(x) = – |x – 1| is
(a) continuous as well as differentiable at x = 1
(b) not continuous but differentiable at x = 1
(c) continuous but not differentiable at x = 1
(d) neither continuous nor differentiable at x = 1
Ans: (c) continuous but not differentiable at x = 1

9. $\tan^{-1}3 + \tan^{-1}\lambda = \tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ is valid for what values of λ?

(a) $\lambda \in \left(-\frac{1}{3}, \frac{1}{3}\right)$ (b) $\lambda > \frac{1}{3}$ (c) $\lambda < \frac{1}{3}$ (d) All real values of λ

Ans: (c) $\lambda < \frac{1}{3}$

10. If the equation of a line AB is $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, find the direction ratios of a line parallel to AB.
(a) 1, 2, 4 (b) 1, 2, –4 (c) 1, –2, –4 (d) 1, –2, 4
Ans: (d) The direction ratios of line parallel to AB is 1, –2 and 4.

11. If y = $\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then value of dy/dx at x = π/6 is:
(a) 1/2 (b) −1/2 (c) 1 (d) -1
Ans: (b) −1/2

Given: $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}} = \tan^{-1}\sqrt{\frac{1-\cos\left(\frac{x}{2}-x\right)}{1+\cos\left(\frac{x}{2}-x\right)}} = \tan^{-1}\left|\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| = \frac{dy}{dx} = \frac{-1}{2}$

12. Area of parallelogram, whose diagonals are along vectors $\hat{i} + 2\hat{k}$ and $2\hat{j} - 3\hat{k}$ is

(a) $\sqrt{29}$ (b) $-4\hat{i} + 3\hat{j} + 2\hat{k}$ (c) $\frac{1}{2}\sqrt{29}$ (d) none of these

Ans: (c) $\frac{1}{2}\sqrt{29}$

Area of Parallelogram = $\frac{1}{2}\left|(\hat{i}+2\hat{k})\times(2\hat{j}-3\hat{k})\right| = \frac{1}{2}\sqrt{29}$

13. The value of $\int_0^{2\pi}\frac{dx}{e^{\sin x}+1}$ is

(a) π (b) 0 (c) 3 π (d) π/2

Ans: (a) π

$$\text{Let } I=\int_0^{2\pi}\frac{dx}{e^{\sin x}+1} \quad \ldots(\text{i})$$

$$\Rightarrow I=\int_0^{2\pi}\frac{dx}{e^{\sin(2\pi-x)}+1} \qquad \left(\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{2\pi}\frac{dx}{e^{-\sin x}+1} \Rightarrow I=\int_0^{2\pi}\frac{e^{\sin x}}{e^{\sin x}+1}dx$$

Adding (i) and (ii), we get

$$2I=\int^{2\pi}1\cdot dx=2\pi \quad \therefore \quad I=\pi$$

14. If $|\vec{a}\times\vec{b}|=4$ and $|\vec{a}.\vec{b}|=2$ then $|\vec{a}|^2|\vec{b}|^2$ is equal to

(a) 2 (b) 6 (c) 8 (d) 20

Ans: (d) 20

15. $\int_1^3\frac{3\cos(\log x)}{x}dx$ is equal to

(a) sin (log 3) (b) cos (log 3) (c) 1 (d) π/4

Ans: (a) sin (log 3)

16. If y = $\sqrt{a^2-x^2}$, then $y\frac{dy}{dx}$ is:

(a) 0 (b) x (c) -x (d) 1

Ans: (c) -x

We have, $y=\sqrt{a^2-x^2}$ ...(i)

$$\frac{dy}{dx}=\frac{1}{2\sqrt{a^2-x^2}}(0-2x)=\frac{-x}{\sqrt{a^2-x^2}}$$

$$\sqrt{a^2-x^2}\frac{dy}{dx}=-x \Rightarrow y\frac{dy}{dx}=-x \quad \text{(from (i))}$$

17. If $\begin{bmatrix}2x+y & 4x\\ 5x-7 & 4x\end{bmatrix}=\begin{bmatrix}7 & 7y-13\\ y & x+6\end{bmatrix}$, then the value of x + y is

(a) x = 3, y = 1 (b) x = 2, y = 3 (c) x = 2, y = 4 (d) x = 3, y = 3

Ans: (b) x = 2, y = 3

18. The integral $\int\frac{x^9}{(4x^2+1)^6}dx$ is equal to

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+C$ (b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+C$ (c) $\frac{1}{10x}(5)^{-5}+C$ (d) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+C$

Ans: (d) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+C$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A): Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, then P(E|F) = 2/3

Reason (R): Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, then P(E|F) = 1/3

Ans. (c) A is true but R is false.

Given that, P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2

$$P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{0.2}{0.3}=\frac{2}{3}$$

Hence, Assertion is true and Reason is false.

20. Assertion(A): $\int_{-3}^{3}(x^3+5)dx=30$

Reason(R): $f(x) = x^3 + 5$ is an odd function.

Ans. (c) A is true but R is false.

Let $f(x) = x^3 +5$

$f(-x) = (-x)^3 +5 = -x^3 +5$

f(x) is neither even nor odd.

Hence R is false.

$$\int_{-3}^{3}(x^3+5)dx=\int_{-3}^{3}x^3dx+\int_{-3}^{3}5dx=0+5\left[x\right]_{-3}^{3}=30$$

Hence A is true.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Find the values of k so that the function f $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & if \quad x\neq\frac{\pi}{2}\\ 3, & if \quad x=\frac{\pi}{2}\end{cases}$ is continuous at point $x=\frac{\pi}{2}$

Ans: Here, $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & if \quad x\neq\frac{\pi}{2}\\ 3, & if \quad x=\frac{\pi}{2}\end{cases}$

$$LHL = \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{k\cos x}{\pi - 2x}$$

Putting $x = \frac{\pi}{2} - h$ as $x \to \frac{\pi}{2}^-$ when $h \to 0$

$$\therefore \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{h \to 0} \frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = \lim_{h \to 0} \frac{k\sinh}{2h} = \frac{k}{2} \times \lim_{h \to 0} \frac{\sinh}{h} = \frac{k}{2} \times 1 = \frac{k}{2}$$

Since f(x) is continuous at $x = \frac{\pi}{2}$, therefore LHL = $f\left(\frac{\pi}{2}\right)$

Also, $f\left(\frac{\pi}{2}\right)$ = 3 $\Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6$

OR

If $e^y (x + 1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

Ans: Given that $e^y (x + 1) = 1$

$$\Rightarrow e^y = \frac{1}{x+1}$$

Differentiating both sides w.r.t. x, we get

$$e^y \frac{dy}{dx} = -\frac{1}{(x+1)^2} \Rightarrow \frac{1}{x+1}\frac{dy}{dx} = -\frac{1}{(x+1)^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$$

Again, Differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} = \left(-\frac{1}{x+1}\right)^2 = \left(\frac{dy}{dx}\right)^2$$

22. Find the value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Ans:

As $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\frac{\sin 2\pi}{3}\right)$

$$= \frac{2\pi}{3} + \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \frac{2\pi}{3} + \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{2\pi}{3} + \frac{\pi}{3} = \frac{3\pi}{3} = \pi$$

23. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stich 6 shirts and 4 pants per day while B can stich 10 shirts and 4 pants per day. Form a linear programming problem to minimise the labour cost to produce at least 60 shirts and 52 pants.

Ans : Let A works for x days and B works for y days

Then LPP is

To Minimise cost Z = 150x + 200y subject to constraints,

$x \geq 0$, $y \geq 0$

$6x + 10y \geq 60$

4x + 4y ≥ 52

24. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).
Ans: Let A (1, –1, 2) and B (3, 4, – 2) be given points.
Direction ratios of AB are
(3 – 1), {(4 – (–1)}, (–2 –2) i.e., 2, 5, – 4.
Let C (0, 3, 2) and D (3, 5, 6) be given points.
Direction ratios of CD are
(3 – 0), (5 – 3), (6 – 2) i.e., 3, 2, 4.
We know that two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.
∴ 2 × 3 + 5 × 2 + (– 4) × 4 = 6 +10 –16 = 0, which is true.
It will shows that lines AB and CD are perpendicular.

25. Find the general solution of the differential equation $\frac{dy}{dx} + 2y = e^{3x}$

Ans:
Integrating factor is $e^{\int 2dx} = e^{2x}$
∴ Required solution is:

$$y.e^{2x} = \int e^{3x}.e^{2x}dx \Rightarrow y.e^{2x} = \frac{e^{5x}}{5} + C$$

$$\Rightarrow y = \frac{e^{3x}}{5} + Ce^{-2x}$$

OR

Show that differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$ is a homogenous equation.

Ans:

$$x\frac{dy}{dx} = y(\log y - \log x + 1) \Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$$

Let $f(x, y) = \frac{y}{x}\left[\log\left(\frac{y}{x} + 1\right)\right]$

$$f(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x}\left(\log\frac{\lambda y}{\lambda x} + 1\right) = \frac{y}{x}\left(\log\frac{y}{x} + 1\right) = f(x, y)$$

∴ function is homogeneous.
Hence, equation is homogeneous.

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. A random variable x has the following probability distribution.

X	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	$2k^2$	$7k^2 + k$

Determine: (i) k, (ii) P(x < 3); (iii) P(x > 6); (iv) P(0 < x < 3)
Ans: (i) It is known that the sum of a probability distribution of random variable is one
i.e., ∑P(x) = 1, therefore P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 1
$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$
$\Rightarrow 10k^2 + 9k - 1 = 0$
⇒ (k + 1)(10k – 1) = 0
⇒ k = – 1 or k = 1/10
k = – 1 is not possible as the probability of an event is never negative.

∴ k = 1/10

(ii) P(x < 3) = P(0) + P(1) + P(2) = 0 + k + 2k = 3k = 3/10

(iii) P(x > 6) = P(7) = 7k2 + k = 7/100 + 1/10 = (7 + 10)/100 = 17/100

(iv) P(0 < x <3) = P(1) + P(2) = k + 2k = 3k = 3/10

27. Find the general solution of differential equation:

$$\left[x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right]ydx=\left[y\sin\left(\frac{y}{x}\right)-\cos\left(\frac{y}{x}\right)\right]xdy$$

Ans:

Given differential equation is:

$$\left[x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right]ydx = \left[y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right]xdy$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)\left[x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right]}{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)} = \frac{\left(\frac{y}{x}\right)\left[\cos\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)\sin\left(\frac{y}{x}\right)\right]}{\left(\frac{y}{x}\right)\sin\left(\frac{y}{x}\right)-\cos\left(\frac{y}{x}\right)}$$

putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i), we get

$$v+x\frac{dv}{dx} = \frac{v[\cos v+v\sin v]}{v\sin v-\cos v} \Rightarrow x\frac{dv}{dx} = \frac{v[\cos v+v\sin v]}{v\sin v-\cos v}-v = \frac{2v\cos v}{v\sin v-\cos v}$$

$$\Rightarrow \frac{v\sin v-\cos v}{v\cos v}dv = \frac{2}{x}dx$$

Integrating both sides, we get $\int\frac{v\sin v-\cos v}{v\cos v}dv = 2\int\frac{1}{x}dx$

$$\Rightarrow \int\left(\tan v-\frac{1}{v}\right)dv = 2\int\frac{1}{x}dx \Rightarrow \log|\sec v|-\log|v| = 2\log|x| + \log C$$

$$\Rightarrow \log\left|\sec\left(\frac{y}{x}\right)\right|-\log\left|\frac{y}{x}\right| = 2\log|x| + \log C, \Rightarrow \sec\frac{y}{x} = Cxy$$

which is the required general solution.

OR

Solve the differential equation: $ydx + (x - y^2)\, dy = 0$

Ans:

$ydx + (x-y^2)dy = 0$

Reducing the given differential equation to the form $\frac{dx}{dy}+Px=Q$

we get, $\frac{dx}{dy}+\frac{x}{y}=y$

$$\text{I.F.} = e^{\int P\,dy} = e^{\int\frac{1}{y}dy} = e^{\log y} = y$$

The general solution is given by $x.IF = \int Q.IFdy \Rightarrow xy = \int y^2 dy$

$\Rightarrow xy = \frac{y^3}{3}+C$, which is the required general solution

28. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

Ans: Let $y = x^{\sin x} + (\sin x)^{\cos x}$

Let $u = x^{\sin x}$, $v = (\sin x)^{\cos x}$ then we have $y = u + v$

Therefore, $\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$ -------------------- (1)

Now, $u = x^{\sin x}$

Taking logarithm on both sides, we have log u = (sinx) logx.

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\frac{du}{dx}=\sin x\frac{d}{dx}\log x+\log x\frac{d}{dx}(\sin x)=\sin x\frac{1}{x}+\log x\cos x$$

$$\Rightarrow \frac{du}{dx}=u\left[\frac{\sin x}{x}+\log x\cos x\right]=x^{\sin x}\left[\frac{\sin x}{x}+\log x\cos x\right] \quad \text{---------------------- (2)}$$

Again $v = \sin x^{\cos x}$

Taking logarithm on both sides, we have log v = (cos$x$) log(sinx)

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\frac{dv}{dx}=cosx\frac{d}{dx}\log(\sin x)+\log(\sin x)\frac{d}{dx}(\cos x)=\cos x\frac{1}{\sin x}\times\cos x+\log(\sin x)(-\sin x)$$

$$\Rightarrow \frac{dv}{dx}=v[\cot x\cos x-\sin x\log(\sin x)]=\sin x^{\cos x}[\cot x\cos x-\sin x\log(\sin x)] \quad \text{----------- (3)}$$

From (1), (2) and (3)

$$\frac{dy}{dx}=x^{\sin x}\left[\frac{\sin x}{x}+\log x\cos x\right]+\sin x^{\cos x}[\cot x\cos x-\sin x\log(\sin x)]$$

29. If the position vectors of the vertices of a triangle are $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $3\hat{i}+\hat{j}+2\hat{k}$, show that triangle is equilateral.

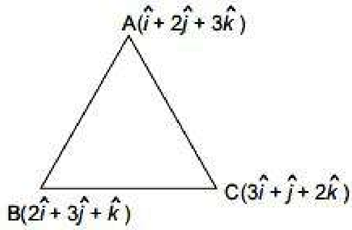
Ans:

$\overrightarrow{AB}$ = Position vector of B – Position Vector of A

$= \hat{i}+\hat{j}-2\hat{k} \quad \Rightarrow |\overrightarrow{AB}| = \sqrt{1+1+4}=\sqrt{6}$

$\overrightarrow{BC} = \hat{i}-2\hat{j}+\hat{k} \quad \Rightarrow |\overrightarrow{BC}| = \sqrt{1+4+1}=\sqrt{6}$

$\overrightarrow{CA} = -2\hat{i}+\hat{j}+\hat{k} \quad \Rightarrow |\overrightarrow{CA}| = \sqrt{4+1+1}=\sqrt{6}$



As $|\overrightarrow{AB}|=|\overrightarrow{BC}|=|\overrightarrow{CA}|$ $\Rightarrow$ triangle ABC is an equilateral triangle.

OR

Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

Ans:

Unit vector perpendicular to $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ = $\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|}$ ... (*i*)

Now $\vec{a}+\vec{b} = 4\hat{i}+4\hat{j}; \vec{a}-\vec{b}=2\hat{i}+4\hat{k}$

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 16\hat{i}-16\hat{j}-8\hat{k}=8(2\hat{i}-2\hat{j}-\hat{k})$$

$$|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})| = |8(2\hat{i}-2\hat{j}-\hat{k})| = 8\sqrt{4+4+1} = 24.$$

Substituting in (*i*), we get

Unit vector perpendicular to $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is $\frac{8(2\hat{i}-2\hat{j}-\hat{k})}{24}=\frac{2}{3}\hat{i}-\frac{2}{3}\hat{j}-\frac{1}{3}\hat{k}.$

30. Evaluate: $\int(\sqrt{\cot x}+\sqrt{\tan x})dx$

Ans:

Let $I = \int (\sqrt{\cot x} + \sqrt{\tan x})\, dx$

$$I = \int \left(\frac{\sqrt{\cos x}}{\sqrt{\sin x}} + \frac{\sqrt{\sin x}}{\sqrt{\cos x}} \right) dx = \int \frac{(\cos x + \sin x)}{\sqrt{\sin x . \cos x}} dx$$

Let $(\sin x - \cos x) = t \Rightarrow (\cos x + \sin x)\, dx = dt$

Also $(\sin x - \cos x)^2 = t^2 \Rightarrow \sin^2 x + \cos^2 x - 2 \sin x . \cos x = t^2$

$\Rightarrow 1 - 2 \sin x . \cos x = t^2 \Rightarrow \sin x . \cos x = \frac{1-t^2}{2}$

Therefore, $I = \int \frac{dt}{\sqrt{\frac{1-t^2}{2}}} = \sqrt{2} \int \frac{dt}{\sqrt{1-t^2}}$

$$= \sqrt{2} \sin^{-1} t + C = \sqrt{2} \sin^{-1} (\sin x - \cos x) + C$$

OR

Evaluate: $\int_1^3 | x^2 - 2x |\, dx.$

Ans:

Consider $I = \int_1^3 | x^2 - 2x |\, dx$

$$|x^2 - 2x| = \begin{cases} -(x^2 - 2x) \text{ where } 1 \le x < 2 \\ (x^2 - 2x) \text{ where } 2 \le x \le 3 \end{cases}$$

$$I = \int_1^2 | x^2 - 2x |\, dx + \int_2^3 | x^2 - 2x |\, dx$$

$$I = \int_1^2 -(x^2 - 2x)\, dx + \int_2^3 (x^2 - 2x)\, dx$$

$$I = -\left[\frac{x^3}{3} - x^2\right]_1^2 + \left[\frac{x^3}{3} - x^2\right]_2^3$$

$$I = -\left[\frac{8}{3} - 4 - \frac{1}{3} + 1\right] + \left[9 - 9 - \frac{8}{3} + 4\right]$$

$$= -\frac{7}{3} + 3 - \frac{8}{3} + 4$$

$$I = \frac{6}{3} = 2$$

31. Minimise Z = 13x – 15y subject to the constraints x + y ≤ 7, 2x – 3y + 6 ≥ 0, x ≥ 0 and y ≥ 0.

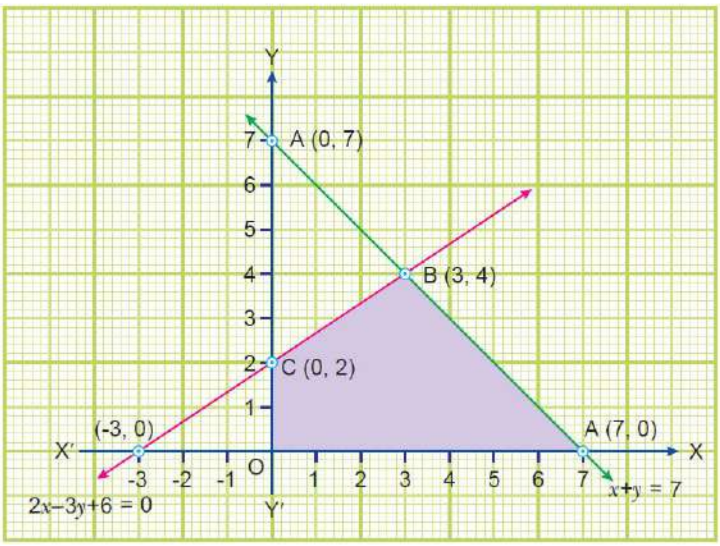
Ans: Minimise Z = 13x – 15y ...(i)

Subject to the constraints

x + y ≤ 7 ...(ii)

2x – 3y + 6≥ 0 ...(iii)

x ≥ 0, y ≥ 0 ...(iv)



Shaded region shown as OABC is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively.

Corner Points	Z = 13x – 15y
O (0, 0)	0
A (7, 0)	91
B (3, 4)	–21
C (0, 2)	–30

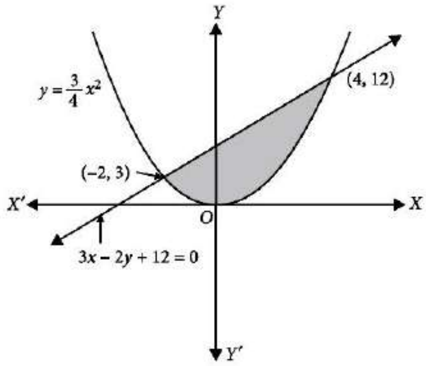
Hence, the minimum value of Z is –30 at (0, 2).

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x – 2y + 12 = 0$.

Ans:



Given equations are $y=\dfrac{3x^2}{4}$ ...(i)

and $3x-2y+12=0 \Rightarrow y=\dfrac{3x+12}{2}$ ...(ii)

Solving equations (i) and (ii), we get

$\dfrac{3x^2}{4}=\dfrac{3x+12}{2}$

$\Rightarrow x^2-2x-8=0 \Rightarrow (x+2)(x-4)=0 \Rightarrow x=-2, 4$

When $x=-2 \Rightarrow y=3$

When $x=4 \Rightarrow y=12$

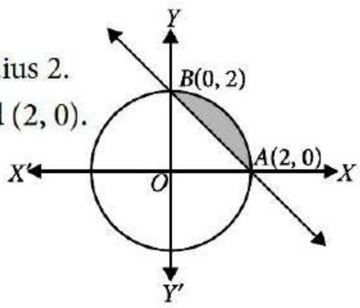
$\therefore$ Required area $=\displaystyle\int_{-2}^{4}\left(\frac{3x+12}{2}-\frac{3}{4}x^2\right)dx=\left[\frac{3}{4}x^2+6x-\frac{x^3}{4}\right]_{-2}^{4}$

$=\left[\dfrac{3\times16}{4}+6\times4-\dfrac{64}{4}\right]-\left[\dfrac{3}{4}\times4-6\times2+\dfrac{8}{4}\right]=27$ sq. units.

OR

Find the area of the region $\{(x, y) : x^2 + y^2 \le 4, x + y \ge 2\}$.

Ans:



The given curves are

$x^2+y^2=4$ ...(i) and $x+y=2$ ...(ii)

Now, $x^2+y^2=4$ is a circle having centre at (0, 0) and radius 2.

And $x+y=2$ is a straight line passing through (0, 2) and (2, 0).

$\therefore$ Required area $=\displaystyle\int_0^2\left[\sqrt{4-x^2}-(2-x)\right]dx$

$=\left[\dfrac{x\sqrt{4-x^2}}{2}+\dfrac{4}{2}\sin^{-1}\dfrac{x}{2}-2x+\dfrac{x^2}{2}\right]_0^2$

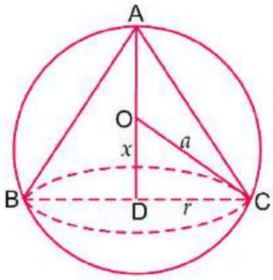
$=0+2\sin^{-1}(1)-4+2-0=2\cdot\dfrac{\pi}{2}-2=(\pi-2)$ sq.units.

33. Prove that the volume of the largest cone that can be inscribed in a sphere of radius 'a' is 8/27 of the volume of the sphere.

Ans:

Consider a sphere of radius a with centre at O such that $OD = x$ and $DC = r$.

Let h be the height of the cone.



Then $h = AD = AO + OD = a + x$ ...(*i*)

($OA = OC$ = radius)

In the right angle ΔODC,

$a^2 = r^2 + x^2$ (By Pythagoras theorem) ...(*ii*)

Let V be the volume the cone, then $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V(x) = \frac{1}{3}\pi(a^2 - x^2)(a + x)$ [From (*i*) and (*ii*)]

$\Rightarrow V'(x) = \frac{1}{3}\pi\left[(a^2 - x^2)\frac{d}{dx}(a + x) + (a + x)\frac{d}{dx}(a^2 - x^2)\right]$

$= \frac{1}{3}\pi[(a^2 - x^2)(1) + (a + x)(-2x)] = \frac{1}{3}\pi[(a + x)(a - x - 2x)] = \frac{1}{3}\pi(a + x)(a - 3x)$

Also, $V''(x) = \frac{1}{3}\pi\left[(a + x)\frac{d}{dx}(a - 3x) + (a - 3x)\frac{d}{dx}(a + x)\right]$

$\Rightarrow V''(x) = \frac{1}{3}\pi[(a + x)(-3) + (a - 3x)(1)]$

For maximum or minimum value, we have $V'(x) = 0$

$\frac{1}{3}\pi(a + x)(a - 3x) = 0 \Rightarrow x = -a$ or $x = \frac{a}{3}$

Neglecting $x = -a$ $[\because x > 0]$

$V''\left(\frac{a}{3}\right) = \frac{1}{3}\pi\left[\left(a + \frac{a}{3}\right)(-3) + \left(a - 3\left(\frac{a}{3}\right)\right)\right] = \frac{-4\pi a}{3} < 0$

$\therefore$ Volume is maximum when $x = \frac{a}{3}$.

Putting $x = \frac{a}{3}$ in equation (*i*) and (*ii*), we get $h = a + \frac{a}{3} = \frac{4a}{3}$ and $r^2 = a^2 - \frac{a^2}{9} = \frac{8a^2}{9}$

Now, volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{8a^2}{9}\right)\left(\frac{4a}{3}\right) = \frac{8}{27}\left(\frac{4}{3}\pi a^3\right)$

Thus, volume of the cone $= \frac{8}{27}$ (volume of the sphere).

34. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Ans:

According to question

$x + y + z = 12$

$2x + 3y + 3z = 33$

$x - 2y + z = 0$

The above system of linear equation can be written in matrix form as $AX = B$ where

$$A=\begin{bmatrix}1&1&1\\2&3&3\\1&-2&1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}12\\33\\0\end{bmatrix}$$

$$Here, |A|=\begin{vmatrix}1&1&1\\2&3&3\\1&-2&1\end{vmatrix}$$

= 1 (3 + 6) –1 (2–3) + 1 (– 4 –3) = 9 + 1 – 7 = 3

$\therefore$ A^{-1} exists.

$A_{11} = 9, A_{12} = 1, A_{13} = -7$

$A_{21} = -3, A_{22} = 0, A_{23} = 3$

$A_{31} = 0, A_{32} = -1, A_{33} = 1$

$$adj(A)=\begin{bmatrix}9&1&-7\\-3&0&3\\0&-1&1\end{bmatrix}^T=\begin{bmatrix}9&-3&0\\1&0&-1\\-7&3&1\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{3}\begin{bmatrix}9&-3&0\\1&0&-1\\-7&3&1\end{bmatrix}$$

$$\text{Now, } X=A^{-1}B\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{3}\begin{bmatrix}9&-3&0\\1&0&-1\\-7&3&1\end{bmatrix}\begin{bmatrix}12\\33\\0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{3}\begin{bmatrix}108-99\\12+0+0\\-84+99\end{bmatrix}=\frac{1}{3}\begin{bmatrix}9\\12\\15\end{bmatrix}=\begin{bmatrix}3\\4\\5\end{bmatrix}$$

$\Rightarrow x = 3, y = 4, z = 5$

No. of awards for honesty = 3

No. of awards for helping others = 4

No. of awards for supervising = 5.

The persons, who work in the field of health and hygiene should also be awarded.

35. Find the equation of the line which intersects the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ passes through the point (1, 1, 1).

Ans: General point on the first line is (λ – 2, 2λ + 3, 4λ – 1).

General point on the second line is (2μ + 1, 3μ + 2, 4μ + 3).

Direction ratios of the required line with first line are (λ – 3, 2λ + 2, 4λ – 2).

Direction ratios of the same line with second line are (2μ, 3μ + 1, 4μ + 2).

Therefore, $\frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{4\lambda-2}{4\mu+2}$ ....(1) $\Rightarrow \frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{2\lambda-1}{2\mu+1}=k$ (say)

$\Rightarrow \lambda - 3 = 2\mu k,\ 2\lambda + 2 = (3\mu + 1) k, 2\lambda - 1 = (2\mu + 1)k$

$\Rightarrow \frac{\lambda-3}{2} = \mu k, 2\lambda + 2 = 3\left(\frac{\lambda-3}{2}\right) + k,$ and $2\lambda - 1 = 2\left(\frac{\lambda-3}{2}\right)+k$

$\Rightarrow k = \frac{4\lambda+4-3\lambda+9}{2} = \lambda + 2$ or $\lambda = 9,\ \mu = \frac{3}{11}$, which satisfy (1).

Therefore, the direction ratios of the required line are (6, 20, 34) or, (3, 10, 17).

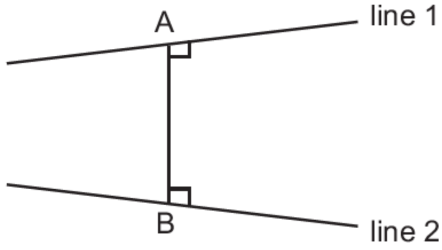
Hence, the required equation of line passes through (1, 1, 1) is $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$

OR

Find the shortest distance between the following lines :

$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

Ans: Let $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}=\lambda$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}=k$



Now, let's take a point on first line as

A(λ + 3, -2λ + 5, λ + 7) and let

B(7k - 1, - 6k - 1, k - 1) be point on the second line

The direction ratio of the line AB

7k - λ - 4, - 6k + 2λ - 6, k - λ - 8

Now as AB is the shortest distance between line 1 and line 2 so,

(7k - λ - 4) × 1 + (- 6k + 2λ - 6) × (-2) + (k - λ - 8) × 1 = 0 ...(i)

and (7k - λ - 4) × 7 + (- 6k + 2λ - 6) × (-6) + (k - λ - 8) × 1 = 0 ...(ii)

Solving equation (i) and (ii) we get

λ = 0 and k = 0

∴ A = (3, 5, 7) and B = (- 1, - 1, - 1)

∴ AB = $\sqrt{(3+1)\ 2+(5+1)\ 2+(7+1)\ 2}=\sqrt{16+36+64}=\sqrt{116}=2\sqrt{29}$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1:

Soumya was doing a project related to the average number of hours spent on study by students selected at random. At the end of the survey, she prepared the report related to the data.

Let X denotes the average number of hours spent on study by students.

The probability that X can take the values x, has the following form, where k is some unknown constant

$$P(X=x)=\begin{cases} k, & if \quad x=0 \\ 2k, & if \quad x=1 \\ 3k, & if \quad x=2 \\ 0, & otherwise \end{cases}$$

Based on the above information, answer the following questions:
(i) What is the value of k? [1]
(ii) What is the value P(X = 2)? [1]
(iii) What is the probability that average study time of students is atleast 1 hours. [2]

OR

(iii) Find the mean of the given data. [2]
Ans: (i) Since, $\sum P(x) = 1$
∴ k + 2k + 3k + 0 = 1
⇒ 6k = 1 ⇒ k = 1/6
(ii) From (i), k = 1/6
∴ P(x = 2) = 3k = 3/6 = 1/2
(iii) If a students has study time at least 1 hr then either he/she has studies for 1 hour or 2 hour.
∴ Required Probability = P(x=1) + P(x=2)
= 2k + 3k = 5k = 5/6

OR

Mean = $\sum xP(x) = 0\times\frac{1}{6}+1\times\frac{2}{6}+2\times\frac{3}{6}=0+\frac{2}{6}+1=\frac{1}{3}+1=\frac{4}{3}$

37. Case-Study 2:
Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes.
A = {S, D}, B = {1, 2, 3, 4, 5, 6}

(i) Show that relation R : B ∈ B be defined by R = {(x, y) : y is divisible by x} is reflexive and symmetric but not transitive.
(ii) Let R be a relation on B defined by R = {(1, 2), (2,2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)}. Then R is show that R is neither reflexive nor symmetric nor transitive.
Ans: (i) R is reflexive, since every element of B i.e.,
B = {1, 2, 3, 4, 5, 6} is divisible by itself.
i.e., (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) ∈ R
further, (1, 2) ∈ R
but (2, 1) ∈ R
Moreover, (1, 2), (2, 4) ∈ R
⇒ (1, 4) ∈ R ⇒ R is transitive.
Therefore, R is reflexive and transitive but not symmetric.

(ii) R = {(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)} ⇒ R is not reflexive.
Since, (1, 1), (3, 3), (4, 4), (6, 6) ∉ R ⇒ R is not symmetric.
Because, for (1, 2) ∈ R there (2, 1) ∉ R. ⇒ R is not transitive.
Because for all element of B there does not exist, (a, b)(b, c) ∈ R and (a, c) ∈ R.

38. Case-Study 3:

Two schools A and B want to award their selected students on the values of Honesty, Hard work and Punctuality. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of ₹ 2200. School B wants to spend ₹ 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school A). The total amount of award for one prize on each value is ₹ 1200.

Using the concept of matrices and determinants, answer the following questions.
(i) What is the award money for Honesty? [1]
(ii) What is the award money for Punctuality? [1]
(iii) What is the award money for Hard work? [1]
(iv) If a matrix P is both symmetric and skew-symmetric, then find |P|. [1]
Ans: Three equations are formed from the given statements:

$$3x + 2y + z = 2200$$
$$4x + y + 3z = 3100$$
$$\text{and } x + y + z = 1200$$

Converting the system of equations in matrix form, we get

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

i.e. PX = Q

$$\text{where } P = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } Q = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$|P| = 3(1 – 3) –2(4 – 3) + 1(4 – 1) = –6 – 2 + 3 = –5 \neq 0$
⇒ X = P–1 Q, provided P–1 exists.

$$\therefore \quad \text{adj } P = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore\ P^{-1} = \frac{1}{|P|}(\text{adj } P) = \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

$$\therefore X = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4400+3100-6000 \\ 2200-6200+6000 \\ -6600+3100+6000 \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

$$\Rightarrow \quad x = 300, y = 400 \text{ and } z = 500$$

(i) ₹ 300 (ii) ₹ 500 (iii) ₹ 400

(iv) If a matrix P is both symmetric and skew symmetric matrix then it will be a zero matrix. So, |P| = 0.