SAMPLE PAPER TEST 04 FOR BOARD EXAM 2025

SUBJECT: MATHEMATICS (041) | **MAX. MARKS : 80**
CLASS : XII | **DURATION: 3 HRS**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. Find the cofactor of a_{12} in the following: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

(a) -46 (b) 46 (c) 0 (d) 1

2. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the value of x.

(a) 1 (b) 2 (c) 3 (d) 4

3. Evaluate: $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$

(a) tanx – cotx + C (b) –tanx + cotx + C
(c) tanx + cotx + C (d) –tanx – cotx +C

4. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

(a) $\cos^{-1}\left(-\frac{1}{2}\right)$ (b) 60° (c) $\cos^{-1}\left(-\frac{1}{3}\right)$ (d) $\cos^{-1}\left(-\frac{2}{3}\right)$

5. If y = $\sqrt{a^2 - x^2}$, then $y\frac{dy}{dx}$ is:

(a) 0 (b) x (c) -x (d) 1

6. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m + n.

(a) 1 (b) 2 (c) 3 (d) 4

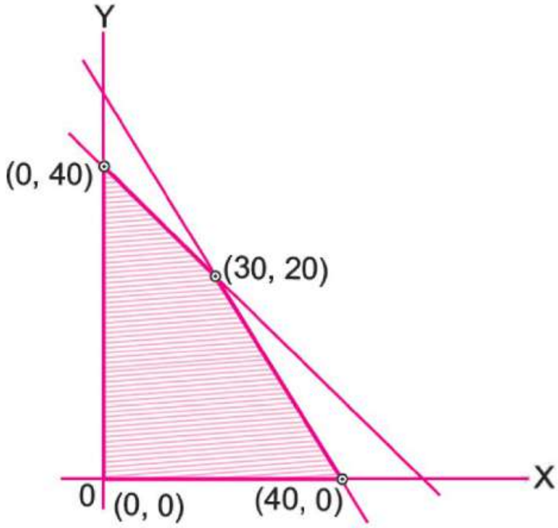
7. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F =4x + 6y be the objective function. The minimum value of F occurs at
(a) Only (0, 2)
(b) Only (3, 0)

(c) the mid-point of the line segment joining the points (0, 2) and (3, 0)
(d) any point on the line segment joining the points (0, 2) and (3, 0)

8. The magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is 9/2, is
(a) 2 (b) 3 (c) 4 (d) 5

9. Which of the following is a homogeneous differential equation?
(a) $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$
(b) $(xy)dx - (x^3 + y^3)dy = 0$
(c) $(x^3 + 2y^2)dx + 2xy\,dy = 0$
(d) $y^2dx + (x^2 - xy - y^2)dy = 0$

10. Feasible region (shaded) for a LPP is shown in the given figure.
The maximum value of the $Z = 0.4x + y$ is



(a) 45 (b) 40 (c) 50 (d) 41

11. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to
(a) 6 (b) ± 6 (c) -6 (d) 0

12. If $y = 5 \cos x - 3 \sin x$, then $\frac{d^2y}{dx^2}$ is equal to:
(a) $-y$ (b) y (c) $25y$ (d) $9y$

13. If a line makes angles α, β, γ with the positive direction of co-ordinates axes, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.
(a) 1 (b) 2 (c) 3 (d) 4

14. The points (1, 2, 3), (4, 0, 4), (– 2, 4, 2), (7, – 2, 5) are:
(a) collinear (b) are the vertices of a square
(c) are the vertices of a rectangle (d) None of these

15. If A is a square matrix of order 3, such that A(adjA) = 10 I, then $|adj\ A|$ is equal to
(a) 1 (b) 10 (c) 100 (d) 101

16. Given two independent events A and B such that P(A) =0.3, P(B) = 0.6 and $P(A' \cap B)$ is
(a) 0.42 (b) 0.18 (c) 0.28 (d) 0.12

17. The value of $\int_2^3 \frac{\log x^2}{x} dx$ is

(a) $\log 6 \log\left(\frac{3}{2}\right)$ (b) $\log\left(\frac{3}{2}\right)$ (c) 2 log 3 (d) $\left(\frac{1}{3}\right)\log 6$

18. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then write the value of x and y.

(a) x = 3, y = 3 (b) x = 3, y = 2 (c) x = 2, y = 2 (d) x = 2, y = 3

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): The domain of the function $sec^{-1}2x$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

Reason (R): $sec^{-1}(-2) = -\frac{\pi}{4}$

20. Assertion(A) : The pair of lines given by $\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect .

Reason(R) : Two lines intersect each other, if they are not parallel and shortest distance = 0.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Prove that the Greatest Integer Function $f : R \to R$, given by $f(x) = [x]$ is neither one-one nor onto. Where $[x]$ denotes the greatest integer less than or equal to x.

OR

If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of x.

22. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

OR

Find the coordinates of the point where the line $\frac{x+3}{3} = \frac{y-1}{-1} = \frac{z-5}{-5}$ cuts the XY plane.

23. If $x\sin(a+y) + \sin a\cos(a+y) = 0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

24. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

25. For what value of 'k' is the function $f(x) = \begin{cases} \frac{\sin 5x}{3x} + \cos x, & if \quad x \neq 0 \\ k, & if \quad x = 0 \end{cases}$ continuous at x = 0?

SECTION – C

Questions 13 to 22 carry 3 marks each.

26. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid.

OR

An urn contains 5 white and 8 white black balls. Two successive drawing of three balls at a time are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and second draw gives 3 black balls.

27. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

OR

Evaluate: $\int_1^3 |x^2 - 2x| \, dx.$

28. Evaluate: $\int \frac{dx}{9x^2+6x+10}$.

29. Solve : $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$

OR

Solve : $(x^2 + y^2)\, dx - 2xydy = 0$

30. Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

31. Solve the following Linear Programming Problem graphically:
Minimise $Z = 13x - 15y$ subject to the constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0$ and $y \ge 0$.

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$.

33. Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b) : |a - b|$ is a multiple of $4\}$ is an equivalence relation.
Find the set of all elements related to 1 in each case.

OR

Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, then show that f is one-one and onto.

34. Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, verify that AB = 6I, how can we use the result to find the values of x, y, z from given equations $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 17$

35. Find the shortest distance between the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$. If the lines intersect find their point of intersection.

OR

Find the vector equation of the line passing through (1, 2, – 4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

SECTION – E(Case Study Based Questions)

Questions 35 to 37 carry 4 marks each.

36. Case-Study 1: Read the following passage and answer the questions given below.

In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

(i) Find the conditional probability that an error is committed in processing given that Sonia processed the form.

(ii) Find the probability that Sonia processed the form and committed an error.

(iii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is not processed by Vinay.

OR

If the form selected at random has an error, find the probability that the form is processed by Sonia

37. Case-Study 2: Read the following passage and answer the questions given below.

In an elliptical sport field the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x.
(ii) Find the critical point of the function.
(iii) Use First derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

OR

(iii) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

38. Case-Study 3:

One day Shweta's Mathematics teacher was explaining the topic Increasing and decreasing functions in the class. He explained about different terms like stationary points, turning points etc. He also explained about the conditions for which a function will be increasing or decreasing. He took examples of different functions to make it more clear to the students. He then took the function $f(x) = (x + 1)^3(x - 3)^3$ and ask the students to answer the following questions. With Shweta, you can also test your knowledge by answering the questions

(i) Find the stationary points on the curve. [2]
(ii) Find the intervals where the function is increasing and decreasing? [2]

SAMPLE PAPER TEST 04 FOR BOARD EXAM 2025
(ANSWERS)

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SECTION – A

Questions 1 to 20 carry 1 mark each.

1. Find the cofactor of a_{12} in the following: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

(a) -46 (b) 46 (c) 0 (d) 1

Ans: (b) 46

Minor of a_{12} is $M_{12} = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} = -42 - 4 = -46$

Cofactor $C_{12} = (-1)^{1+2} M_{12} = (-1)^3 (-46) = 46$

2. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the value of x.

(a) 1 (b) 2 (c) 3 (d) 4

Ans:

We have, $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow 2x - y = 10$ and $3x + y = 5$

$\Rightarrow 5x = 15 \Rightarrow x = 3$

3. Evaluate: $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$

(a) tanx – cotx + C (b) –tanx + cotx + C
(c) tanx + cotx + C (d) –tanx – cotx +C

Ans: (c) tanx + cotx + C

4. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

(a) $\cos^{-1}\left(-\frac{1}{2}\right)$ (b) 60° (c) $\cos^{-1}\left(-\frac{1}{3}\right)$ (d) $\cos^{-1}\left(-\frac{2}{3}\right)$

Ans: (c) $\cos^{-1}\left(-\frac{1}{3}\right)$

$\vec{a} = \hat{i} - \hat{j} + \hat{k} \Rightarrow |\vec{a}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$

$\vec{b} = \hat{i} + \hat{j} - \hat{k} \Rightarrow |\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-1)^2} = \sqrt{3}$

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 1 - 1 - 1 = \sqrt{3}\sqrt{3}\cos\theta \Rightarrow -1 = 3\cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(-\frac{1}{3}\right)$

5. If y = $\sqrt{a^2 - x^2}$, then $y\frac{dy}{dx}$ is:

(a) 0 (b) x (c) -x (d) 1

Ans: (c) -x

We have, $y = \sqrt{a^2 - x^2}$ ...(i)

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}}(0 - 2x) = \frac{-x}{\sqrt{a^2 - x^2}}$$

$\sqrt{a^2 - x^2}\frac{dy}{dx} = -x \Rightarrow y\frac{dy}{dx} = -x$ (from (i))

6. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m + n.

(a) 1 (b) 2 (c) 3 (d) 4

Ans:

$m + n = 4$

$\because$ $m = 2$ (second order derivative)

$\therefore$ $n = 2$ (degree of the highest order derivative)

7. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F =4x + 6y be the objective function. The minimum value of F occurs at

(a) Only (0, 2)

(b) Only (3, 0)

(c) the mid-point of the line segment joining the points (0, 2) and (3, 0)

(d) any point on the line segment joining the points (0, 2) and (3, 0)

Ans: (d) any point on the line segment joining the points (0, 2) and (3, 0)

Corner points	Corresponding value of $F = 4x + 6y$
(0, 2)	12← Minimum
(3, 0)	12← Minimum
(6, 0)	24
(6, 8)	72← Maximum
(0, 5)	30

Hence, minimum value of F occurs at any points on the line segment joining the points (0, 2) and (3, 0).

8. The magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is 9/2, is

(a) 2 (b) 3 (c) 4 (d) 5

Ans: (b) 3

Given, $|\vec{a}| = |\vec{b}|, \theta = 60°$ and $\vec{a}\cdot\vec{b} = \frac{9}{2}$

Now, $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \cos 60° = \frac{9/2}{|\vec{a}|^2} \Rightarrow \frac{1}{2} = \frac{9/2}{|\vec{a}|^2}$

$\Rightarrow |\vec{a}|^2 = 9 \Rightarrow |\vec{a}| = 3 \therefore |\vec{a}| = |\vec{b}| = 3$

9. Which of the following is a homogeneous differential equation?

(a) $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

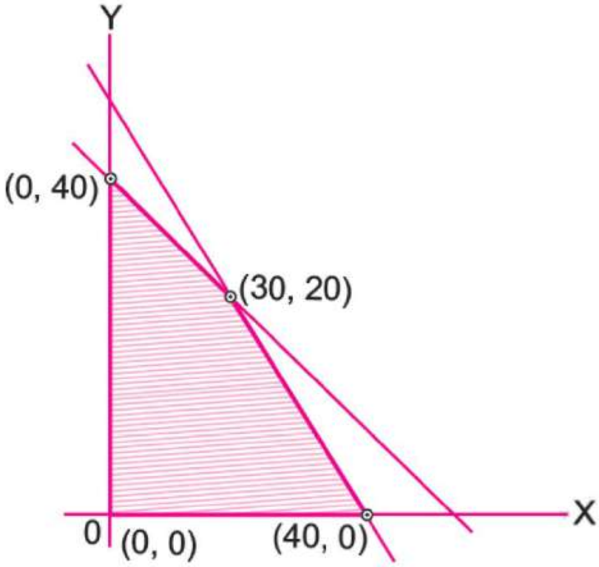
(b) $(xy)dx - (x^3 + y^3)dy = 0$

(c) $(x^3 + 2y^2)dx + 2xy\, dy = 0$

(d) $y^2dx + (x^2 - xy - y^2)dy = 0$

Ans: (d) $y^2dx + (x^2 - xy - y^2)dy = 0$

10. Feasible region (shaded) for a LPP is shown in the given figure.
The maximum value of the Z = 0.4x + y is



(a) 45 (b) 40 (c) 50 (d) 41

Ans: (d) 41

11. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(a) 6 (b) ± 6 (c) -6 (d) 0

Ans: (b) ± 6

12. If y = 5 cos x – 3 sin x, then $\frac{d^2y}{dx^2}$ is equal to:

(a) –y (b) y (c) 25y (d) 9y

Ans: (a) –y

$y = 5\cos x - 3\sin x$

$\Rightarrow \frac{dy}{dx} = -5\sin x - 3\cos x \Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -y$

13. If a line makes angles α, β, γ with the positive direction of co-ordinates axes, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

(a) 1 (b) 2 (c) 3 (d) 4

Ans:

We know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

or $(1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

14. The points (1, 2, 3), (4, 0, 4), (– 2, 4, 2), (7, – 2, 5) are:

(a) collinear (b) are the vertices of a square

(c) are the vertices of a rectangle (d) None of these

Ans: (a) collinear

15. If A is a square matrix of order 3, such that A(adjA) = 10 I, then $|adj\ A|$ is equal to

(a) 1 (b) **10** (c) 100 (d) 101

Ans: (c) 100

16. Given two independent events A and B such that P(A) =0.3, P(B) = 0.6 and P($A' \cap B$) is

(a) 0.42 (b) 0.18 (c) 0.28 (d) 0.12

Ans: (a) 0.42

17. The value of $\int_2^3 \frac{\log x^2}{x} dx$ is

(a) $\log 6 \log\left(\frac{3}{2}\right)$ (b) $\log\left(\frac{3}{2}\right)$ (c) 2 log 3 (d) $\left(\frac{1}{3}\right)\log 6$

Ans: (a) $\log 6 \log\left(\frac{3}{2}\right)$

Let $I = \int \frac{\log x^2}{x} dx = 2\int \frac{\log x}{x} dx$

Let $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$I = 2\int t.dt = \frac{t^2}{2} \times 2 = (\log x)^2$

$\therefore \int_2^3 \frac{\log x^2}{x} dx = \left[(\log x)^2\right]_2^3 = (\log 3)^2 - (\log 2)^2$

$= (\log 3 + \log 2)(\log 3 - \log 2) = (\log 3 \times 2)\log\left(\frac{3}{2}\right) = \log 6 \log\left(\frac{3}{2}\right)$

18. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then write the value of x and y.

(a) x = 3, y = 3 (b) x = 3, y = 2 (c) x = 2, y = 2 (d) x = 2, y = 3

Ans: (a) x = 3, y = 3

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing both matrices

$2 + y = 5$ and $2x + 2 = 8 \Rightarrow y = 3$ and $2x = 6$

$\Rightarrow x = 3, y = 3.$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): The domain of the function $sec^{-1}2x$ is $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$

Reason (R): $sec^{-1}(-2)=-\frac{\pi}{4}$

Ans: (c) A is true but R is false.

$sec^{-1}x$ is defined if $x\le -1$ *or* $x\ge 1$. Hence, $sec^{-1}2x$ will be defined if $x\le -\frac{1}{2}$ or $x\ge\frac{1}{2}$.

Hence, A is true.

The range of the function $secx$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

R is false.

20. Assertion(A) : The pair of lines given by $\vec{r}=\hat{i}-\hat{j}+\lambda(2\hat{i}+\hat{k})$ and $\vec{r}=2\hat{i}-\hat{k}+\mu(\hat{i}+\hat{j}-\hat{k})$ intersect .

Reason(R) : Two lines intersect each other, if they are not parallel and shortest distance = 0.

Ans: (a) Both A and R are true and R is the correct explanation of A.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Prove that the Greatest Integer Function $f: R\to R$, given by $f(x)=[x]$ is neither one-one nor onto. Where $[x]$ denotes the greatest integer less than or equal to x.

Ans: Given $f: R\to R$ defined by $f(x)=[x]$

For one-one: We know by definition that for $a\le x<a+1$, $f(x)=a$, a is an integer,

i.e. for $x_1, x_2\in[a, a+1)$, $x_1\ne x_2$, $f(x_1)=f(x_2)=a$.

Hence, not one-one.

For onto: For y (non-integer) $\in R$ in co-domain there does not exist $x\in R$ in domain such that $f(x)=y$. Hence, not onto.

OR

If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x.

Ans:

$$\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1=\sin\frac{\pi}{2}$$

$$\Rightarrow\ \sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}\Rightarrow x=\frac{1}{5}\quad\left[\text{as}\sin^{-1}\theta+\cos^{-1}\theta=\frac{\pi}{2}\right].$$

22. Find the angle between the vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

The angle θ between the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$i.e., \cos\theta = \frac{(\hat{i}+\hat{j}-\hat{k}).(\hat{i}-\hat{j}+\hat{k})}{\sqrt{(1)^2+(1)^2+(-1)^2}.\sqrt{(1)^2+(-1)^2+(1)^2}}$$

$$i.e., \cos\theta = \frac{1-1-1}{\sqrt{3}.\sqrt{3}}$$

$$i.e., \cos\theta = -\frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(-\frac{1}{3}\right).$$

OR

Find the coordinates of the point where the line $\frac{x+3}{3} = \frac{y-1}{-1} = \frac{z-5}{-5}$ cuts the XY plane.

Ans:

Given. As line cuts the xy plane $z = 0$

We have, $\frac{x+3}{3} = \frac{y-1}{-1} = \frac{z-5}{-5} \Rightarrow \frac{x+3}{3} = \frac{y-1}{-1} = \frac{z-5}{-5} = \lambda$

Now, Put $z = 0$

$$\frac{z-5}{-5} = \lambda \Rightarrow z = -5\lambda + 5 \Rightarrow 0 = -5\lambda + 5 \Rightarrow \lambda = 1$$

Now, $\frac{x+3}{3} = 1 \Rightarrow x = 3 - 3 = 0$ | $\frac{y-1}{-1} = 1 \Rightarrow y = -1 + 1 = 0$

$\therefore x = 0, y = 0, z = 0$ $\therefore$ **Required Point (0, 0, 0)**

23. If $x\sin(a+y)+\sin a\cos(a+y)=0$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

Ans: Given that $x\sin(a+y)+\sin a\cos(a+y)=0$

$\Rightarrow x\sin(a+y) = -\sin a\cos(a+y)$

$\Rightarrow x = \frac{-\sin a\cos(a+y)}{\sin(a+y)} \Rightarrow x = -\sin a.\cot(a+y)$

Differentiating with respect to y, we get $\frac{dx}{dy} = -\sin a\left[-\operatorname{cosec}^2(a+y)\right].\frac{d}{dy}(a+y)$

$= -\sin a\left[-\operatorname{cosec}^2(a+y)\right].(0+1) = \frac{\sin a}{\sin^2(a+y)}$

$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

24. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

Ans: Let A (1, –1, 2) and B (3, 4, – 2) be given points.

Direction ratios of AB are

(3 – 1), {(4 – (–1)}, (–2 –2) i.e., 2, 5, – 4.

Let C (0, 3, 2) and D (3, 5, 6) be given points.

Direction ratios of CD are

(3 – 0), (5 – 3), (6 – 2) i.e., 3, 2, 4.

We know that two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$\therefore$ 2 × 3 + 5 × 2 + (– 4) × 4 = 6 +10 –16 = 0, which is true.

It will shows that lines AB and CD are perpendicular.

25. For what value of 'k' is the function $f(x) = \begin{cases} \dfrac{\sin 5x}{3x} + \cos x, & if \quad x \neq 0 \\ k, & if \quad x = 0 \end{cases}$ continuous at x = 0?

Ans:

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h)$$

$$= \lim_{h \to 0} f(h) = \lim_{h \to 0}\left(\frac{\sin 5h}{3h} + \cos h\right)$$

$$= \lim_{h \to 0} \frac{\sin 5h}{5h} \times \frac{5}{3} + \lim_{h \to 0} \cos h = 1 \times \frac{5}{3} + 1 \quad [\because \ h \to 0 \Rightarrow 5h \to 0]$$

$$\Rightarrow \lim_{x \to 0^+} f(x) = \frac{8}{3}$$

Also, $f(0) = k$

Since, $f(x)$ is continuous at $x = 0$.

$$\Rightarrow \lim_{x \to 0^+} f(x) = f(0) \Rightarrow \frac{8}{3} = k$$

SECTION – C

Questions 13 to 22 carry 3 marks each.

26. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid.

Ans: Let *X* be no. of selected scouts who are well trained in first aid. Here random variable *X* may have value 0, 1, 2.

Now, P(X = 0) = $\dfrac{{}^{20}C_2}{{}^{50}C_2} = \dfrac{20 \times 19}{50 \times 49} = \dfrac{38}{245}$

P(X = 1) = $\dfrac{{}^{20}C_1 \times {}^{30}C_1}{{}^{50}C_2} = \dfrac{20 \times 30 \times 2}{50 \times 49} = \dfrac{120}{245}$

P(X = 2) = $\dfrac{{}^{30}C_2}{{}^{50}C_2} = \dfrac{30 \times 29}{50 \times 49} = \dfrac{87}{245}$

Now probability distribution table is

X	0	1	2
P(x)	$\frac{38}{245}$	$\frac{120}{245}$	$\frac{87}{245}$

OR

An urn contains 5 white and 8 white black balls. Two successive drawing of three balls at a time are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and second draw gives 3 black balls.

Consider the following events

$A =$ Drawing 3 white balls in first draw

$B =$ Drawing 3 black balls in the second draw.

Required probability $= P(A \cap B) = P(A)\,P(B/A)$ ...(i)

Now, $P(A) = \frac{^5C_3}{^{13}C_3} = \frac{10}{286} = \frac{5}{143}$

After drawing 3 white balls in first draw 10 balls are left in the bag, out of which 8 are black balls.

$\therefore P(B/A) = \frac{^8C_3}{^{10}C_3} = \frac{56}{120} = \frac{7}{15}$

Substituting these values in Eq. (i), we get

Required probability $= P(A \cap B) = P(A)\,P(B/A)$

$$= \frac{5}{143} \times \frac{7}{15} = \frac{7}{429}$$

27. Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

$I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx \quad ...(i)$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx \quad ...(ii)$

Adding (i) and (ii)

$2I = \int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}dx$

Putting $\cos x = t$ gives $I = \frac{\pi}{2}\int_{-1}^{1} \frac{dt}{1+t^2}$

$\Rightarrow I = \frac{\pi}{2}\left[\tan^{-1} t\right]_{-1}^{1} = \frac{\pi^2}{4}$

OR

Evaluate: $\int_1^3 |x^2 - 2x|\,dx.$

Ans:

Consider $I = \int_1^3 |x^2 - 2x|\,dx$

$$|x^2 - 2x| = \begin{cases} -(x^2 - 2x) & \text{where } 1 \le x < 2 \\ (x^2 - 2x) & \text{where } 2 \le x \le 3 \end{cases}$$

$I = \int_1^2 |x^2 - 2x|\,dx + \int_2^3 |x^2 - 2x|\,dx$

$I = \int_1^2 -(x^2 - 2x)\,dx + \int_2^3 (x^2 - 2x)\,dx$

$I = -\left[\frac{x^3}{3} - x^2\right]_1^2 + \left[\frac{x^3}{3} - x^2\right]_2^3$

$I = -\left[\frac{8}{3} - 4 - \frac{1}{3} + 1\right] + \left[9 - 9 - \frac{8}{3} + 4\right]$

$= -\frac{7}{3} + 3 - \frac{8}{3} + 4$

$I = \frac{6}{3} = 2$

28. Evaluate: $\int \frac{dx}{9x^2 + 6x + 10}$.

Let $I=\int\frac{1}{9x^2+6x+10}dx=\frac{1}{9}\int\frac{1}{x^2+\frac{2}{3}x+\frac{10}{9}}dx$

$$=\frac{1}{9}\int\frac{1}{x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}+\frac{10}{9}}dx$$

$$=\frac{1}{9}\int\frac{1}{\left(x+\frac{1}{3}\right)^2+(1)^2}dx=\frac{1}{9}\times\frac{1}{1}\tan^{-1}\left(\frac{x+\frac{1}{3}}{1}\right)+C$$

$$=\frac{1}{9}\tan^{-1}\left(\frac{3x+1}{3}\right)+C$$

29. Solve : $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$

Ans:

The given differential equation is

$x^2(1-y)dy + y^2(1+x^2)dx = 0$

$\Rightarrow\ x^2(1-y)dy = -y^2(1+x^2)dx$

$\Rightarrow\ \frac{1-y}{y^2}dy=-\left(\frac{1+x^2}{x^2}\right)dx$, if $x, y \neq 0$

$\Rightarrow\ \left(\frac{1}{y}-\frac{1}{y^2}\right)dy=\left(\frac{1}{x^2}+1\right)dx\ \Rightarrow\ \int\left(\frac{1}{y}-\frac{1}{y^2}\right)dy=\int\left(\frac{1}{x^2}+1\right)dx$

$\Rightarrow\ \log|y|+\frac{1}{y}=-\frac{1}{x}+x+C$, which is the general solution of the differential equation.

OR

Solve : $(x^2 + y^2)\,dx - 2xydy = 0$

Ans:

We have, $(x^2+y^2)\,dx-2xydy=0\ \Rightarrow\ \frac{dy}{dx}=\frac{x^2+y^2}{2xy}$ ... (i)

Let $f(x, y)=\frac{x^2+y^2}{2xy}$, so $f(\lambda x, \lambda y)=\frac{\lambda^2x^2+\lambda^2y^2}{2\lambda x\cdot\lambda y}=\frac{\lambda^2}{\lambda^2}f(x, y)=\lambda^0 f(x, y)$

∴ This is homogeneous differential equation.

Let $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

So, equation (i) becomes $v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{2x\cdot vx}=\frac{x^2(1+v^2)}{x^2\cdot 2v}=\frac{1+v^2}{2v}$

$\Rightarrow\ x\frac{dv}{dx}=\frac{1+v^2}{2v}-v=\frac{1+v^2-2v^2}{2v}=\frac{1-v^2}{2v}$

Separating the variables, we get $\frac{2v}{1-v^2}dv=\frac{dx}{x}$

$\Rightarrow\ \int\frac{2v}{1-v^2}dv=\int\frac{dx}{x}\ \Rightarrow\ -\log|1-v^2|=\log x+C\Rightarrow\log x+\log|1-v^2|=-C$

$\Rightarrow\ \log x(1-v^2)=-C\ \Rightarrow\ x\left(1-\frac{y^2}{x^2}\right)=e^{-C}\ \Rightarrow\ x^2-y^2=Ax$, where $e^{-C}=A$

Hence, $x^2-y^2=Ax$ is the general solution of the given differential equation.

30. Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+9)}dx$

Ans:

Let $\frac{x^2}{(x^2+4)(x^2+9)} = \frac{y}{(y+4)(y+9)}$, where $x^2 = y$.

Let $\frac{y}{(y+4)(y+9)} = \frac{A}{(y+4)} + \frac{B}{(y+9)}$.

Then, $y \equiv A(y+9) + B(y+4)$. ... (i)

Putting $y = -4$ on both sides of (i), we get $A = \frac{-4}{5}$.

Putting $y = -9$ on both sides of (i), we get $B = \frac{9}{5}$.

$\therefore \quad \frac{y}{(y+4)(y+9)} = \frac{-4}{5(y+4)} + \frac{9}{5(y+9)} \Rightarrow \frac{x^2}{(x^2+4)(x^2+9)} = \frac{-4}{5(x^2+4)} + \frac{9}{5(x^2+9)}$

$\Rightarrow \int \frac{x^2}{(x^2+4)(x^2+9)}dx = \frac{-4}{5}\int \frac{dx}{(x^2+4)} + \frac{9}{5}\int \frac{dx}{(x^2+9)}$

$= \left(\frac{-4}{5} \times \frac{1}{2}\right)\tan^{-1}\frac{x}{2} + \left(\frac{9}{5} \times \frac{1}{3}\right)\tan^{-1}\frac{x}{3} + C = \frac{-2}{5}\tan^{-1}\frac{x}{2} + \frac{3}{5}\tan^{-1}\frac{x}{3} + C.$

31. Solve the following Linear Programming Problem graphically:

Minimise Z = 13x – 15y subject to the constraints x + y ≤ 7, 2x – 3y + 6 ≥ 0, x ≥ 0 and y ≥ 0.

Ans: Minimise Z = 13x – 15y ...(i)

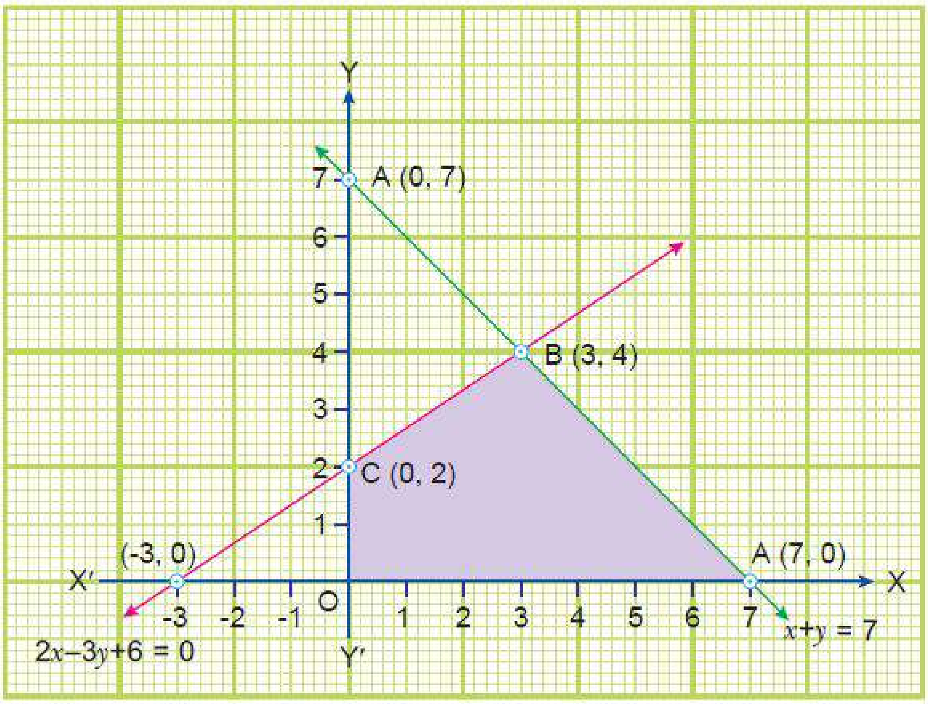
Subject to the constraints

x + y ≤ 7 ...(ii)

2x – 3y + 6≥ 0 ...(iii)

x ≥ 0, y ≥ 0 ...(iv)

Shaded region shown as OABC is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively.



Corner Points	$Z = 13x - 15y$
O (0, 0)	0
A (7, 0)	91
B (3, 4)	–21
C (0, 2)	–30 ← Minimum

Hence, the minimum value of Z is –30 at (0, 2).

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$.

Ans:

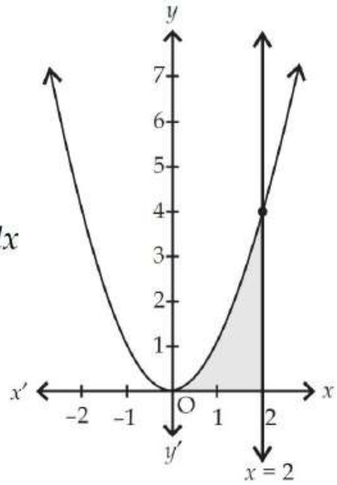
Given. $y^2 = 8x \Rightarrow y = \pm 2\sqrt{2x}$

x	0	1	2
y	0	$+2\sqrt{2}$	± 4

$\therefore$ Required area of shaded region $= 2\int_0^2 2\sqrt{2x}\,dx = 4\sqrt{2}\int_0^2 x^{\frac{1}{2}}\,dx$

$= 4\sqrt{2} \times \frac{2}{3}[x^{3/2}]_0^2 = \frac{8\sqrt{2}}{3}(2^{3/2} - 0)$

$= \frac{8\sqrt{2}}{3}(2\sqrt{2}) = \frac{32}{3}$ **sq. units**



33. Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b) : |a - b|$ is a multiple of 4} is an equivalence relation.

Find the set of all elements related to 1 in each case.

Ans: $A = \{x \in Z : 0 \le x \le 12\}$

$R = \{(a, b) : | a - b |$ is a multiple of 4}

Reflexive: Let $x \in A \Rightarrow |x - x| = 0$, which is a multiple of 4.

$\Rightarrow (x, x) \in R \ \forall \ x \in A$

$\therefore$ R is reflexive.

Symmetric: Let $x, y \in A$ and $(x, y) \in R$

$\Rightarrow |x - y|$ is a multiple of 4

or $x - y = \pm 4p$ {p is any integer}

$\Rightarrow y - x = \mp 4p \Rightarrow | y - x |$ is a multiple of 4. $\Rightarrow (y, x) \in R$

$\Rightarrow$ R is symmetric.

Transitive: Let $x, y, z \in A$, $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow |x - y|$ is multiple of 4 and $| y - z |$is multiple of 4

$\Rightarrow x - y$ is multiple of 4 and $y - z$ is multiple of 4

$\Rightarrow (x - y) + (y - z)$ is multiple of 4 $\Rightarrow (x - z)$ is multiple of 4.

$\Rightarrow |x - z|$ is multiple of 4 $\Rightarrow (x, z) \in R \Rightarrow$ R is transitive.

So, R is an equivalence relation.

Let B be the set of elements related to 1.

$\therefore B = \{a \in A : |a - 1|$ is multiple of 4}

$\Rightarrow$ B = {1, 5, 9} {as |1 – 1| = 0, |1 – 5 |= 4, |1 – 9| = 8}

OR

Let A = R – {2} and B = R – {1}. If f : A → B is a function defined by $f(x) = \dfrac{x-1}{x-2}$, then show that f is one-one and onto.

Ans:

Here, $f: A \to B$ is given by $f(x) = \dfrac{x-1}{x-2}$,

where $A = R - \{2\}$ and $B = R - \{1\}$

Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$ (*i.e.*, $x_1 \neq 2, x_2 \neq 2$)

$\Rightarrow \dfrac{x_1 - 1}{x_1 - 2} = \dfrac{x_2 - 1}{x_2 - 2} \Rightarrow (x_1 - 1)(x_2 - 2) = (x_1 - 2)(x_2 - 1)$

$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - x_1 - 2x_2 + 2 \Rightarrow -2x_1 - x_2 = -x_1 - 2x_2$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

Let $y \in B = R - \{1\}$ *i.e.*, $y \in R$ and $y \neq 1$

such that $f(x) = y$

$\Rightarrow \dfrac{x-1}{x-2} = y \Rightarrow (x-2)y = x - 1$

$\Rightarrow xy - 2y = x - 1 \Rightarrow x(y - 1) = 2y - 1 \Rightarrow x = \dfrac{2y-1}{y-1}$

$\therefore f(x) = y$ when $x = \dfrac{2y-1}{y-1} \in A$ (as $y \neq 1$)

Hence, f is onto.

Thus, f is one–one and onto.

34. Given A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, verify that AB = 6I, how can we use the result to find the values of x, y, z from given equations x – y = 3, 2x + 3y + 4z = 17, y + 2z = 17

Ans: We have A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$$\text{Now, } AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow AB = 61 \Rightarrow A^{-1} = \frac{1}{6}B$$

$$\Rightarrow A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

The given system of linear equations can be written in matrix form as AX = B, where

$$A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}3\\ 17\\ 7\end{bmatrix}$$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow X=\frac{1}{6}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}\begin{bmatrix}3\\ 17\\ 7\end{bmatrix}\Rightarrow X=\frac{1}{6}\begin{bmatrix}2\times3+2\times17-4\times7\\ -4\times3+2\times17-4\times7\\ 2\times3-1\times17+5\times7\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ -1\\ 4\end{bmatrix}\Rightarrow x=2, y=-1, z=4$$

35. Find the shortest distance between the lines $\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ and $\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$. If the lines intersect find their point of intersection.

Ans:

$\vec{r} = 3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k}) = (3+\lambda)\hat{i}+(2+2\lambda)\hat{j}+(-4+2\lambda)\hat{k}$

$\vec{r} = 5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k}) = (5+3\mu)\hat{i}+(-2+2\mu)\hat{j}+6\mu\hat{k}$

If given two lines are intersecting, then for some value of λ and μ

$3+\lambda = 5+3\mu$

$\lambda = 2+3\mu$ ...(*i*)

$2+2\lambda = -2+2\mu$ ...(*ii*)

$2+2(2+3\mu) = -2+2\mu$ **From (*i*)**

$2+4+6\mu = -2+2\mu$

$4\mu = -8$ $\therefore$ $\mu = -2$

$-4+2\lambda = 6\mu$ ...(*iii*)

Putting the value of μ in (*i*), $\lambda = 2+3(-2) = 2-6 = -4$

Putting the value of λ and μ in (*iii*), we get

$-4+2(-4) = 6(-2)$

$-4-8 = -12$

$-12 = -12$, which is true

Hence, the given two lines are intersecting.

For point of intersection:

1st method

$(3+\lambda, 2+2\lambda, -4+2\lambda)$ $(\because \lambda = -4)$

$(3-4, 2-8, -4-8)$

$(-1, -6, -12)$

2nd method

$(5+3\mu, -2+2\mu, 6\mu)$ $(\because \mu = -2)$

$(5-6, -2-4, -12)$

$(-1, -6, -12)$

OR

Find the vector equation of the line passing through (1, 2, – 4) and perpendicular to the two lines:

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

Ans:

Equation of any line through the point (1, 2, – 4) is

$$\frac{x-1}{a}=\frac{y-2}{b}=\frac{z+4}{c} \qquad ...(i)$$

where a, b and c are direction ratios of line (i).

Now the line (i) is perpendicular to the lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

having direction ratios 3, – 16, 7 and 3, 8, – 5 respectively.

$$\therefore \quad 3a-16b+7c=0 \qquad ...(ii)$$

$$3a+8b-5c=0 \qquad ...(iii)$$

Solving (ii) and (iii) by cross-multiplication method, we have

$$\frac{a}{80-56}=\frac{b}{21+15}=\frac{c}{24+48} \Rightarrow \frac{a}{24}=\frac{b}{36}=\frac{c}{72} \Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{6}$$

Let $\frac{a}{2}=\frac{b}{3}=\frac{c}{6}=\lambda \Rightarrow a=2\lambda, b=3\lambda$ and $c=6\lambda$

The equation of required line which passes through point (1, 2, –4) and parallel to vector $2\hat{i}+3\hat{j}+6\hat{k}$ is $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$.

SECTION – E(Case Study Based Questions)

Questions 35 to 37 carry 4 marks each.

36. Case-Study 1: Read the following passage and answer the questions given below.

In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

(i) Find the conditional probability that an error is committed in processing given that Sonia processed the form.

(ii) Find the probability that Sonia processed the form and committed an error.

(iii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is not processed by Vinay.

OR

If the form selected at random has an error, find the probability that the form is processed by Sonia

Ans: Let V : Vinay processes form; S : Sonia processes form; I : Iqbal processes form; E : Error rate

P(V) = 50% = 50/100 = 5/10 ; P(S) = 20% = 20/100 = 2/10 ; P(I) = 30% = 30/100 = 3/10;

P(E/V) = 0.06; P(E/S) = 0.04; P(E/I) = 0.03

(i) Required conditional probability = P(E/S) = 0.04

(ii) P(Sonia processed the form and committed an error) = P(S) + P(E/S) = 2/10 × 0.04 = 0.008
(iii) P(Form is processed by Vinay)

$$P(V/E) = \frac{P(V) \times P(E/V)}{[P(V) \times P(E/V)] + [P(S) \times P(E/S)] + [P(I) \times P(E/I)]}$$

$$= \frac{\frac{5}{10} \times 0.06}{\left(\frac{5}{10} \times 0.06\right) + \left(\frac{2}{10} \times 0.04\right) + \left(\frac{3}{10} \times 0.03\right)} = \frac{0.30}{0.30 + 0.08 + 0.09} = \frac{0.30}{0.47} = \frac{30}{47}$$

$\therefore$ **P(Form is not processed by Vinay) = 1 –** $\frac{30}{47}$ **=** $\frac{\mathbf{17}}{\mathbf{47}}$

OR

P(Form is processed by Sonia)

$$P(S/E) = \frac{P(S) \times P(E/S)}{[P(V) \times P(E/V)] + [P(S) \times P(E/S)] + [P(I) \times P(E/I)]}$$

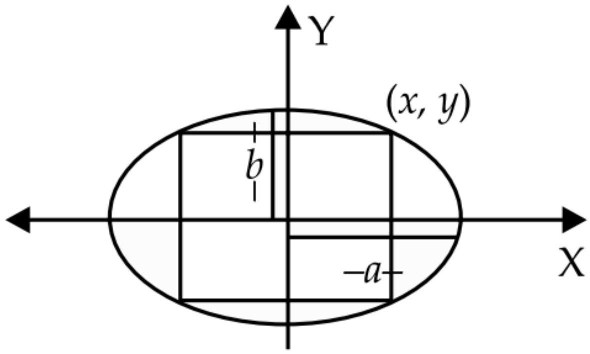
$$= \frac{\frac{2}{10} \times 0.04}{\left(\frac{5}{10} \times 0.06\right) + \left(\frac{2}{10} \times 0.04\right) + \left(\frac{3}{10} \times 0.03\right)} = \frac{0.08}{0.30 + 0.08 + 0.09} = \frac{0.08}{0.47} = \frac{8}{47}$$

37. Case-Study 2: Read the following passage and answer the questions given below.
In an elliptical sport field the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x.
(ii) Find the critical point of the function.
(iii) Use First derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

OR

(iii) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.
Ans:



Let $(x, y) = \left(x, \frac{b}{a}\sqrt{a^2 - x^2}\right)$ be the upper right vertex of the rectangle.

The area function $A = 2x \times 2\frac{b}{a}\sqrt{a^2 - x^2}$

$= \frac{4b}{a}x\sqrt{a^2 - x^2}, x \in (0, a).$

(ii)$\frac{dA}{dx} = \frac{4b}{a}\left[x \times \frac{-x}{\sqrt{a^2-x^2}} + \sqrt{a^2 - x^2}\right]$

$$= \frac{4b}{a} \times \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}} = -\frac{4b}{a} \times \frac{2\left(x + \frac{a}{\sqrt{2}}\right)(x - \frac{a}{\sqrt{2}})}{\sqrt{a^2 - x^2}}$$

$\frac{dA}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}.$

$x = \frac{a}{\sqrt{2}}$ is the critical point.

(iii)For the values of x less than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}, \frac{dA}{dx} > 0$

and for the values of x greater than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}, \frac{dA}{dx} < 0.$

Hence, by the first derivative test, there is a local maximum at the critical point $x = \frac{a}{\sqrt{2}}$. Since there is only one critical point, therefore, the area of the soccer field is maximum at this critical point $x = \frac{a}{\sqrt{2}}$

Thus, for maximum area of the soccer field, its length should be $a\sqrt{2}$ and its width should be $b\sqrt{2}$.

OR

(iii) $A = 2x \times 2\frac{b}{a}\sqrt{a^2 - x^2}, x \in (0, a).$

Squaring both sides, we get

$Z = A^2 = \frac{16b^2}{a^2}x^2(a^2 - x^2) = \frac{16b^2}{a^2}(x^2a^2 - x^4), x \in (0, a).$

A is maximum when Z is maximum.

$$\frac{dZ}{dx} = \frac{16b^2}{a^2}(2xa^2 - 4x^3) = \frac{32b^2}{a^2}x(a + \sqrt{2}x)(a - \sqrt{2}x)$$

$\frac{dZ}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{2}}.$

$$\frac{d^2Z}{dx^2} = \frac{32b^2}{a^2}(a^2 - 6x^2)$$

$$\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{a}{\sqrt{2}}} = \frac{32b^2}{a^2}(a^2 - 3a^2) = -64b^2 < 0$$

Hence, by the second derivative test, there is a local maximum value of Z at the critical point $x = \frac{a}{\sqrt{2}}$. Since there is only one critical point, therefore, Z is maximum at $x = \frac{a}{\sqrt{2}}$, hence, A is maximum at $x = \frac{a}{\sqrt{2}}$.

Thus, for maximum area of the soccer field, its length should be $a\sqrt{2}$ and its width should be $b\sqrt{2}$.

38. Case-Study 3:

One day Shweta's Mathematics teacher was explaining the topic Increasing and decreasing functions in the class. He explained about different terms like stationary points, turning points etc. He also explained about the conditions for which a function will be increasing or decreasing. He took examples of different functions to make it more clear to the students. He then took the function $f(x) = (x + 1)^3(x - 3)^3$ and ask the students to answer the following questions. With Shweta, you can also test your knowledge by answering the questions

(i) Find the stationary points on the curve. [2]

(ii) Find the intervals where the function is increasing and decreasing? [2]

Ans: (i) We have, $f(x) = (x + 1)^3(x - 3)^3$

$\Rightarrow f'(x) = 3(x+1)^2(x-3)^3 + 3(x-3)^2(x+1)^3 \Rightarrow f'(x) = 3(x+1)^2(x-3)^2[x-3+x+1]$

$\Rightarrow f'(x) = 3(x+1)^2(x-3)^2(2x-2) \Rightarrow f'(x) = 6(x+1)^2(x-3)^2(x-1)$

For stationary points, $f'(x) = 0 \Rightarrow x = -1, 3, 1$

(ii) The points x = −1, x = 1, and x = 3 divide the real line into four disjoint intervals i.e.,(−∞, −1), (−1, 1), (1, 3) and (3, ∞).

In intervals (−∞, −1) and (−1, 1), $f'(x) = 6(x + 1)2(x - 3)2(x - 1) < 0$

∴ f is strictly decreasing in intervals (−∞, −1) and (−1, 1).

In intervals (1, 3) and (3, ∞), $f'(x) = 6(x + 1)2(x - 3)2(x - 1) > 0$

∴ f is strictly increasing in intervals (1, 3) and (3, ∞).