SAMPLE PAPER TEST 03 FOR BOARD EXAM 2025

SUBJECT: MATHEMATICS (041) **MAX. MARKS : 80**
CLASS : XII **DURATION: 3 HRS**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. If $\frac{dy}{dx} = y\sin 2x$, y(0) = 1, then solution is

(a) y = $e^{\sin^2 x}$ (b) y = $\sin^2$x (c) y = $\cos^2$x (d) y = $e^{\cos^2 x}$

2. A is a skew-symmetric matrix and a matrix B such that B'AB is defined, then B'AB is a:

(a) symmetric matrix (b) skew-symmetric matrix
(c) Diagonal matrix (d) upper triangular symmetric

3. If $f'(x) = x^2 e^{x^3}$,then f(x) is

(a) $\frac{1}{3}e^{x^3} + C$ (b) $\frac{1}{3}e^{x^4} + C$ (c) $\frac{1}{2}e^{x^3} + C$ (d) $\frac{1}{2}e^{x^2} + C$

4. If $(\hat{i} + 3\hat{j} + 8\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = 0$, then λ and μ are respectively:

(a) 27, -9 (b) 9, 9 (c) -9, 18 (d) -1, 1

5. If m and n are the order and degree, respectively of the differential equation $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$, then write the value of m + n.

(a) 1 (b) 2 (c) 3 (d) 4

6. In an LPP, the objective function is always:

(a) linear (b) quadratic (c) cubic (d) biquadratic

7. The value of is $\int_1^2 \frac{dx}{x\sqrt{x^2 - 1}}$:

(a) π/3 (b) π/2 (c) π/4 (d) π/6

8. If A is a 3 x 3 matrix and |A| = - 2 then value of |A(adjA)| is

(a) -2 (b) 2 (c) -8 (d) 8

9. The value of the expression $|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2$ is

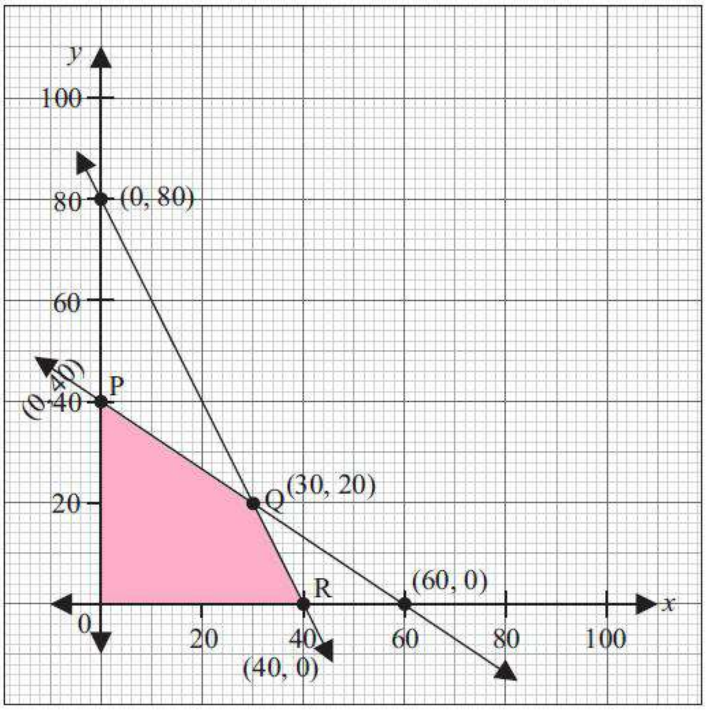
(a) $\vec{a}.\vec{b}$ (b) $|\vec{a}|.|\vec{b}|$ (c) $|\vec{a}|^2|\vec{b}|^2$ (d) none of these

10. For what value of k the function $f(x)=\begin{cases} 2x+1; & if\ x<2 \\ k, & x=2 \\ 3x-1; & x>2 \end{cases}$ is continuous at x = 2 ,

(a) Any real value (b) No real value (c) 5 (d) 1/5

11. For any matrix A = [a_{ij}], if c_{ij} denotes its cofactors then find the value of $a_{11}c_{12} + a_{12}c_{22} + a_{13}c_{32}$.
(a) 1 (b) -1 (c) 0 (d) none of these

12. For an L.P.P. the objective function is Z = 4x + 3y, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Which one of the following statements is true?
(a) Maximum value of Z is at R. (b) Maximum value of Z is at Q.
(c) Value of Z at R is less than the value at P. (d) Value of Z at Q is less than the value at R.

13. If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$, then the value of x is:
(a) 6 (b) 3 (c) 7 (d) 1

14. The projection of $\vec{a}$ on $\vec{b}$, if $\vec{a}.\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

(a) $\frac{8}{7}$ (b) $\frac{2}{3}$ (c) $\frac{2}{9}$ (d) $\frac{4}{5}$

15. If A and B are square matrices of order 3 such that |A| = 1 and |B| = 3, then the value of |3AB| is:
(a) 3 (b) 9 (c) 27 (d) 81

16. If y = $\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then value of dy/dx at x = π/6 is:

(a) 1/2 (b) −1/2 (c) 1 (d) -1

17. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is:

(a) parallel to x-axis (b) parallel to y-axis
(c) parallel to z-axis (d) perpendicular to z-axis

18. If P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6 then P(A U B) is equal to
(a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A) : The angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is 90°.
Reason (R) : Skew lines are lines in different planes which are parallel and intersecting.

20. Assertion (A): Domain of f(x) = $\sin^{-1}x$ + cosx is [-1, 1]
Reason (R): Domain of a function is the set of all possible values for which function will be defined.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. If $|\vec{a}\times\vec{b}|^2+(\vec{a}.\vec{b})^2=144$ and $|\vec{a}|=4$, then find the value of $|\vec{b}|$.

22. Show that the modulus function f : R → R given by f (x) = |x|, is neither one-one nor onto, where |x| is x, if x is positive or 0 and |x| is –x, if x is negative.

OR

Find the value of the $\tan\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right)$, $|x|<1$, $y>0$ and $xy<1$.

23. An edge of a variable cube is increasing at the rate of 5cm per second. How fast is the volume increasing when the side is 15 cm.

24. Given, $\vec{p}=3\hat{i}+2\hat{j}+4\hat{k},\vec{a}=\hat{i}+\hat{j},\vec{b}=\hat{j}+\hat{k},\vec{c}=\hat{i}+\hat{k}$ and $\vec{p}=x\vec{a}+y\vec{b}+z\vec{c}$, then find the value of x, y, z.

OR

Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

25. If y = log(cos e^x), then find $\frac{dy}{dx}$.

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random.
(a) Find the probability that the student reads neither Hindi nor English newspaper.
(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.
(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

OR

The random variable X has a probability distribution P(X) of the following form, where k is some number:

$$P(X)=\begin{cases} k, & if \quad x=0 \\ 2k, & if \quad x=1 \\ 3k, & if \quad x=2 \\ 0, & if \quad otherwise \end{cases}$$

(a) Determine the value of k.
(b) Find $P(X < 2)$, $P(X \le 2)$, $P(X \ge 2)$.

27. Find the particular solution of the differential equation $(1-y^2)(1+\log x)dx + 2xydy = 0$, given that y = 0 when x = 1

OR

Solve the differential equation $xdy - ydx = \sqrt{x^2+y^2}\,dx$

28. Find the value of $\int_1^2 \frac{dx}{x(1+\log x)^2}$.

OR

Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x . \operatorname{cosec} x} dx$

29. Evaluate: $\int \frac{(x^2-3x)}{(x-1)(x-2)} dx$

30. Evaluate: $\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx$

31. Solve the following Linear Programming Problem graphically:
Maximise Z = x + 2y subject to the constraints: $x + 2y \ge 100$; $2x - y < 0$; $2x + y \le 200$; $x, y \ge 0$

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

33. Find the vector equation of the line through the point (1, 2, –4) and perpendicular to the two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

OR

Find the shortest distance between the following lines :
$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + s(2\hat{i} + \hat{j} + \hat{k})$ *and* $\vec{r} = (\hat{i} + \hat{j} + 2\hat{k}) + t(4\hat{i} + 2\hat{j} + 2\hat{k})$

34. Show that the relation R on the set Z of all integers, given by R = {(a, b) : 2 divides (a – b)} is an equivalence relation.

OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Prove that the function $f : A \rightarrow B$ defined by *f(x)* = $\left(\frac{x-2}{x-3}\right)$ is *f* one-one and onto ? Justify your answer.

35. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations: $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$; $x + y - 2z = -3$.

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1: Read the following passage and answer the questions given below.

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6, 0 \le x \le 12$, m being a constant, where f(x) is the temperature in °F at x days.

(i) Is the function differentiable in the interval (0, 12)? Justify your answer.

(ii) If 6 is the critical point of the function, then find the value of the constant

(iii) Find the intervals in which the function is strictly increasing/strictly decreasing.

OR

(iii) Find the points of local maximum/local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum/absolute minimum in the interval [0, 12]. Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function.

37. Case-Study 2: An owner of a car rental company have determined that if they charge customers Rs x per day to rent a car, where $50 \le x \le 200$, then number of cars (n), they rent per day can be shown by linear function $n(x) = 1000 - 5x$. If they charge Rs. 50 per day or less they will rent all their cars. If they charge Rs. 200 or more per day they will not rent any car.

Based on the above information, answer the following question.

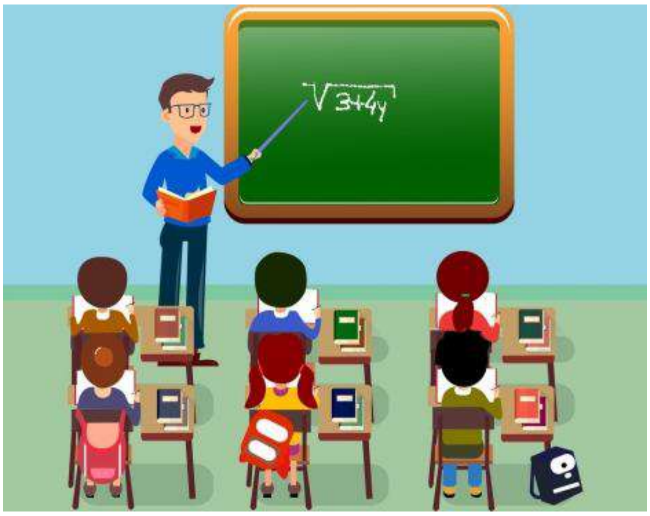
(i) If R(x) denote the revenue, then find the value of x at which R(x) has maximum value.

(ii) Find the Maximum revenue collected by company

OR

Find the number of cars rented per day, if x = 75.

38. Case Study 3: On one day, Maths teacher is conducted Mental Ability test. Anand, Sanjay and Aditya are trying to solve a given Mental ability problem in Mathematics whose respective probabilities of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. They were asked to solve it independently.



Based on the above data, answer any four of the following questions.

(i) Find the probability that Anand alone solves it.

(ii) Find the probability that exactly one of them solves it.

SAMPLE PAPER TEST 03 FOR BOARD EXAM 2025
(ANSWERS)

SUBJECT: MATHEMATICS (041) **MAX. MARKS : 80**
CLASS : XII **DURATION: 3 HRS**

General Instructions:
1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A
Questions 1 to 20 carry 1 mark each.

1. If $\frac{dy}{dx} = y\sin 2x$, y(0) = 1, then solution is

(a) y = $e^{\sin^2 x}$ (b) y = $\sin^2$x (c) y = $\cos^2$x (d) y = $e^{\cos^2 x}$

Ans:

We have $\frac{dy}{dx} = y\sin 2x$

$\Rightarrow \frac{dy}{y} = \sin 2x\, dx \Rightarrow \log y = -\frac{\cos 2x}{2} + c$

Since $y(0) = 1 \Rightarrow x = 0, y = 1 \Rightarrow c = 1/2$

Now, $\log y = \frac{1}{2}(1 - \cos 2x) \Rightarrow \log y = \sin^2 x \Rightarrow y = e^{\sin^2 x}$

2. A is a skew-symmetric matrix and a matrix B such that B'AB is defined, then B'AB is a:

(a) symmetric matrix (b) skew-symmetric matrix
(c) Diagonal matrix (d) upper triangular symmetric

Ans: (b) skew-symmetric matrix
A is a skew-symmetric matrix
⇒ A' = -A
Consider (B'AB)' = (AB)'(B')' = B'A'(B')'
= B'A'B = B'(-A)B = -B'AB
As (B'AB) = -B'AB
Hence, B'AB is a skew-symmetric matrix.

3. If $f'(x) = x^2 e^{x^3}$,then f(x) is

(a) $\frac{1}{3}e^{x^3} + C$ (b) $\frac{1}{3}e^{x^4} + C$ (c) $\frac{1}{2}e^{x^3} + C$ (d) $\frac{1}{2}e^{x^2} + C$

Ans: (a) $\frac{1}{3}e^{x^3} + C$

4. If $(\hat{i} + 3\hat{j} + 8\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = 0$, then λ and μ are respectively:

(a) 27, -9 (b) 9, 9 (c) -9, 18 (d) -1, 1

Ans: (a) 27, -9

Given, $(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k})=0$

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix} = \vec{0}$$

$\hat{i}(3\mu+9\lambda)-\hat{j}(\mu-27)+\hat{k}(-\lambda-9)=0\hat{i}+0\hat{j}+0\hat{k}$

On comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ we get,

$3\mu + 9\lambda = 0$, $-\mu + 27 = 0$ and $-\lambda - 9 = 0$

$\mu = 27$ and $-\lambda = 9$

or $\mu = 27$ and $\lambda = -9$

5. If m and n are the order and degree, respectively of the differential equation $5x\left(\frac{dy}{dx}\right)^2-\frac{d^2y}{dx^2}-6y=\log x$, then write the value of m + n.

(a) 1 (b) 2 (c) 3 (d) 4

Ans: (c) 3

Here, m = 2 and n = 1 then m + n = 2 + 1 = 3

6. In an LPP, the objective function is always:

(a) linear (b) quadratic (c) cubic (d) biquadratic

Ans: (a) linear

7. The value of is $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$:

(a) $\pi/3$ (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/6$

Ans: (a) $\pi/3$

Let $I = \int_1^2 \frac{dx}{x\sqrt{x^2-1}} = [\sec^{-1}x]_1^2 = \sec^{-1}2-\sec^{-1}1 = \frac{\pi}{3}-0=\frac{\pi}{3}$

8. If A is a 3 x 3 matrix and |A| = - 2 then value of |A(adjA)| is

(a) -2 (b) 2 (c) -8 (d) 8

Ans: (c) -8

9. The value of the expression $|\vec{a}\times\vec{b}|^2+|\vec{a}.\vec{b}|^2$ is

(a) $\vec{a}.\vec{b}$ (b) $|\vec{a}|.|\vec{b}|$ (c) $|\vec{a}|^2|\vec{b}|^2$ (d) none of these

Ans: (c) $|\vec{a}|^2|\vec{b}|^2$

10. For what value of k the function $f(x)=\begin{cases} 2x+1; & if\ x<2 \\ k, & x=2 \\ 3x-1; & x>2 \end{cases}$ is continuous at x = 2 ,

(a) Any real value (b) No real value (c) 5 (d) 1/5

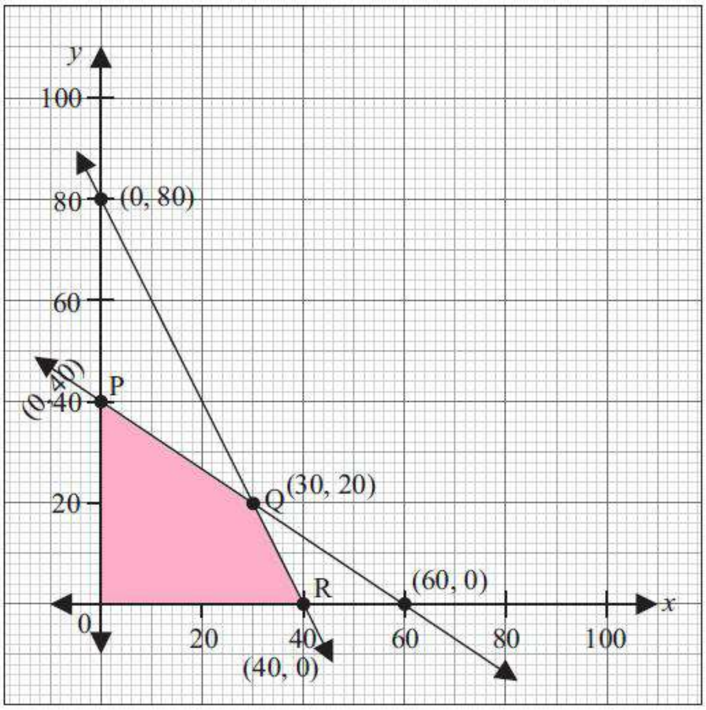
Ans: (c) 5

11. For any matrix A = [a_{ij}], if c_{ij} denotes its cofactors then find the value of $a_{11}c_{12} + a_{12}c_{22} + a_{13}c_{32}$.

(a) 1 (b) -1 (c) 0 (d) none of these

Ans: Zero

12. For an L.P.P. the objective function is Z = 4x + 3y, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Which one of the following statements is true?

(a) Maximum value of Z is at R.

(b) Maximum value of Z is at Q.

(c) Value of Z at R is less than the value at P.

(d) Value of Z at Q is less than the value at R.

Ans: (b) Maximum value of Z is at Q.

Z = 4x + 3y

at P (0, 40), Z = 4(0) + 3(40) = 120

at Q (30, 20), Z = 4(30) + 3(20) = 180

at R (40, 0), Z = 4(40) + 3(0) = 160

$\therefore$ Z_{max} = 180 at Q (30, 20)

13. If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$, then the value of x is:

(a) 6 (b) 3 (c) 7 (d) 1

Ans: (a) 6

$\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix} \Rightarrow (4 - 2)^2 = (3x - 2) - (x + 6)$

$\Rightarrow 4 = 3x - 2 - x - 6 \Rightarrow 2x = 12 \Rightarrow x = 6$

14. The projection of $\vec{a}$ on $\vec{b}$, if $\vec{a}.\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

(a) $\frac{8}{7}$ (b) $\frac{2}{3}$ (c) $\frac{2}{9}$ (d) $\frac{4}{5}$

Ans: (a) $\frac{8}{7}$

15. If A and B are square matrices of order 3 such that |A| = 1 and |B| = 3, then the value of |3AB| is:

(a) 3 (b) 9 (c) 27 (d) 81

Ans: (d) 81

As AB is of order 3 and

$|3AB| = 3^3|AB|$

$= 27|A||B| = 27 \times 1 \times 3 = 81$

16. If y = $\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then value of dy/dx at x = π/6 is:

(a) 1/2 (b) −1/2 (c) 1 (d) -1

Ans: (b) −1/2

$$\text{Given: } y=\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}=\tan^{-1}\sqrt{\frac{1-\cos\left(\frac{x}{2}-x\right)}{1+\cos\left(\frac{x}{2}-x\right)}}=\tan^{-1}\left|\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|=\frac{dy}{dx}=\frac{-1}{2}$$

17. The straight line $\frac{x-3}{3}=\frac{y-2}{1}=\frac{z-1}{0}$ is:

(a) parallel to x-axis (b) parallel to y-axis

(c) parallel to z-axis (d) perpendicular to z-axis

Ans: (c) parallel to z-axis

18. If P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6 then P(A U B) is equal to

(a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96

Ans: (d) 0.96

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}\Rightarrow 0.6=\frac{P(A\cap B)}{0.4}$$

$$\therefore\ P(A\cap B)=0.6\times 0.4=0.24$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=0.4+0.8-0.24=1.20-0.24=0.96$$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A) : The angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is 90°.

Reason (R) : Skew lines are lines in different planes which are parallel and intersecting.

Ans: (c) A is true but R is false.

Assertion is correct.

Give that $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$

Direction ratios of lines are

$$a_1=2,\ b_1=5,\ c_1=4 \text{ and } a_2=1,\ b_2=2,\ c_2=-3$$

As we know, the angle between the lines is given by

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right).\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$$

$$\Rightarrow\quad \cos\theta=\frac{2\times1+5\times2+4\times(-3)}{\left(\sqrt{2^2+5^2+4^2}\right).\left(\sqrt{1^2+2^2+(-3)^3}\right)}=0$$

$$\therefore\quad \theta=90°$$

Reason (R) is wrong.

20. Assertion (A): Domain of $f(x) = \sin^{-1}x + \cos x$ is [-1, 1]
Reason (R): Domain of a function is the set of all possible values for which function will be defined.
Ans: (a) Both A and R are true and R is the correct explanation of A.

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. If $|\vec{a}\times\vec{b}|^2 + (\vec{a}.\vec{b})^2 = 144$ and $|\vec{a}| = 4$, then find the value of $|\vec{b}|$.

We know that, $|\vec{a}\times\vec{b}| + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

$$\Rightarrow \ 144 = (4)^2 |\vec{b}|^2$$

$$\Rightarrow |\vec{b}|^2 = \frac{144}{16} = 9$$

$$\therefore \ |\vec{b}| = 3$$

22. Show that the modulus function f : R → R given by f (x) = |x|, is neither one-one nor onto, where |x| is x, if x is positive or 0 and |x| is –x, if x is negative.

Ans: $f(x) = |x| = \begin{cases} x, & if \quad x \geq 0 \\ -x, & if \quad x < 0 \end{cases}$

One-one: Let $x_1 = 1$, $x_2 = -1$ be two elements belongs to R
$f(x_1) = f(1) = |1|$ and $f(x_2) = f(-1) = -(-1) = 1$
$\Rightarrow f(x_1) = f(x_2)$ for $x_1 \neq x_2$
⇒ f(x) is not one-one.
Onto: Let $f(x) = -1 \Rightarrow |x| = -1 \in R$, which is not possible.
⇒ f(x) is not onto.
Hence, f is neither one-one nor onto function.

OR

Find the value of the $\tan\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right)$, $|x| < 1$, $y > 0$ and $xy < 1$.

Ans:

$$\tan\left\{\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right)\right\} = \tan\left\{\frac{1}{2}(2\tan^{-1}x + 2\tan^{-1}y)\right\}$$

$$= \tan\left\{\frac{1}{2}\times 2(\tan^{-1}x + \tan^{-1}y)\right\} \qquad \left[\because \ \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x \text{ and } \cos^{-1}\frac{1-y^2}{1+y^2} = 2\tan^{-1}y\right]$$

$$= \tan\left\{\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\} = \frac{x+y}{1-xy}.$$

23. An edge of a variable cube is increasing at the rate of 5cm per second. How fast is the volume increasing when the side is 15 cm.
Ans: Let x be the edge of the cube and V be the volume of the cube at any time t. Given,
$\frac{dx}{dt} = 5cm/s, \ \ x = 15cm$
Since we know the volume of cube $= (\,side\,)^3$ i.e., $V = x^3$.

$$\Rightarrow \frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

$$\frac{dV}{dt} = 3 \cdot (15)^2 \times 5 = 3375\ cm^3/sec$$

24. Given, $\vec{p} = 3\hat{i} + 2\hat{j} + 4\hat{k}, \vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{i} + \hat{k}$ and $\vec{p} = x\vec{a} + y\vec{b} + z\vec{c}$, then find the value of x, y, z.

Ans:

We have, $\vec{p} = x\vec{a} + y\vec{b} + z\vec{c}$

$\Rightarrow\ 3\hat{i} + 2\hat{j} + 4\hat{k} = x(\hat{i} + \hat{j}) + y(\hat{j} + \hat{k}) + z(\hat{i} + \hat{k})$

$\Rightarrow\ 3\hat{i} + 2\hat{j} + 4\hat{k} = (x+z)\hat{i} + (x+y)\hat{j} + (y+z)\hat{k}$

$\Rightarrow\ x + z = 3$ ...(i), $x + y = 2$

and $y + z = 4$ ...(iii)

On solving (i), (ii) and (iii), we get $x = \frac{1}{2}, y = \frac{3}{2}, z = \frac{5}{2}$

OR

Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

Ans:

Given, ΔABC with vertices $A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$

Now, $\overrightarrow{AB}(2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$,

and $\overrightarrow{AC}(1-1)\hat{i} + (5-1)\hat{j} + (5-2)\hat{k} = 4\hat{j} + 3\hat{k}$.

$$\therefore\ (\overrightarrow{AB} \times \overrightarrow{AC}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

Hence, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{(-6)^2 + (-3)^2 + 4^2} = \frac{1}{2}\sqrt{61}$ sq. units

25. If y = log(cos e^x), then find $\frac{dy}{dx}$.

Ans:

$y = \log(\cos e^x)$

$$\Rightarrow\ \frac{dy}{dx} = \frac{d}{dx}(\log(\cos e^x)) = \frac{1}{\cos(e^x)} \times (-\sin(e^x)) \times e^x = \frac{-e^x \sin(e^x)}{\cos(e^x)} = -e^x \tan(e^x)$$

$$\therefore\ \frac{dy}{dx} = -e^x \tan(e^x)$$

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random.

(a) Find the probability that the student reads neither Hindi nor English newspaper.

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Ans: Let A be the event that a student reads Hindi newspaper and B be the event that a student reads English newspaper.

P(A) = 60/100 = 0.6, P(B)= 40/100 = 0.4 and P(A ∩ B) = 20/100 = 0.2

(a) Now P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

= 0.6 + 0.4 – 0.2 = 0.8

Probability that she reads neither Hindi nor English newspaper

= 1 – P(A ∪ B) = 1– 0.8 = 0.2 = 1/5

(b) $P(B/A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{0.2}{0.6}=\dfrac{1}{3}$

(c) $P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.2}{0.4}=\dfrac{1}{2}$

OR

The random variable X has a probability distribution P(X) of the following form, where k is some number:

$$P(X)=\begin{cases} k, & if \quad x=0 \\ 2k, & if \quad x=1 \\ 3k, & if \quad x=2 \\ 0, & if \quad otherwise \end{cases}$$

(a) Determine the value of k.

(b) Find $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$.

Ans: (a) k + 2k + 3k =1 $\quad [\because p_1 + p_2 + p_3 + ... + p_n =1]$

$\Rightarrow$ 6k = 1 $\Rightarrow$ k = 1/6

(b) P (X < 2) = k + 2k = 3k = $3\times\dfrac{1}{6}=\dfrac{1}{2}$

P (X $\leq$ 2) = 3k + 2k + k = 6k = $6\times\dfrac{1}{6}=1$

P (X $\geq$ 2) = 3k = $3\times\dfrac{1}{6}=\dfrac{1}{2}$

27. Find the particular solution of the differential equation $(1-y^2)(1+\log x)dx+2xydy=0$, given that y = 0 when x = 1

Ans: $(1-y^2)(1+\log x)dx+2xydy=0 \Rightarrow \dfrac{(1+\log x)dx}{x}=\dfrac{2ydy}{(1-y^2)}$

Integrating we get, $\displaystyle\int\frac{(1+\log x)dx}{x}=\int\frac{2ydy}{(1-y^2)}$

Let logx = t, so $\dfrac{dx}{x}=dt$

Also, let y^2 = s, so 2ydy = ds

$\Rightarrow \displaystyle\int(1+t)dt=\int\frac{ds}{1-s} \Rightarrow t+\frac{t^2}{2}=-\log(1-s)+c$

When y = 0, x = 1; we get 0 + 0 = 0 + c $\Rightarrow$ c = 0

$\Rightarrow \log x+\dfrac{(\log x)^2}{2}=-\log(1-y^2)$

OR

Solve the differential equation $xdy-ydx=\sqrt{x^2+y^2}\,dx$

Ans: $xdy-ydx=\sqrt{x^2+y^2}\,dx$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{\sqrt{x^2+y^2}+y}{x}, x\neq 0$

It is a homogeneous differential Equation

Put y = vx $\Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

$\Rightarrow \dfrac{dv}{\sqrt{1+v^2}}=\dfrac{dx}{x}$

Integrated on both sides, we get

$$\log|v+\sqrt{1+v^2}|=\log|x|+\log C$$

$$\Rightarrow\left|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right|=|Cx|\Rightarrow(y+\sqrt{x^2+y^2})^2=C^2x^2$$

28. Find the value of $\int_1^2\frac{dx}{x(1+\log x)^2}$.

Ans:

Let $I=\int_1^2\frac{dx}{x(1+\log x)^2}$

Put $1+\log x=t\ \Rightarrow\ \frac{dx}{x}=dt$

When $x=1$, $t=1$ and when $x=2$, $t=1+\log 2$

$$\therefore\quad I=\int_1^{1+\log 2}\frac{dt}{t^2}=\left[\frac{-1}{t}\right]_1^{1+\log 2}=-\left[\frac{1}{1+\log 2}-1\right]=-\left[\frac{1-1-\log 2}{1+\log 2}\right]=\frac{\log 2}{1+\log 2}$$

OR

Evaluate: $\int_0^{\pi}\frac{x\tan x}{\sec x.\cos ecx}dx$

Ans:

$$\text{Let } I=\int_0^{\pi}\frac{x\tan x}{\sec x.\operatorname{cosec}x}dx=\int_0^{\pi}\frac{x.\frac{\sin x}{\cos x}}{\frac{1}{\cos x}.\frac{1}{\sin x}}dx$$

$$I=\int_0^{\pi}x\sin^2 x dx=\int_0^{\pi}(\pi-x)\sin^2(\pi-x)dx\qquad\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$I=\int_0^{\pi}\pi\sin^2 x dx-\int_0^{\pi}x\sin^2 x\,dx\ \Rightarrow\ 2I=\frac{\pi}{2}\int_0^{\pi}2\sin^2 x\,dx$$

$$=\frac{\pi}{2}\int_0^{\pi}(1-\cos 2x)dx=\frac{\pi}{2}\int_0^{\pi}dx-\frac{\pi}{2}\int_0^{\pi}\cos 2x dx=\frac{\pi}{2}[x]_0^{\pi}-\frac{\pi}{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi}$$

$$\Rightarrow\ 2I=\frac{\pi}{2}(\pi-0)-\frac{\pi}{4}(\sin 2\pi-\sin 0)\ \Rightarrow\ 2I=\frac{\pi^2}{2}-0\ \Rightarrow\ I=\frac{\pi^2}{4}$$

29. Evaluate: $\int\frac{(x^2-3x)}{(x-1)(x-2)}dx$

Ans:

$$\text{Let } I=\int\frac{(x^2-3x)}{(x-1)(x-2)}dx=\int\frac{(x^2-3x)}{x^2-3x+2}dx$$

$$=\int\frac{x^2-3x+2-2}{x^2-3x+2}dx=\int dx-\int\frac{2dx}{x^2-3x+2}$$

$$=x-2\int\frac{dx}{x^2-2.x.\frac{3}{2}+\frac{9}{4}-\frac{9}{4}+2}=x-2\int\frac{dx}{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}$$

$$=x-2\log\left|\frac{x-\frac{3}{2}-\frac{1}{2}}{x-\frac{3}{2}+\frac{1}{2}}\right|+C\qquad\left[\because\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|\right]$$

$$= x - 2\log\left|\frac{x-2}{x-1}\right| + C$$

30. Evaluate: $\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx$

Ans:

Put $x^2 = y$ to make partial fractions

$$\frac{x^2+1}{(x^2+2)(x^2+3)} = \frac{y+1}{(y+2)(y+3)}$$

Let $\frac{y+1}{(y+2)(y+3)} = \frac{A}{y+2} + \frac{B}{y+3}$

$\Rightarrow \quad y+1 = A(y+3) + B(y+2)$...(i)

Comparing coefficients of y and constant terms on both sides of (1) we get

$$A + B = 1 \quad \text{...(ii)}$$

and $$3A + 2B = 1 \quad \text{...(iii)}$$

Solving equation (ii) and (iii), we get $A = -1, B = 2$

$$\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx = \int \frac{-1}{x^2+2} dx + 2\int \frac{1}{x^2+3} dx$$

$$= -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

31. Solve the following Linear Programming Problem graphically:

Maximise Z = x + 2y subject to the constraints: $x + 2y \geq 100$; $2x - y < 0$; $2x + y \leq 200$; $x, y \geq 0$

Ans: Maximise Z = x + 2y

Subject to constraints : $x + 2y \geq 100$, $2x - y < 0$, $2x + y \leq 200$ and $x, y \geq 0$.

Converting the inequations into equations, we obtain the lines

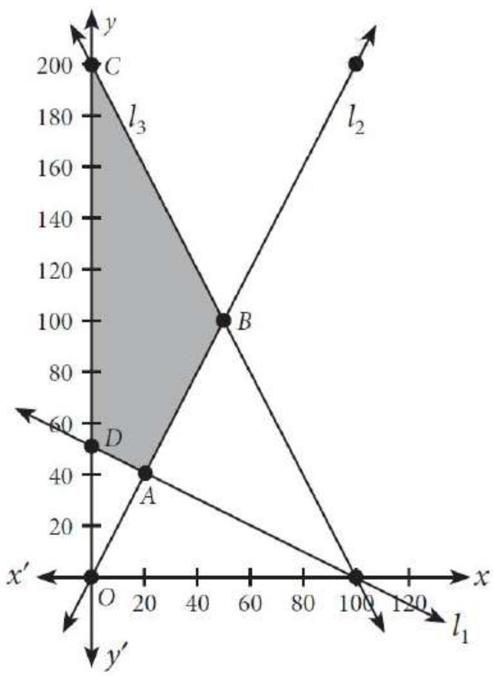
$l_1 : x + 2y = 100$ ...(i)

$l_2 : 2x - y = 0$ ...(ii)

$l_3 : 2x + y = 200$ ...(iii)

$l_4 : x = 0$ ...(iv)

and $l_5 : y = 0$ ...(v)



By intercept form, we get $\frac{x}{100}+\frac{y}{50}=1$

⇒ The line l_1 meets the coordinate axes at (100, 0) and (0, 50).

l_2 : 2x = y ⇒ The line l_2 passes through origin and (50, 100), (100, 200)

$l_3 : \frac{x}{100}+\frac{y}{200}=1$ ⇒ The line l_3 meets the coordinates axes at (100, 0) and (0, 200).

l_4 : x = 0 is the y-axis, l_5 : y = 0 is the x-axis

Now, plotting the above points on the graph, we get the feasible region of the LPP as shaded region ABCD. The coordinates of the corner points of the feasible region ABCD are A(20, 40), B(50, 100), C(0, 200), D(0, 50).

Now, $Z_A = 20 + 2 \times 40 = 100$

$Z_B = 50 + 2 \times 100 = 250$,

$Z_C = 0 + 2 \times 200 = 400$

$Z_D = 0 + 2 \times 50 = 100$

Hence, Z is maximum at C(0, 200) and having value 400.

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

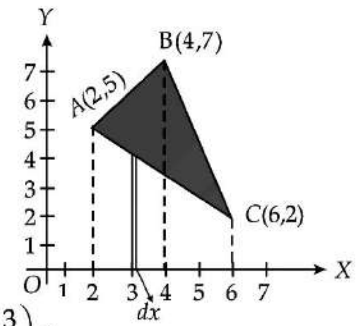
Ans:

From the given points,

Equation of AB : $y = x+3$

Equation of BC : $y = \frac{-5x}{2}+17$

Equation of AC : $y = \frac{-3x}{4}+\frac{13}{2}$



Required Area of ΔABC

$$= \int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2}+17\right)dx - \int_2^6 \left(\frac{-3x}{4}+\frac{13}{2}\right)dx$$

$$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{4}+17x\right]_4^6 - \left[\frac{-3x^2}{8}+\frac{13x}{2}\right]_2^6$$

$$= \left(\frac{49}{2}-\frac{25}{2}\right)+(57-48)-\left(\frac{51}{2}-\frac{23}{2}\right) = 12+9-14 = 21-14 = 7 \text{ sq. units}$$

33. Find the vector equation of the line through the point (1, 2, –4) and perpendicular to the two lines $\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and $\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

Ans:

The given lines are

$\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and $\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

Equation of any line through (1, 2, –4) with d.r's

l, m, n is $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+t(l\hat{i}+m\hat{j}+n\hat{k})$ ...(i)

Since, the required line is perpendicular to both the given lines.

∴ $3l - 16m + 7n = 0$ and $3l + 8m - 5n = 0$

$$\Rightarrow \frac{l}{80-56}=\frac{m}{21+15}=\frac{n}{24+48} \Rightarrow \frac{l}{2}=\frac{m}{3}=\frac{n}{6}$$

$\therefore$ From (i), the required line is $\vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + t(2\hat{i}+3\hat{j}+6\hat{k})$

Here, the position vector of passing point is $\vec{a} = \hat{i}+2\hat{j}-4\hat{k}$ and parallel vector is $\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k}$

OR

Find the shortest distance between the following lines :

$\vec{r} = (\hat{i}+\hat{j}-\hat{k}) + s(2\hat{i}+\hat{j}+\hat{k})$

$\vec{r} = (\hat{i}+\hat{j}+2\hat{k}) + t(4\hat{i}+2\hat{j}+2\hat{k})$

Ans:

Let $\vec{a}_1 = \hat{i}+\hat{j}-\hat{k}, \vec{a}_2 = \hat{i}+\hat{j}+2\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}+\hat{k}$

Here, the lines are parallel.

$\therefore$ Shortest distance between lines $= \dfrac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|} = \dfrac{|(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k})|}{\sqrt{4+1+1}}$

Now, $(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{i}+6\hat{j}$

Hence, the required shortest distance $= \dfrac{3\sqrt{5}}{\sqrt{6}}$ units .

34. Show that the relation R on the set Z of all integers, given by R = {(a, b) : 2 divides (a – b)} is an equivalence relation.

Ans: Given relation R = {(a, b) : 2 divides (a – b)} on the set Z of all integers

Reflexive: Let a ∈ Z

Since (a – a) = 0, which is divisible by 2 i.e., (a, a) ∈ R

∴ R is reflexive.

Symmetric: Let a, b ∈ Z

such that (a, b) ∈ R ⇒ (a – b) is divisible by 2

⇒ – (a – b) is also divisible by 2

⇒ (b – a) is divisible by 2 ⇒ (b, a) ∈ R

i.e., (a, b) ∈ R ⇒ (b, a) ∈ R

∴ R is symmetric.

Transitive: Let a, b, c ∈ Z

such that (a, b) ∈ R ⇒ (a – b) is divisible by 2

Let a – b = $2k_1$ where k_1 is an integer ...(i)

and (b, c) ∈ R ⇒ (b – c) is divisible by 2 ⇒ b – c = 2 k_2 where k_2 is an integer ...(ii)

Adding (i) and (ii), we have

(a – b) + (b – c) = 2 (k_1 + k_2) ⇒ a – c = 2 (k_1 + k_2) ⇒ (a – c) is divisible by 2.

OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Prove that the function $f : A \to B$ defined by $f(x) = \left(\dfrac{x-2}{x-3}\right)$ is f one-one and onto ? Justify your answer.

Ans: Here, $A = R - \{3\}$, $B = R - \{1\}$ and $f : A \to B$ is defined as $f(x) = \left(\dfrac{x-2}{x-3}\right)$

Let $x, y \in A$ such that $f(x) = f(y)$

$\Rightarrow \dfrac{x-2}{x-3} = \dfrac{y-2}{y-3} \Rightarrow (x-2)(y-3) = (y-2)(x-3)$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow -3x - 2y = -3y - 2x$

$\Rightarrow 3x-2x=3y-2y \Rightarrow x=y$

Therefore, f is one- one. Let $y \in B = R-\{1\}$. Then, $y \neq 1$

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now, $f(x) = y$

$\Rightarrow \dfrac{x-2}{x-3} = y \Rightarrow x-2 = xy-3y$

$\Rightarrow x(1-y) = -3y+2$

$\Rightarrow x = \dfrac{2-3y}{1-y} \in A \;\; [y \neq 1]$

Thus, for any $y \in B$, there exists $\dfrac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$

Therefore, f is onto. Hence, function f is one-one and onto.

35. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations: 2x – 3y + 5z = 11, 3x + 2y – 4z = –5; x + y – 2z = –3.

Ans:

$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2(-4+4) + 3(-6+4) + 5(3-2) = 0 - 6 + 5 = -1 \neq 0.$

$\therefore$ A is non-singular matrix and A^{-1} exists.

$C_{11} = 0$; $C_{12} = 2$; $C_{13} = 1$; $C_{21} = -1$; $C_{22} = -9$; $C_{23} = -5$; $C_{31} = 2$; $C_{32} = 23$; $C_{33} = 13$.

So, $adj A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{adj(A)}{|A|} = \dfrac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

The given system of equations can be expressed as AX = B, where

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Now AX = B $\Rightarrow$ X = A^{-1}B

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow x = 1, y = 2, z = 3$

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. Case-Study 1: Read the following passage and answer the questions given below.

The temperature of a person during an intestinal illness is given by
$f(x) = -0.1x^2 + mx + 98.6, 0 \leq x \leq 12$, m being a constant, where f(x) is the temperature in °F at x days.
(i) Is the function differentiable in the interval (0, 12)? Justify your answer.
(ii) If 6 is the critical point of the function, then find the value of the constant
(iii) Find the intervals in which the function is strictly increasing/strictly decreasing.

OR

(iii) Find the points of local maximum/local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum/absolute minimum in the interval [0, 12]. Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function.

Ans: (i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12)
(ii)$f'(x) = -0.2x + m$
Since, 6 is the critical point,
$f'(6) = 0 \Rightarrow m = 1.2$
(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$
$f'(x) = -0.2x + 1.2 = -0.2(x - 6)$

In the Interval	f'(x)	Conclusion
(0, 6)	+ve	f is strictly increasing in [0, 6]
(6, 12)	-ve	f is strictly decreasing in [6, 12]

OR

(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$,
$f'(x) = -0.2x + 1.2$, $f'(6) = 0$,
$f''(x) = -0.2$
$f''(6) = -0.2 < 0$
Hence, by second derivative test 6 is a point of local maximum. The local maximum value = $f(6) = -0.1 \times 6^2 + 1.2 \times 6 + 98.6 = 102.2$
We have $f(0) = 98.6$, $f(6) = 102.2$, $f(12) = 98.6$
6 is the point of absolute maximum and the absolute maximum value of the function = 102.2.
0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.

37. Case-Study 3: An owner of a car rental company have determined that if they charge customers Rs x per day to rent a car, where $50 \leq x \leq 200$, then number of cars (n), they rent per day can be shown by linear function $n(x) = 1000 - 5x$. If they charge Rs. 50 per day or less they will rent all their cars. If they charge Rs. 200 or more per day they will not rent any car.

Based on the above information, answer the following question.
(i) If R(x) denote the revenue, then find the value of x at which R(x) has maximum value.
(ii) Find the Maximum revenue collected by company

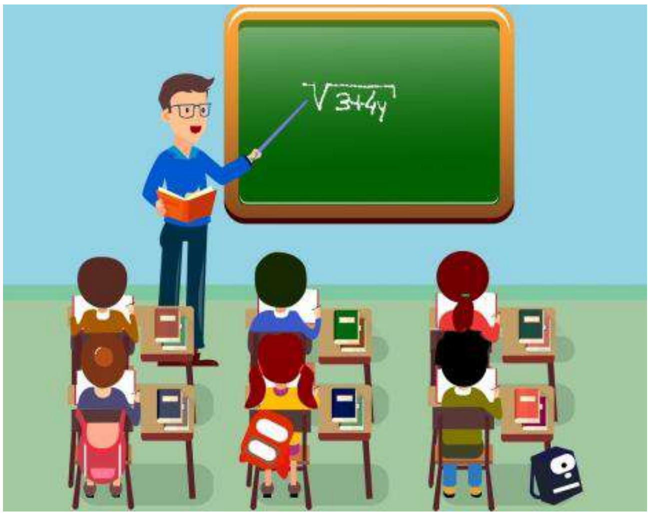
OR

Find the number of cars rented per day, if x = 75.
Ans: (i) Let x be the price charge per car per day and n be the number of cars rented per day.
$R(x) = n \times x = (1000 - 5x)\, x = -5x^2 + 1000x$
$\Rightarrow R'(x) = 1000 - 10x$
For R(x) to be maximum or minimum, $R'(x) = 0$
$\Rightarrow -10x + 1000 = 0 \Rightarrow x = 100$
Also, $R''(x) = -10 < 0$
Thus, R(x) is maximum at x = 100
(ii) At x = 100, R(x) is maximum.
Maximum revenue = R(100) = –5(100)2 + 1000(100) = Rs. 50,000

OR

If x = 75, number of cars rented per day is given by
$n = 1000 - 5 \times 75 = 625$

38. Case Study 1 : On one day, Maths teacher is conducted Mental Ability test. Anand, Sanjay and Aditya are trying to solve a given Mental ability problem in Mathematics whose respective probabilities of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. They were asked to solve it independently.



Based on the above data, answer any four of the following questions.

(i) Find the probability that Anand alone solves it.
(ii) Find the probability that exactly one of them solves it.
Ans: (i) Let A → event that Anand solves

B → event that Sanjay solves
C → event that Aditya solves

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ \ P(C) = \frac{1}{4}$$

$$\therefore\ P(A') = \frac{1}{2},\ \ P(B') = \frac{2}{3},\ \ P(C') = \frac{3}{4}$$

$$P(A \cap B' \cap C') = P(A)\ P(B')\ P(C') = \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = \frac{1}{4}$$

(ii) $P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$$= P(A)\ P(B')\ P(C') + P(A')\ P(B)\ P(C') + P(A')\ P(B')\ P(C)$$

$$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}+\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4} = \frac{11}{24}$$